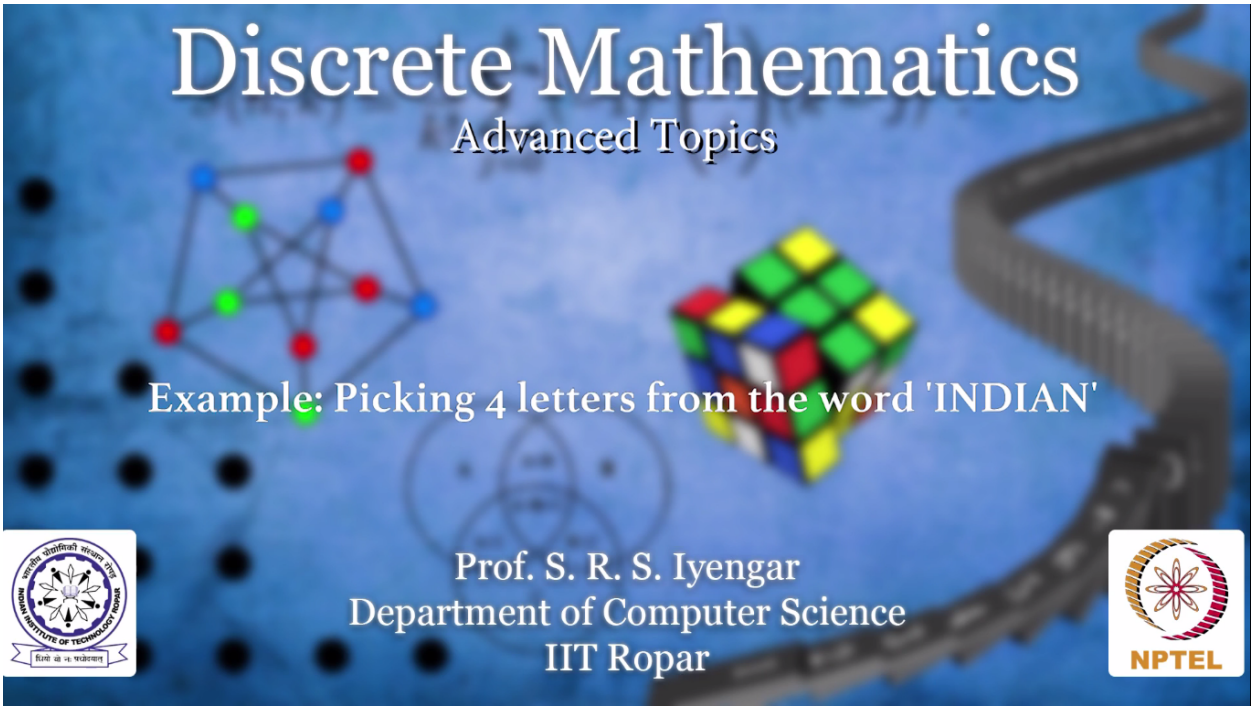
Discrete Mathematics

Functions

Advanced Topics



Example Picking 4 letters from the word 'INDIAN'

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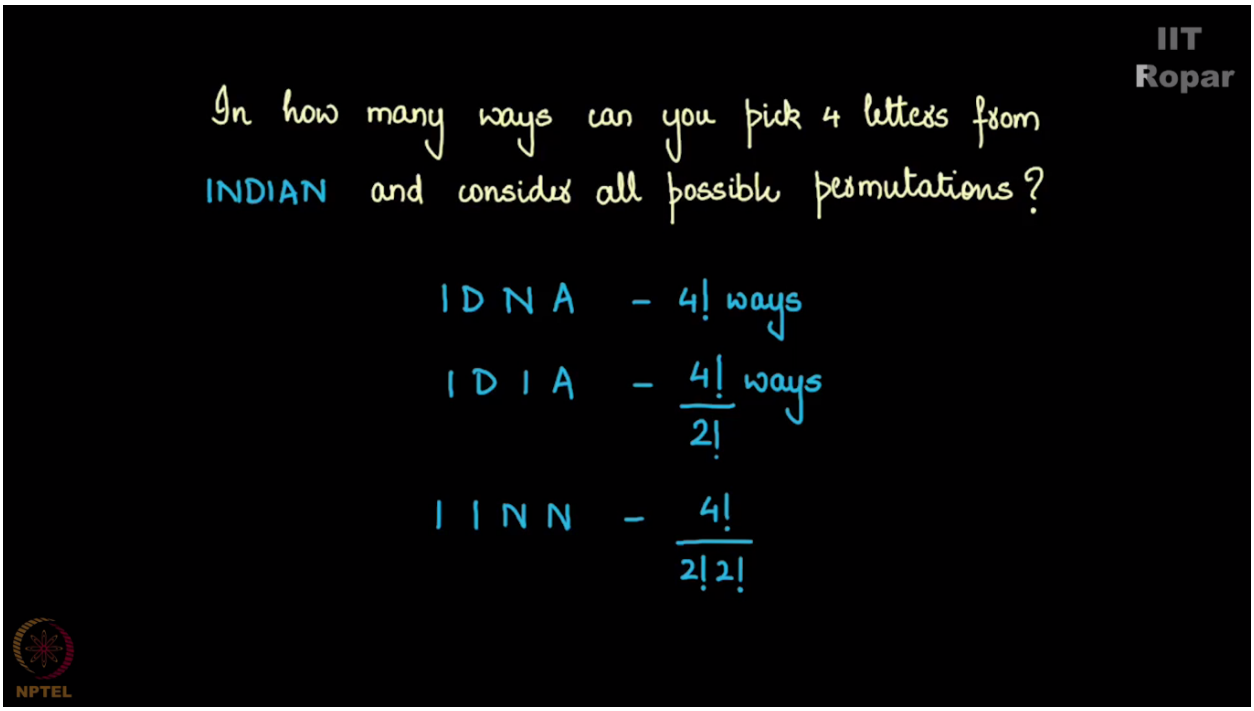
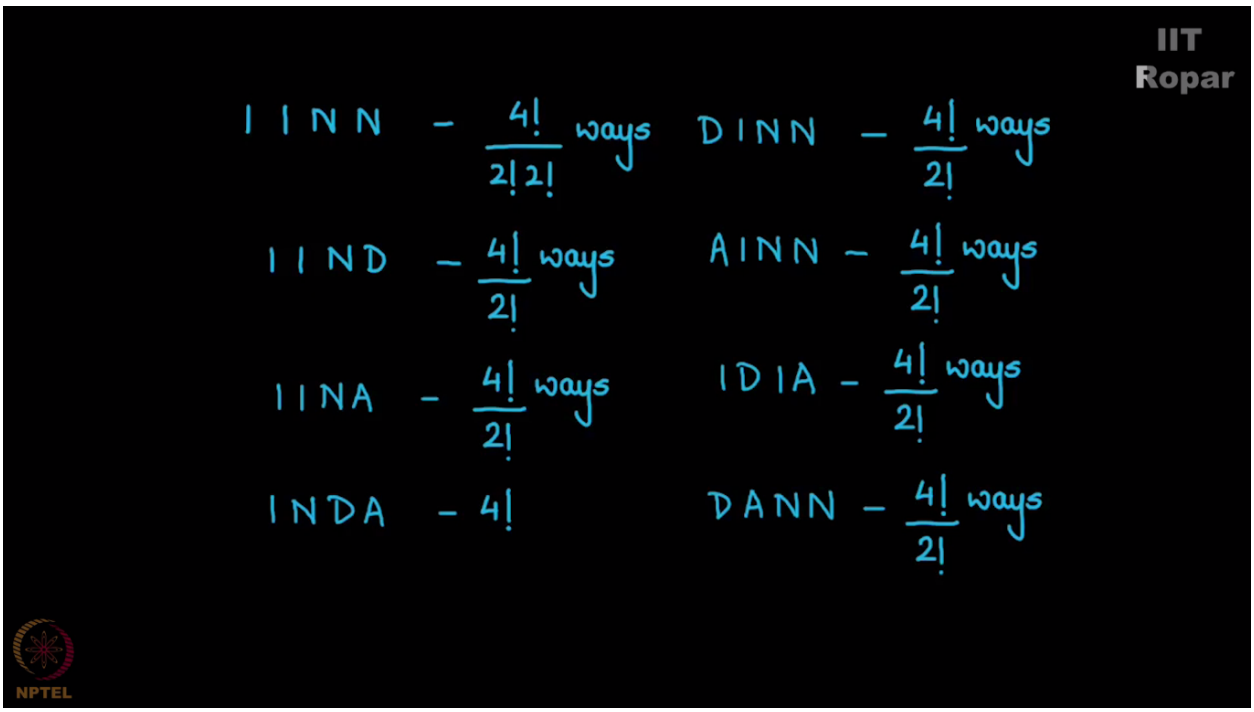
Look at the word Indian. In how many ways can you permute this? You observe that the word Indian can be arranged in one, two, three, four, five, six . Six factorial ways but I is repeated twice. N is repeated twice so divided by two factorial times two factorial. Pretty straightforward and easy application of multinomial theorem basically.

Now let me ask a very end of this question. In how many ways can you pick four letters from Indian and then consider all possible permutations. For example you could consider IDNA this can be permuted in four factorial ways. What about IDIA? Four factorial by two factorial. How about IINN. Four factorial by two factorial times two factorial. Let me write all possibilities now.



IINN, four factorial by two factorial into two factorial. IIND four factorial by two factorial. You see why. So I is repeated twice so two factorial is the denominator. That's it. IINA four factorial by two factorial. INDA four factorial. DINN four factorial by two factorial. AINN four factorial by two factorial. IDIA four factorial by two factorial. DANN four factorial by two factorial base. This is all possible picking of four letters from Indian and trying all possible permutations of these four letters.

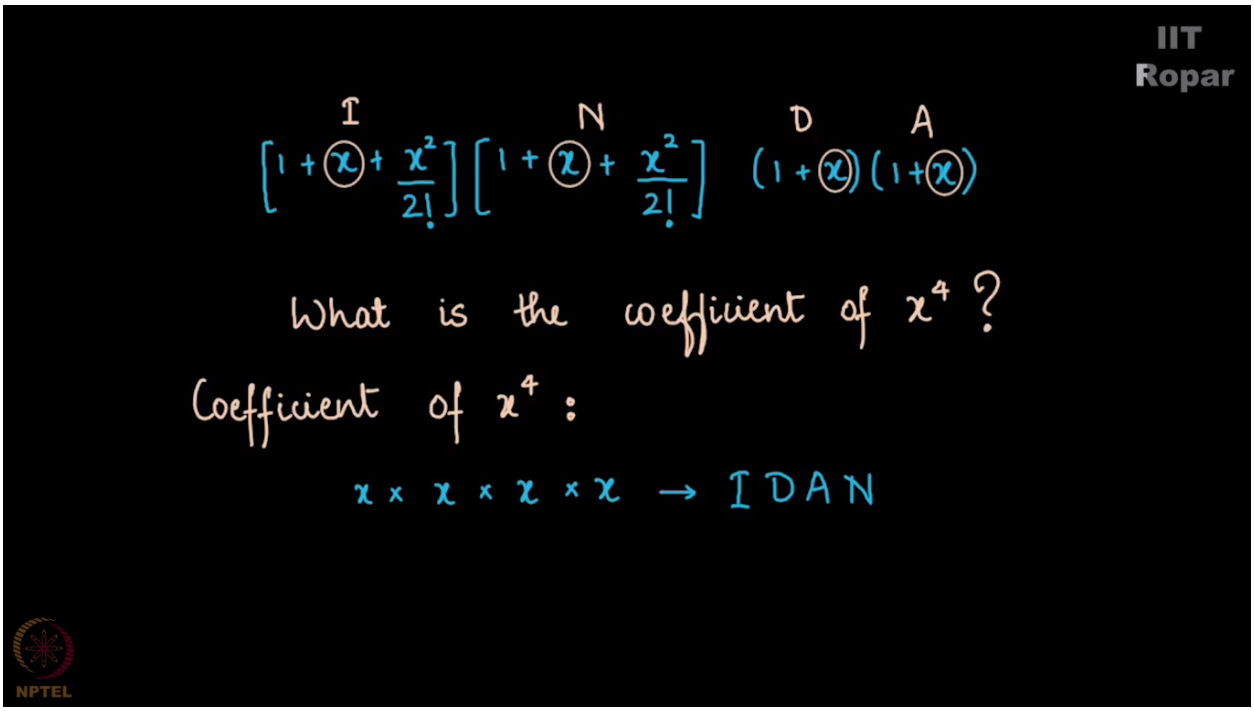


Sum total is the answer that we want. So what is the sum total? Let me write this down. I compute and I observe that it is actually this big sum is going to be simply one not two. Here is the explanation. You can take a look at it. It is one not two. Okay.

now let me look at a different way to answer this question because I need to learn a brand new technique here. Now let me write down I which is repeated twice so I will write 1 plus X plus X square by two factorial for I. This is I's house. What is a house? Why this for I you will get to know soon. Again see sometimes it's tough to explain the details of why we do something but with time you will realize why we are doing it. You need to have a keen sense of observation. For D D is repeated only once so it's 1 plus X. For A it is 1 plus

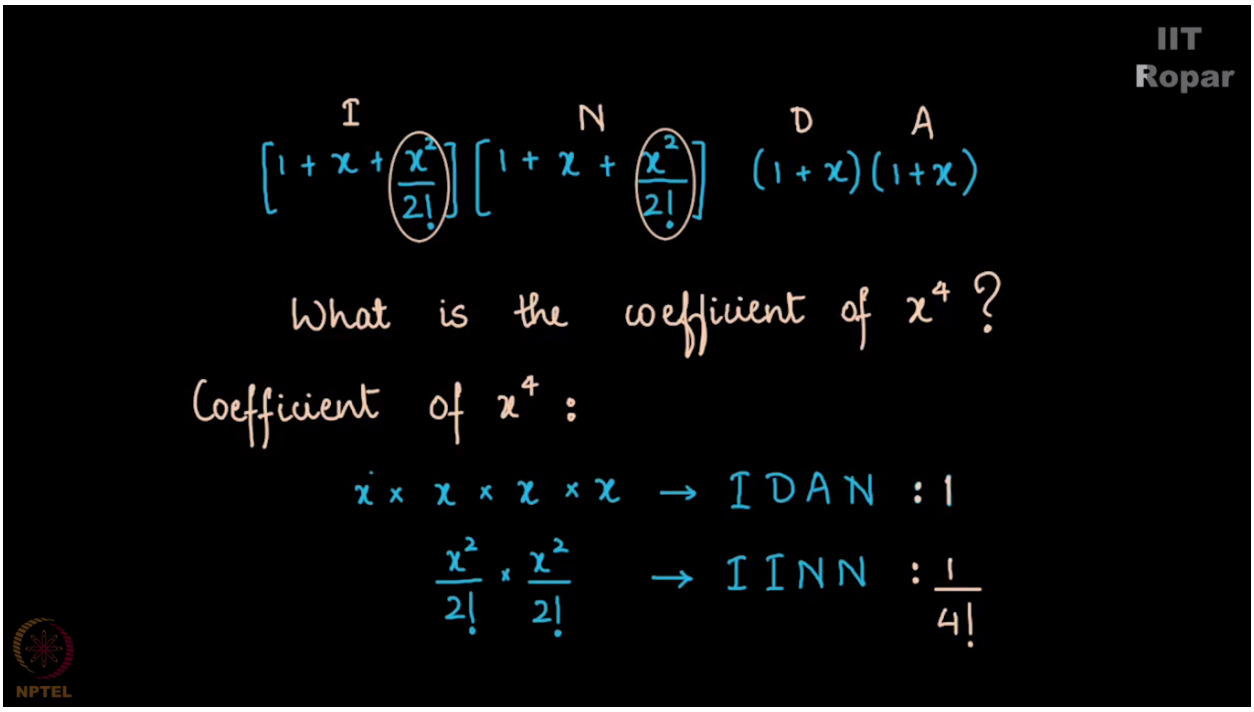
X. for N it's going to be 1 plus X plus X square by two factorial. So when it is repeated K number of times you simply write 1 plus X plus X square by two factorial plus X cube by three factorial plus X to the 4 by four factorial upto X to the K by K factorial. Why? You will get to know soon.

And then so we are done for IDAN. Now let me multiply all these things and then take a look. I get 1 plus X plus X square by two factorial the whole square times 1 plus X into 1 plus X. My question would be what is the coefficient of X to the 4 here?



Let me compute. The coefficient of X to the 4 is going to be simply how will you get X to the 4 here by picking an X from the first house. Basically there are four houses here. 1 plus X plus X square by two factorial. First house. 1 plus X plus X square by two factorial second house. First one is for I. Second one is for N. Next would be 1 plus X for D 1 plus X for A. And now how do I get X to the 4 here? By picking X, X, X, X in all four. This corresponds to the first one is the house of I, the second one is the house of N. the third one is the house of D. The fourth one is the house of A. this corresponds to if you pick X, X, X, X here, very similar to the partitions generating functions that we discussed this corresponds to one I because you are picking X from the

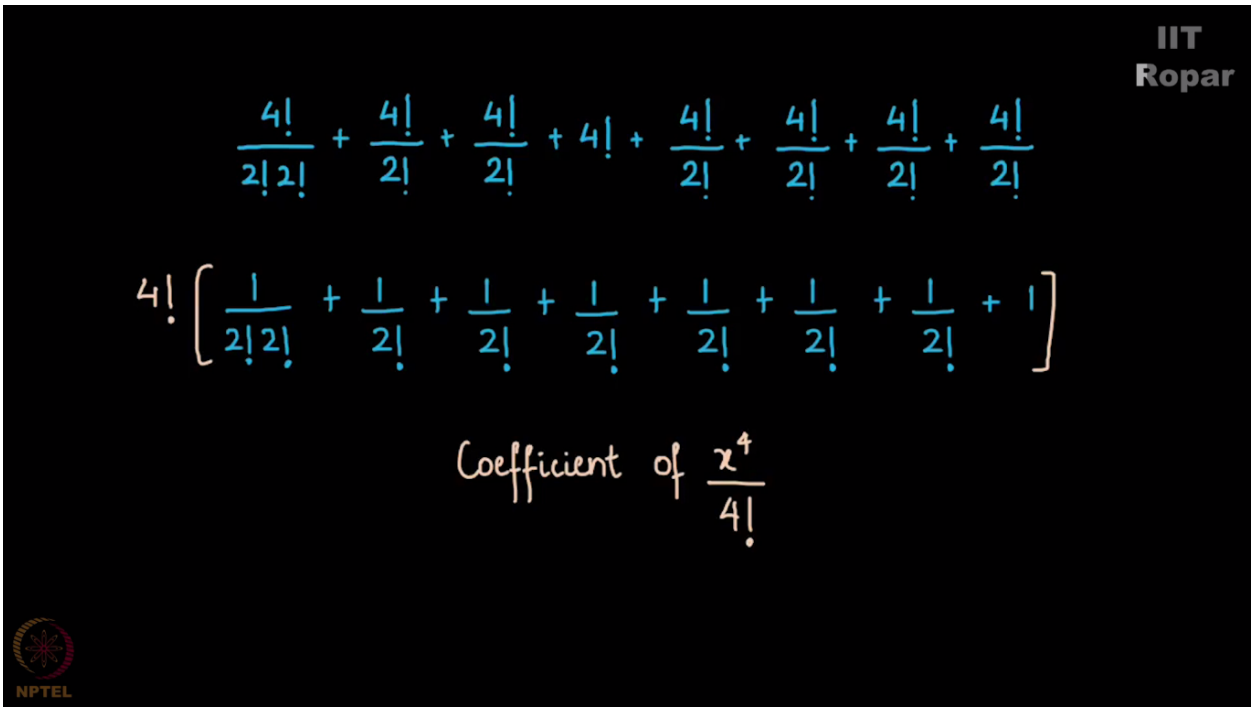
first house. One D because you are picking X from the D's house, second house. One A and one N this corresponds to IDAN. And if you look at the coefficient of X to the 4 it will be 1, simply 1. So for IDAN it will be 1. So if you look at let's if I pick X square by two factorial from the first house. X square by two factorial from the second house it will correspond to you picking I, I because you are picking X square by two factorial. X square if you pick it will – it means I is repeated twice. And again you are picking X square from the last one and you are picking one and one from the D's house and A's house which means D is not picked, A is not picked. I is picked twice. N is picked twice so it is IINN. And with that comes divided by two factors into two factorial so coefficient of X to the power of 4 when it's picked this way corresponding to IINN will given you X to the 4 by two factorial into two factorial.



Now if I compute the coefficient of X to the power of 4 overall I will get this big expression. Look at this. 1 by two factorial into two factorial plus 1 by two factorial plus 1 by two factorial plus 1 by two factorial plus 1 by two factorial plus 1 by two factorial plus 1 by two factorial. Why? Observe and then plus one at the end which corresponds to IDNA. Why does this happen? This simply corresponds to every possible way in which you can pick X to

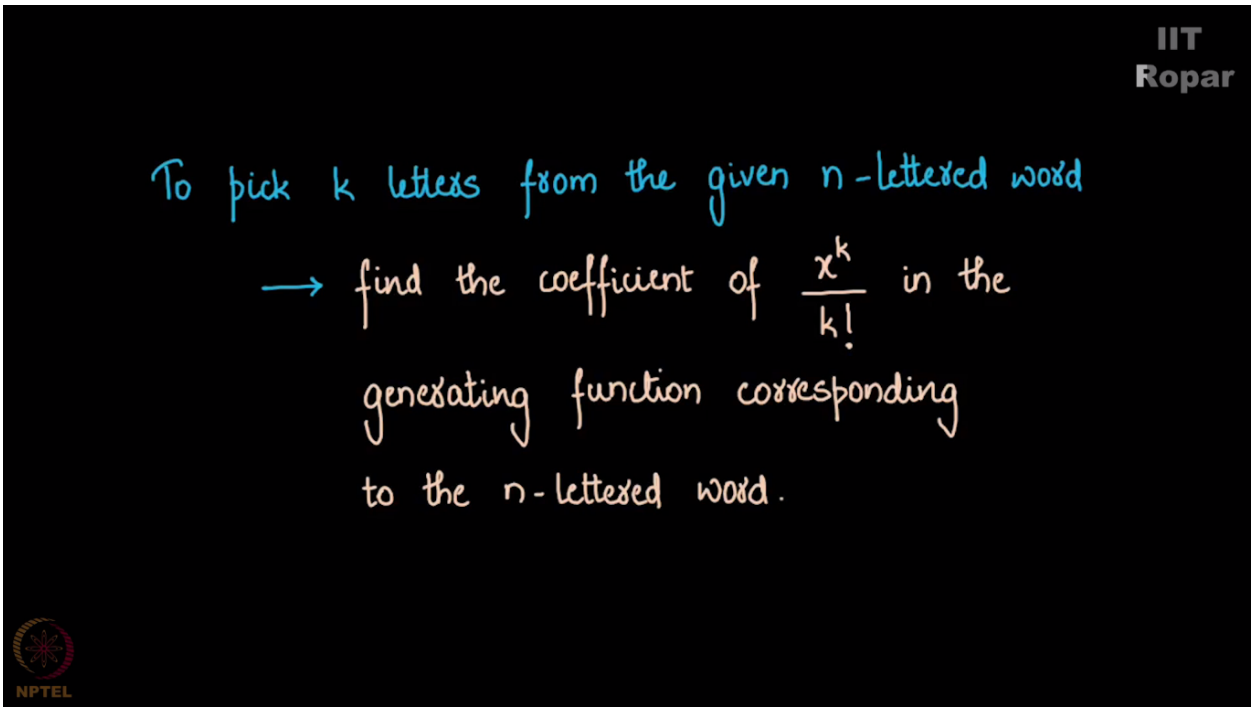
the 4 as simple as that. Sum total is something which is fractional. Correct. All these fractions add up to some number. But then if you look at the actual answer that we computed carefully we found the answer to be this four factorial by two factorial, two factorial plus four factorial by two factorial etc. etc. etc. do you compare these two things what is missing here is simply four factorial. Otherwise denominators are taken care of. So what we will do is we will rewind back and say we do not want the coefficient of X to the power of 4 here.

We want the coefficient of X to the 4 divided by four factorial. Why? That way it forces the four factorial the numerator very very very small trick. Think about it. This is the place where you might possibly have a confusion. So you must pause the video and think what I just now said.



If you want a four factorial in the numerator you should put a four factorial in the denominator to X to the 4. SO if you ask this question what is the coefficient of X to the 4 you will get 1 by two factorial into two factorial plus 1 by two factorial plus 1 by two factorial so on upto 1. that's the answer. But if you ask what is the coefficient of X to the 4 by four factorial? Why did four factorial come here. Whatever I don't care. If you have four factorial here I

will put four factorial the numerator there. Correct. So it will be four factorial whole into the entire thing which again as you can see look at the step, look at the step, and look at this step. You will get 1 not 2 precisely what we did. What is the moral of the story? The moral of the story is that if you want to compute in how many possible ways you can pick K letters from the given n letter word it is simply find the coefficient of X to the K by K factorial in the expansion of generating function corresponding to the n letter word. Correct. We did for the word Indian it is the same for the word engine.



Now let's see as a problem what is it for Mississippi. In how many ways can you pick let's say five letters from Mississippi and find out the answer. Let us look at this problem and apply our explanation to find the solution.

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MISSISSIPPI

In how many ways can you pick 5 letters from MISSISSIPPI and find o