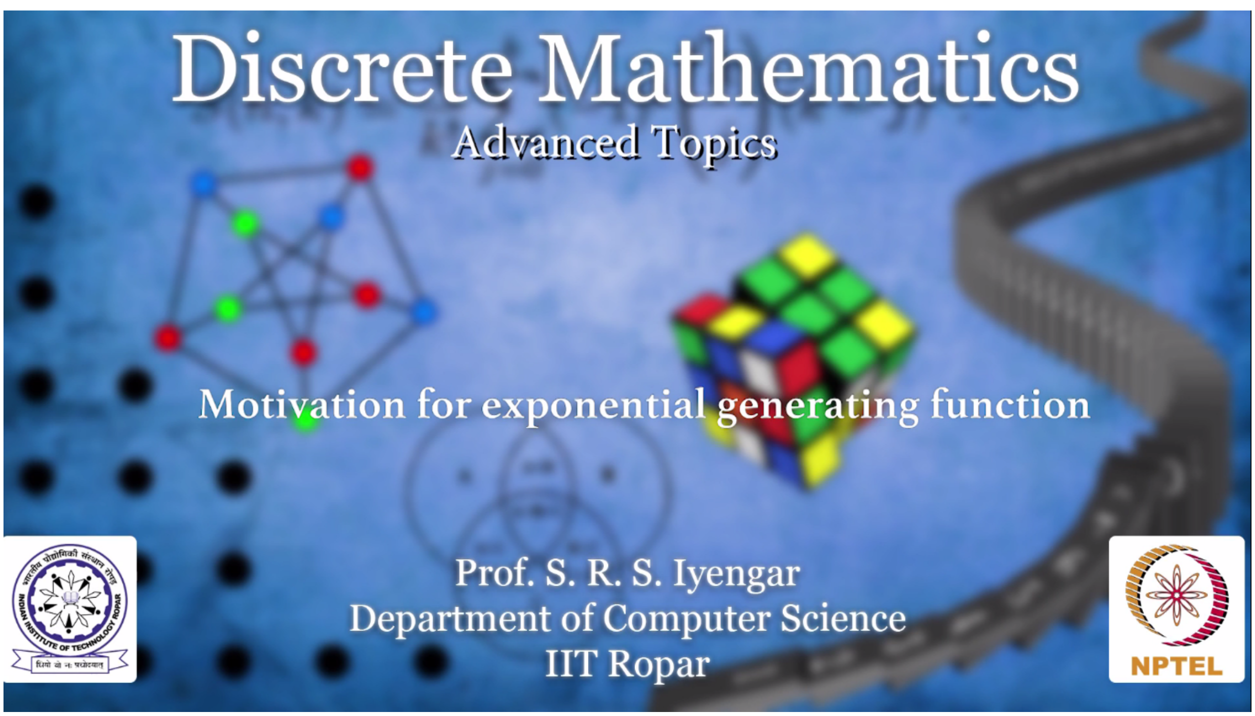
Discrete Mathematics

Functions

Advanced Topics



Motivation for exponential generating function

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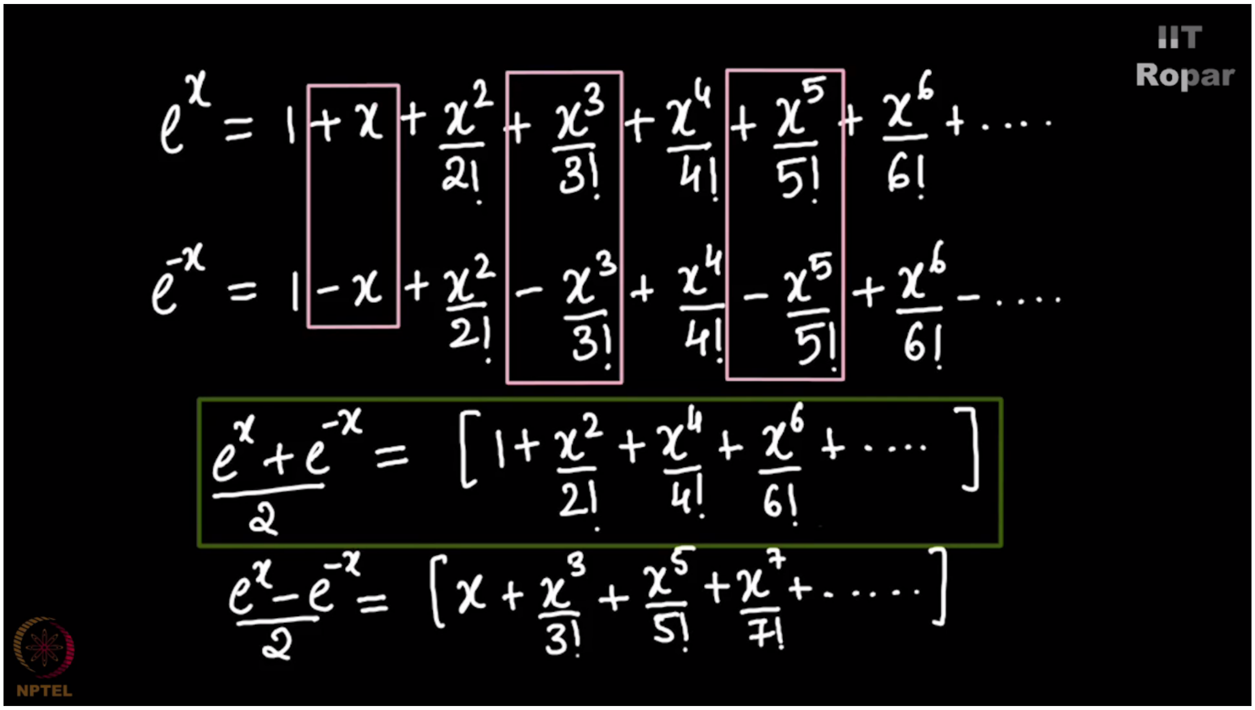
We all have studied some basic calculus where we have observed that E to the X can actually be written as 1 plus X plus X square by two factorial plus so on. This is actually called this is derived from teller series. Technicality aside, you just need to know that E to the X is so much.

If E to the X is so much what is the E to the minus X? You will simply get this. In place of X you have minus X. And X square simply be as it is because when you put X equals minus X, X square will continue to be X square but then X cube becomes minus X cube and so on. So E to the minus X will be 1 minus X plus X square by two factorial minus X cube by three factorial plus X to the 4 by four factorial minus X to the 5 by 5 factorial and so on.

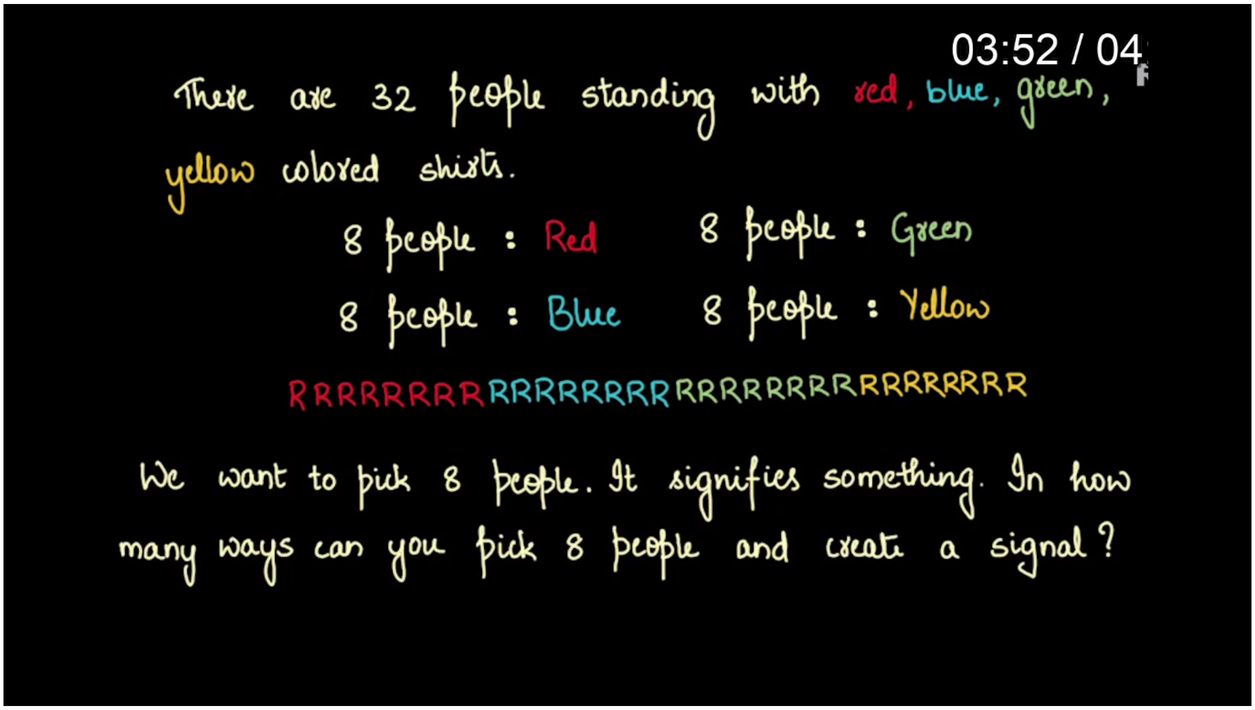
You have minus sign in front of X to the I if I is odd and plus sign if I is even. As simple as that. Now look at this. Let me do E to the X plus E to the minus X by two. You see these things get cancel like this and you will get two times 1 plus X square by two factorial plus X to the 4 by four factorial plus X to the 6 by 6 factorial and so on only the E 1 terms you will get if you add E to the X and E to the minus X. So 2 can come to the left hand side, the denominator and this becomes E to the X plus E to the minus X by 2 which is equal to so much. Let us box this for future reference.



Similarly you can observe that E to the X minus E to the minus X by 2 is actually equal to X plus X cube by three factorial plus X to the 5 by five factorial and so on. Why are we even seeing this? How will this help us in counting? That's a surprising part in discrete math where something very unrelated sometimes helps us in counting something that's very advanced the question. And here goes your question.

The question is there are 32 people standing with red, blue, green, yellow colored shirts. Their entire wear is red, blue, yellow, and green. 8 people are wearing red. 8 people are wearing blue. 8 people are wearing green and 8 are wearing yellow totaling to 32.

Do you see all of them in a sequence standing in a line. Now I want to pick some 8 people from here, some 8 people and make them stand. And this 8 people in a particular combination of colors signify something. Maybe it's a some sort of a coding of soldiers standing in the border or it could be some kind of a signal that you may want to give to your neighboring city or maybe you are going in a ship and you are hosting flag colors like this in a sequence to send out a message. It can be anything. Whatever. But the question here is simply this. In how many ways can you pick 8 flags with this 8 plus 8 plus 8 plus 8 of red, blue, green and yellow respectively and create a signal? How many such signals can you create. This is the question and surprisingly we will see how we can use this E to the X E to the minus X, E to the X plus E to the minus X by 2 etc. in solving this problem.



The key point a good advice please understand the problem carefully before understanding the solution. So now let's look at the solution for this question.