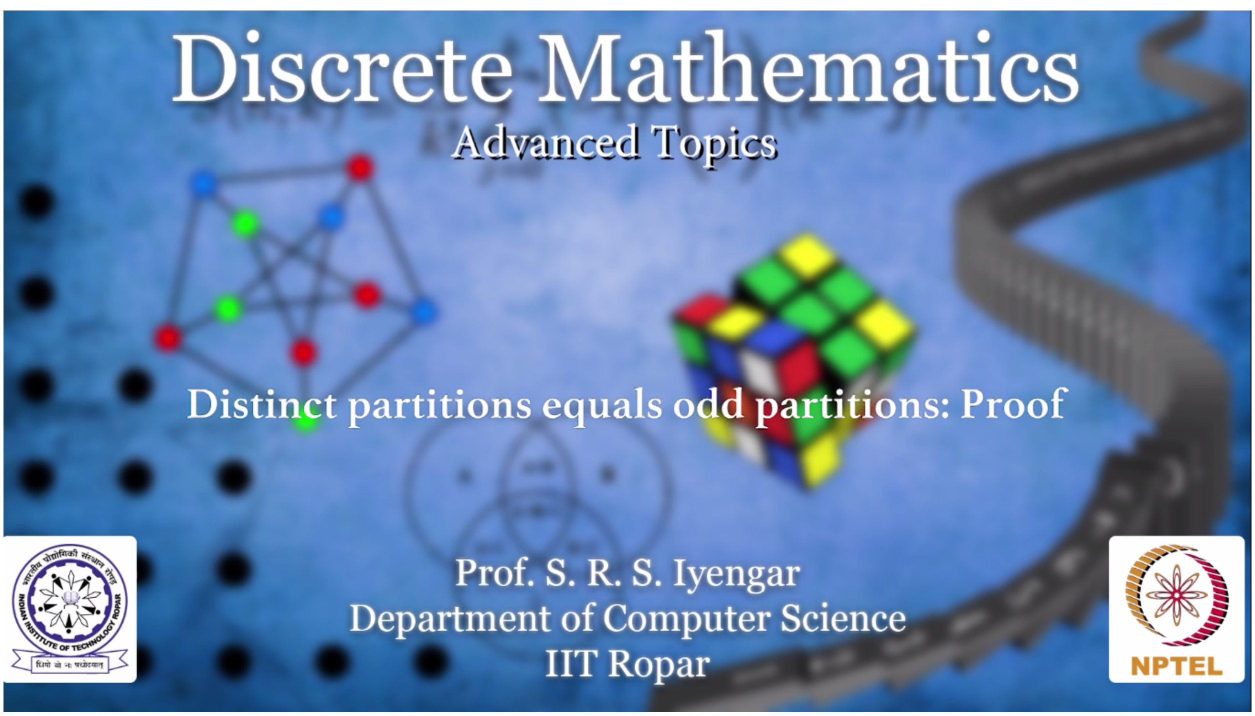
Discrete Mathematics

Functions

Advanced Topics



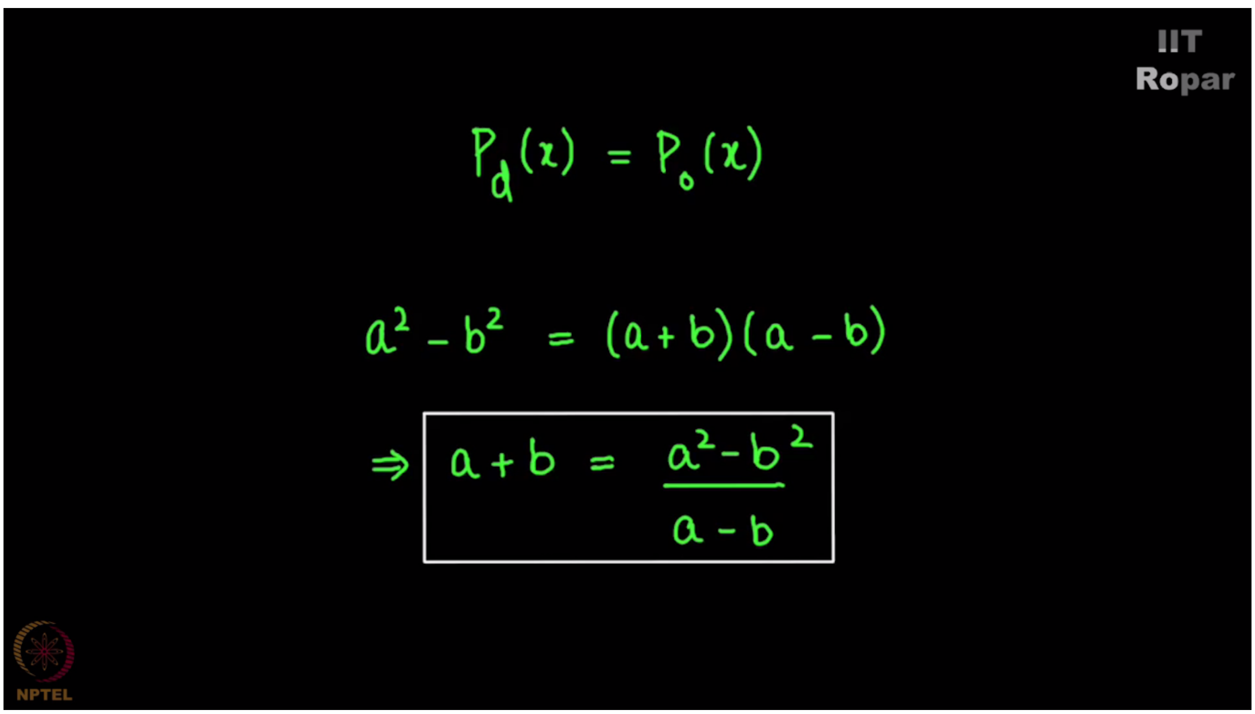
Distinct partitions equals odd partitions Proof

Prof S.R.S. Iyengar

Department of Computer Science

IIT Ropar

So now let us prove the observation that we saw that Pd of X equals Po of X. Before formally proving it. Let me just invoke some high school mathematics which we had seen. What is that? If you remember this algebric identity A square minus B square is A plus B into A minus B. we had seen this. Now in this case what is A plus B? It is A square minus B square by A minus B. Box this and keep. We will be using this to prove the observation. So we have seen that A plus B is A square minus B square by A minus B.



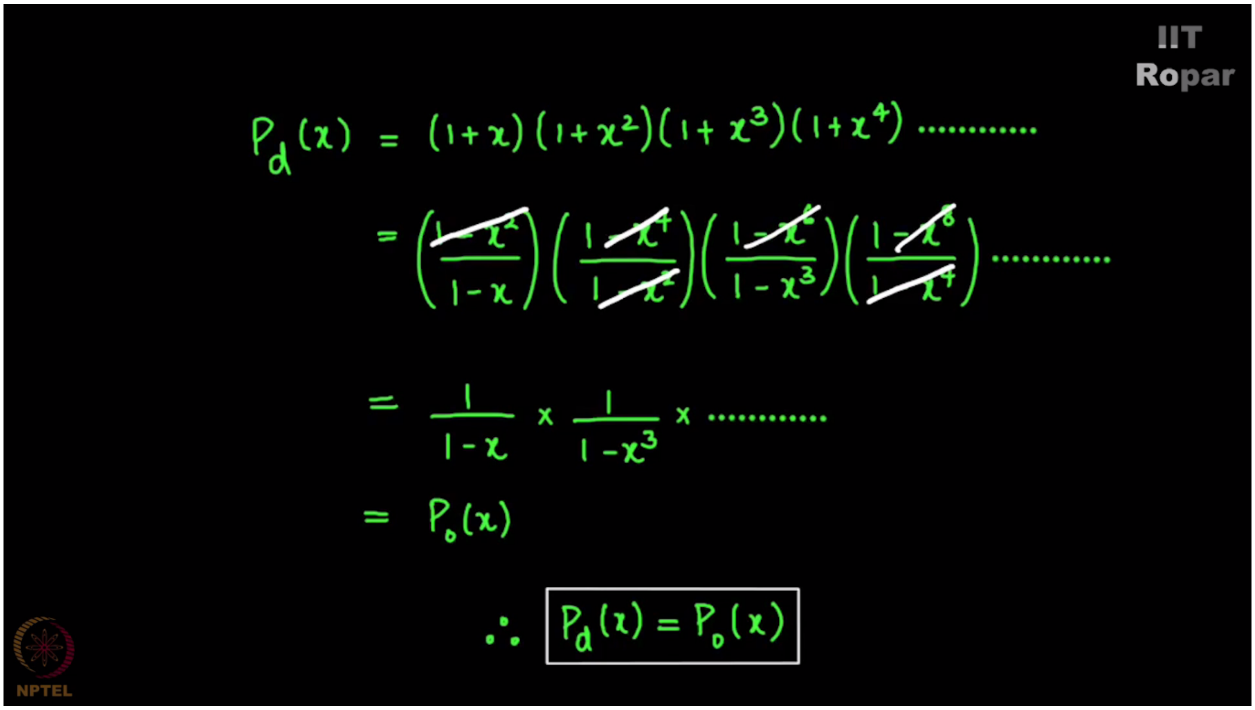
Now we have seen that Pd of X is equal to 1 plus X the generating function that is, into 1 plus X square into 1 plus X cube into 1 plus X to the 4 and so on. We have seen this.

Now the identity will come to our help here. What can I write 1 plus X as? I can write 1 plus X as 1 minus X square by 1 minus X and I can write 1 plus X square as 1 minus X to the 4 by 1 minus X square. Do you see that? Because 1 minus X to the 4 is 1 plus X square into 1 minus X square. And hence 1 plus X square is the following; 1 minus X to the 4 by 1 minus X square.

Now having the same logic 1 plus X cube is 1 minus X cube 1 minus X to the 6 by 1 minus X cube and 1 plus X to the 4 you must be very well versed with it. It is 1 minus X to the 8 by 1 minus X to the 4 and so on. Now do you see you can actually cancel 1 minus X square here and here. 1 minus X to the 4 here and here.

Now do you see you can also cancel 1 minus X to the 6 here with another term after 1 minus X to the 8 by 1 minus X to the 4 and the same holds true for 1 minus X to the 8 as well. You can cancel it with the term which follows in the product.

So what remains 1 minus 1 1 by 1 minus 6, 1 by 1 minus X cube and if you can further write the product you will see that 1 minus 1 by 1 minus X to the 5, 1 by 1 minus X to the 7 all these will remain. We are canceling all the terms with even powers here. 1 minus X square, X to the 4, X to the 6 and so on. And hence this becomes 1 by 1 minus X into 1 by 1 minus X cube so on.



Now if you remember this is the generating function of Po of X and hence we see that Pd of X equals Po of X. So we have now proved it. So this was one proof. There is also another bijective proof which you can see for Pd of X to prove that it is equal to Po of X. what do I mean by bijective proof you can show that there is a bijection between these two. We leave it to you as an exercise.