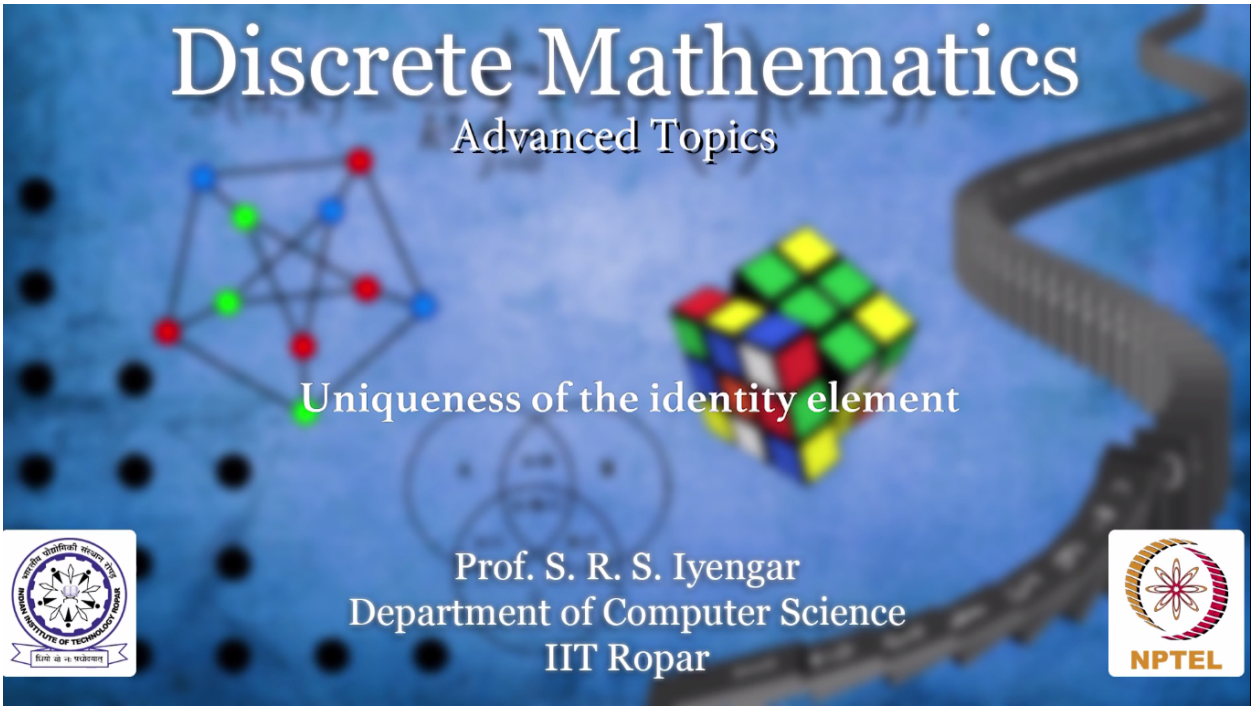
Discrete Mathematics

Functions

Advanced Topics



Uniqueness of the identity element

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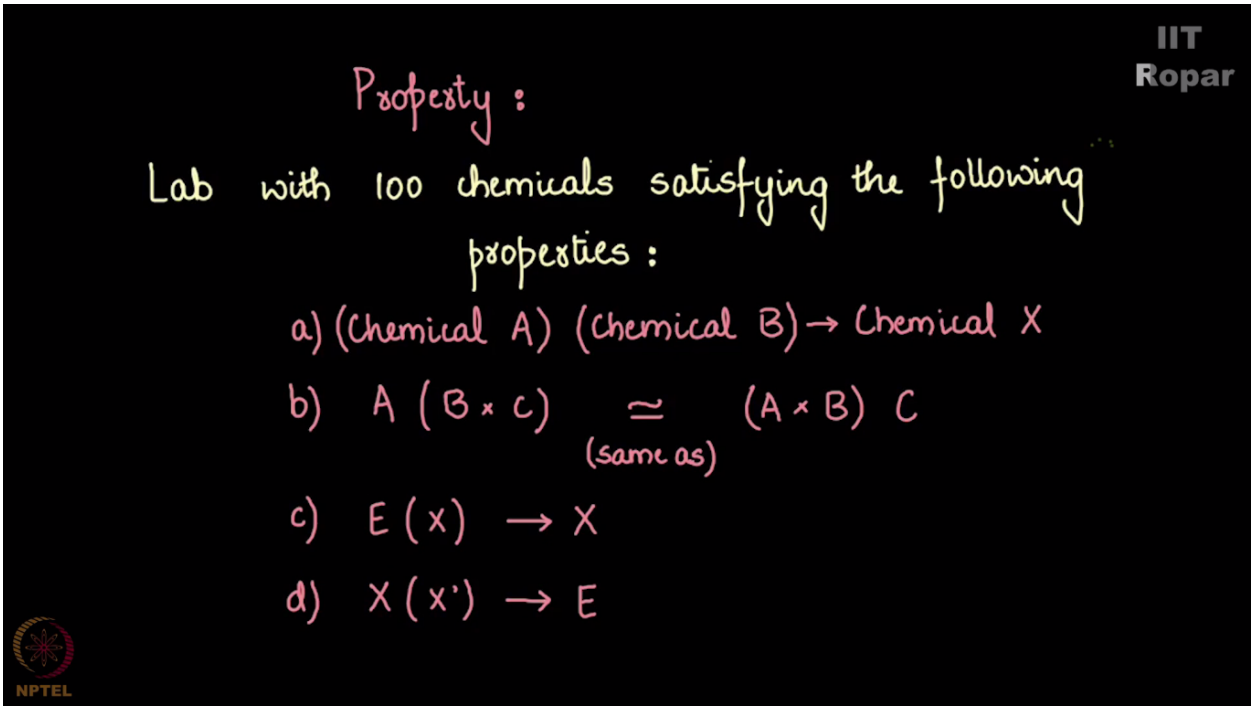
Well now let us look at the nice property I was talking. Something that is very counter-intuitive. Now I told you I had this lab with 100 chemicals satisfying these four properties mixing two chemicals gives back some chemical in my labrotory. The order in which you mix doesn't matter. A mixed with B and C is same as AB mixed with C. it is not actually order. This is the sequence in which you mix. The third one was there is this universal chemical which when mixed with chemical always gives you the same chemical. E times X is same as X and finally we told you given any chemical X there is always another chemical X – in my labortory which when mixed gives you back this E.



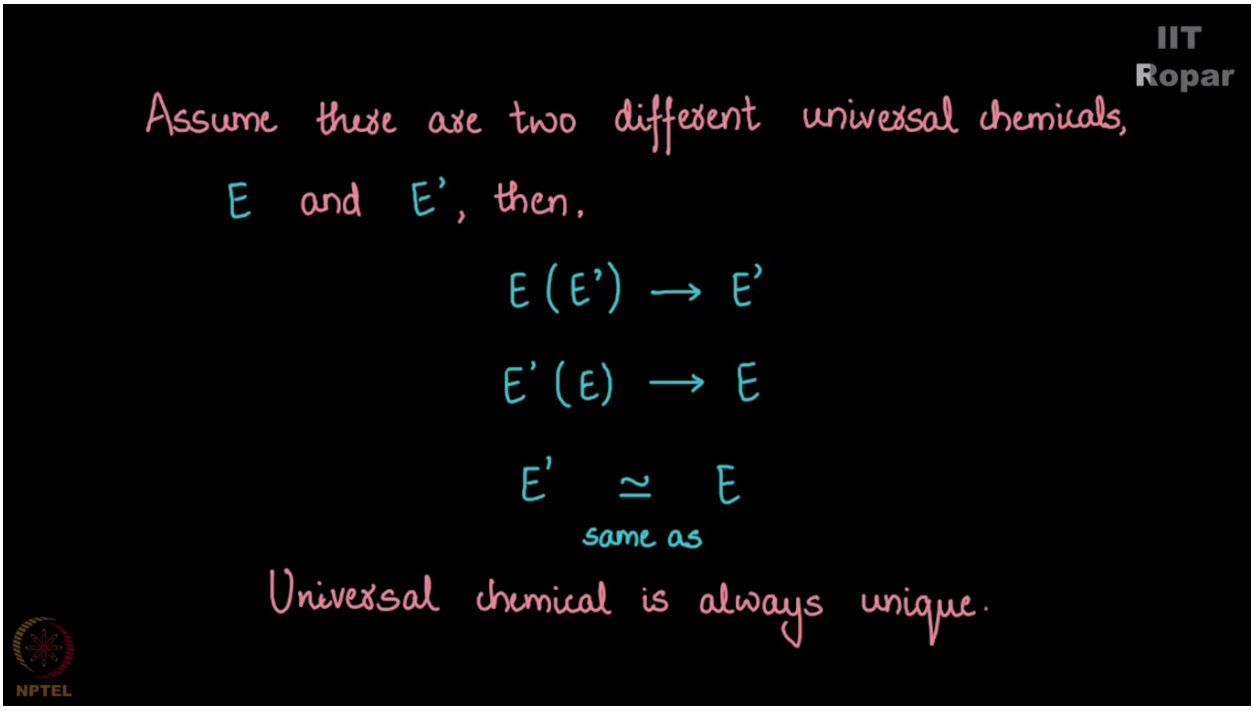
Now here is a cute result. A cute observation. In such a laboratory this chemical E is going to be unique. What do I mean by that?

By that I mean there cannot be two such chemicals E and E – with the same property that when you mix this with the chemical you get back the same chemical. What do I mean by that? By that I mean you cannot have a E which when mixed with X gives you X always for all X, for all

possible chemicals X and E mixed with any other chemical and there is another chemical E – with the same property. By that I mean when E – is mixed with X where X is any chemical you get back the same chemical. This is impossibly I think. Why? Let's see.

If there were two different universal chemicals E and E – then let us mix E with E – that's the climax. You mix E with E – now keep E – closed you are mixing E with something. You see E – is another chemical. You should get back whatever this something is. so you should get back E -. But then let us look at it the other way, E – is getting mixed with something. So E – also has its property because it's a universal chemical. So E – mixed with anything should give me back that anything. So it should also – I can also conclude that it should be E. So E into E mixed with E – gives me E – also gives me E which means my E and E – should be the same. Do you see the point here?

My laboratory if it satisfies these four properties you need not worry about the uniqueness of the universal chemical. It will always be unique. Take up any entity which satisfies these four properties. This particular third property whatever it says there is a universal entity, universal element. It's always unique.



Now you see the point. It's important to study such structures with these four properties because if it satisfies these four properties it satisfied optim number of properties.