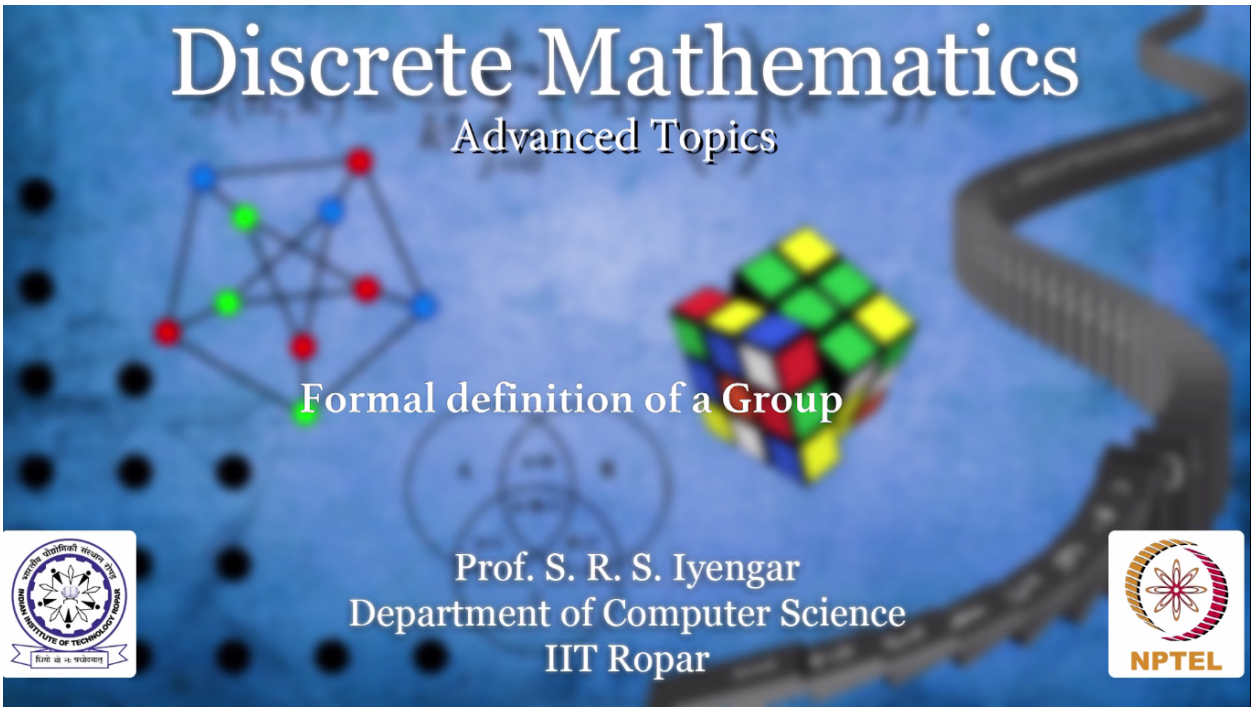
Discrete Mathematics

Functions

Advanced Topics

Formal definition of a Group



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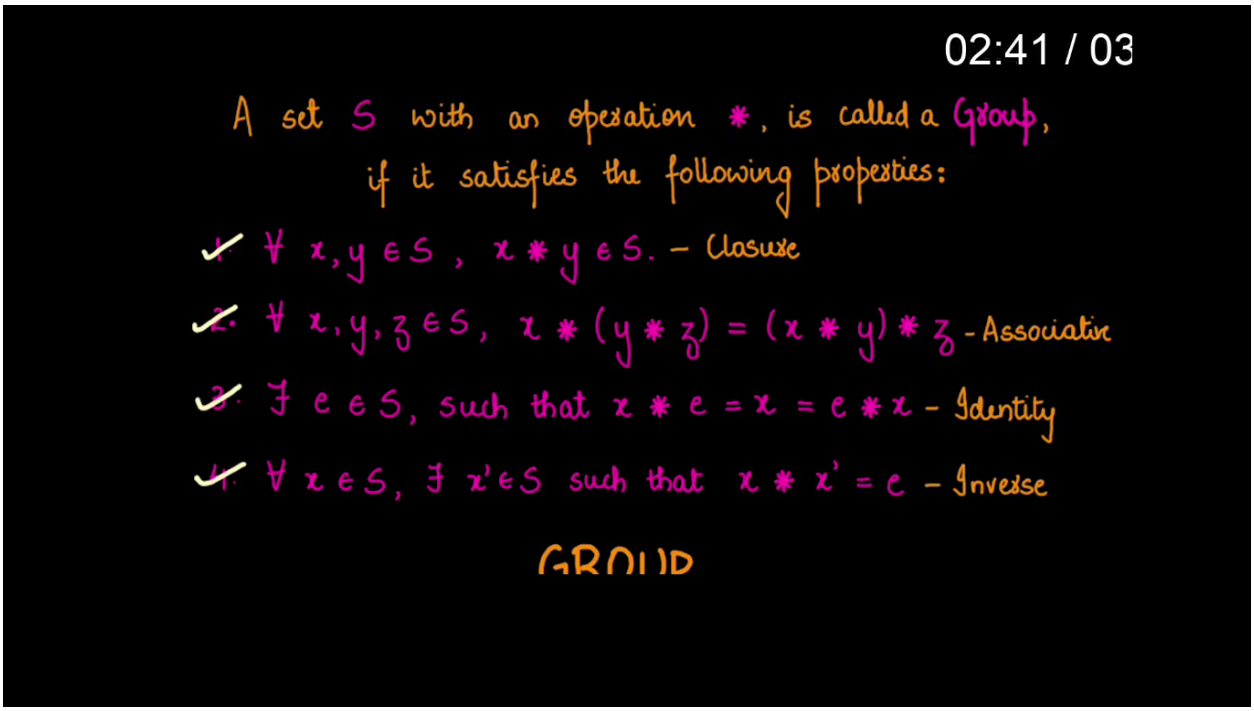
You are now motivated to see an abstract set like that of my lab. Let us define this formulae. A set S with an operation star is called a group if it satisfies the following four properties. The first property is for every element X and Y in S X * Y shoud also belong to S.

Do you think it's the same as what I said in the first property of my lab?

Second property is for every X, Y, Z in S X * of Y * Z should be equal to X * Y * Z as you would have observed the * here means mix the chemicals X and Y or add two numbes or multiply two numbers. It could be anything you see. Anything absolutely.

Third property is there exists an element E in S such that for every element X in S X * E is same as X. We also actually write. E * X is also X. X * E and E * X is also X. this may not be true. When you multiply A with B it may not always be equal to multiplying B with A. You see when you take matrices you know this. When you multiply A and B you might get something. When you multiply B and A you may get something else. In fact, sometimes A into B of two matrices A and B is possible B into A may not even be possible. Think about it. There are many such examples.



Okay. Fourth property what was it? Let us try to translate it. For every chemical there is a another chemical X – so that X one mixed with X – gives me the universal chemical E. So that translated look something like this. For every X in S there exists an X – in S such that X * X – is my E from the previous property. If S is such that with the * operation if S satisfies these four properties, one, two, three and four, what are they? Closure, associative, identity and inverse then S is called a group, a mathematical group. I prefixed the word mathematical just to make you realize that it's not the same group from the English dictionary. This is called a group in mathematics. Okay.

Let us see some nice examples. Some obvious examples. And not so obvious examples.