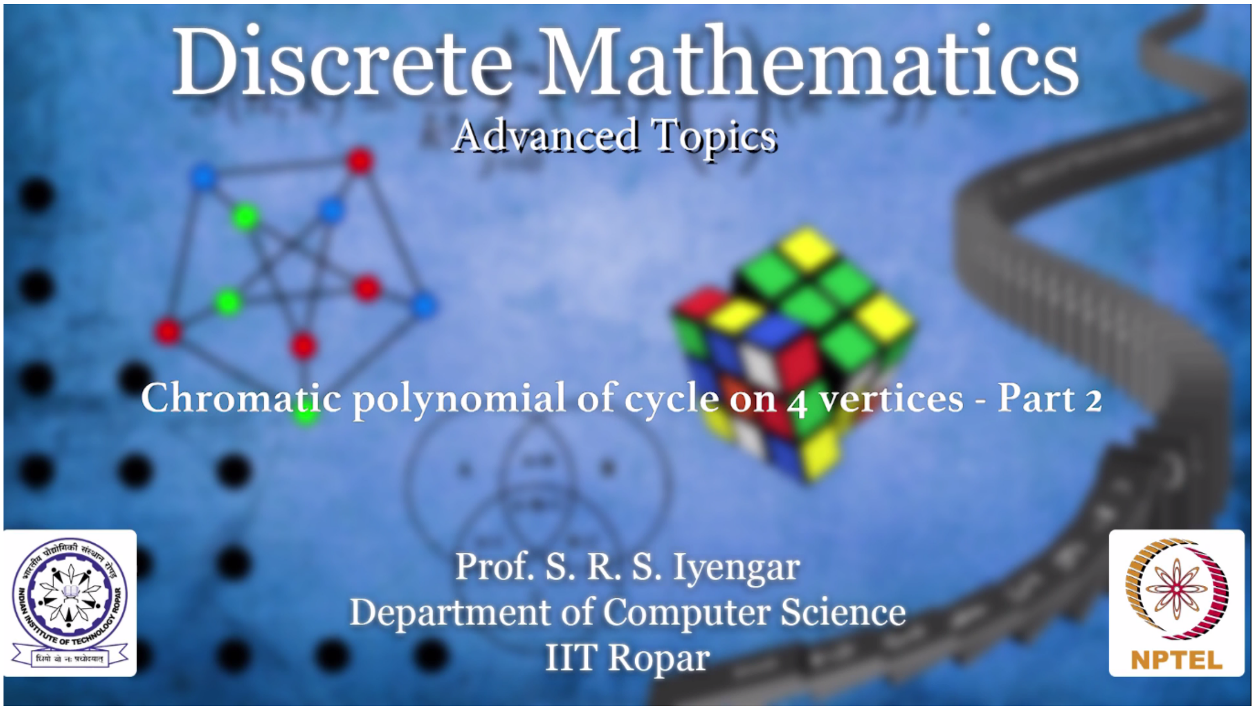
Discrete Mathematics

Functions

Advanced Topics

Chromatic polynomial of cycle on 4 vertices   Part 2

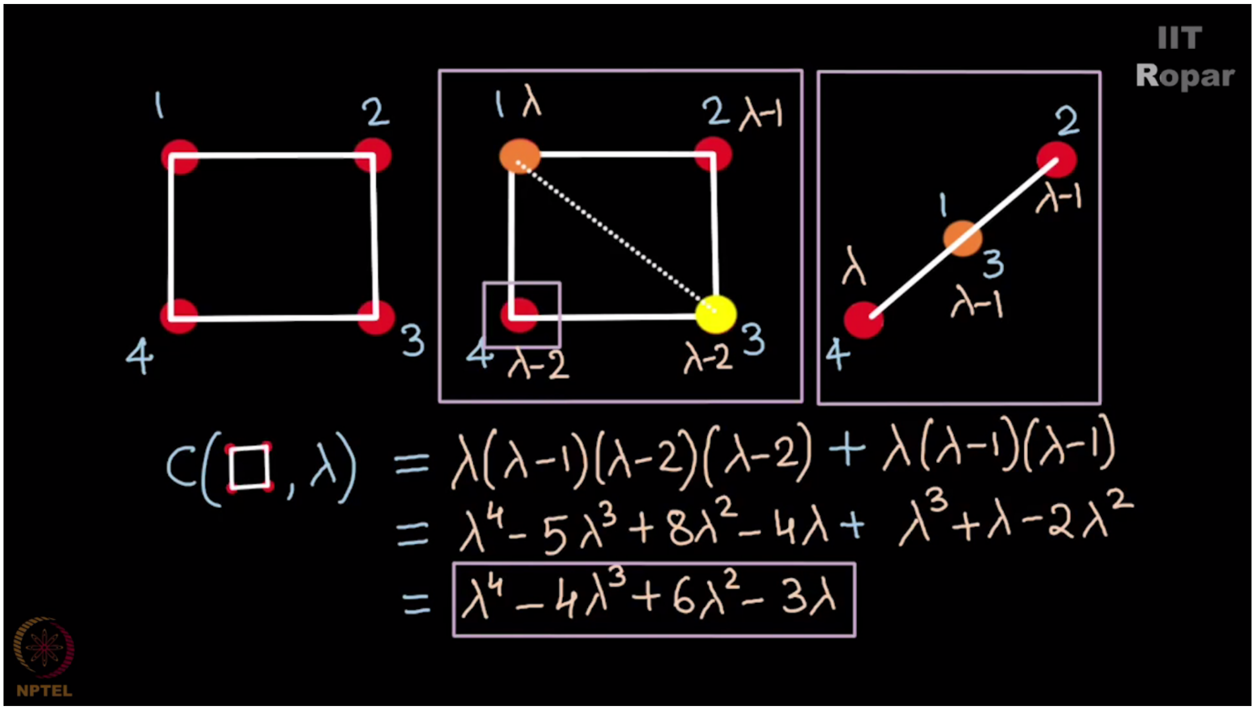


Prof S.R.S. Iyengar

Department of Computer Science

IIT Ropar

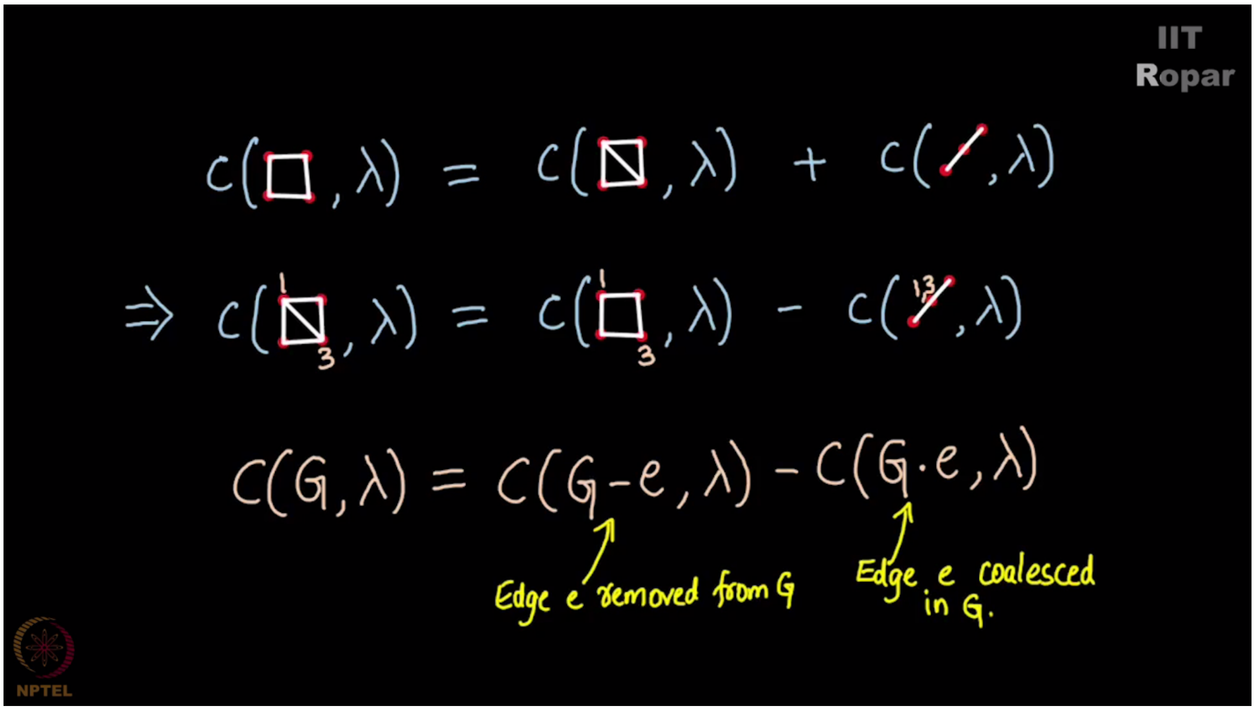
Look at this what is the polynomial for this graph? It's going to be lambda times lambda minus one times lambda minus two times there is no confusion here. You know very well one has taken a color and three has taken a different color and four cannot take these two colors. So four has only the option of choosing lambda minus two colors. So it's into lambda minus two. So lambda into lambda minus one into lambda minus two into lambda minus two and look at this graph which is just a simple path graph. We know the answer for this lambda times lambda minus one times lambda minus one and of course a plus symbol here simplify this and then simplify this you get this and add these two things you will get lambda to the four minus four lambda cube plus six lambda square minus three lambda I did it quite first you can try to see how I arrived at this final form.



Now let us take a back step and try to see what just happened here. Left hand side is equal to something on the right hand side plus something on the right hand side right. I mean the C4 is equal to C4 with a diagonal plus a path now let me see something what is this C4 with a diagonal? The chromatic polynomial of this is without diagonal that is C4 correct minus this goes from the left hand side minus the chromatic polynomial of the path. So which means when you have a graph and you want to remove an edge please observe very carefully if you want to remove this edge connecting 1, 3 right chromatic polynomial of such a graph is chromatic polynomial of a graph without that edge minus the possibility when the chromatic polynomial of a graph when these two nodes become the same which is here a path graph.

So let me write that down neatly. Chromatic polynomial of a graph G is settled down on some edge and remove that edge. So it will be chromatic polynomial of a graph the same graph G with that edge removed correct. This is how I represent with the edge removed minus chromatic polynomial of the graph with that edge glued together. By making that edge vanish what do you do the vertices alongside the edge becomes one. These two vertices become one vertex.

So chromatic polynomial of G is chromatic polynomial of G without that edge minus chromatic problem the G chromatic polynomial of the G of G with that edge this is called coalescing. Okay. you fuse these two vertices alongside the edge.



So now you see let me put this in a square and tell you I now have devised the way in which chromatic polynomial of a given graph is some difference of two chromatic polynomials of graphs which are smaller than this graph correct. We saw something like this in Routh polynomials if you remember but don't worry if you cannot connect that with this all I'm saying is you can find the chromatic polynomial of a graph by seeing a chromatic polynomial of a smaller graph so on and so forth.

Now it's time for us to see at least some three different problems in this direction and try to understand how to find the chromatic polynomial of a graph.