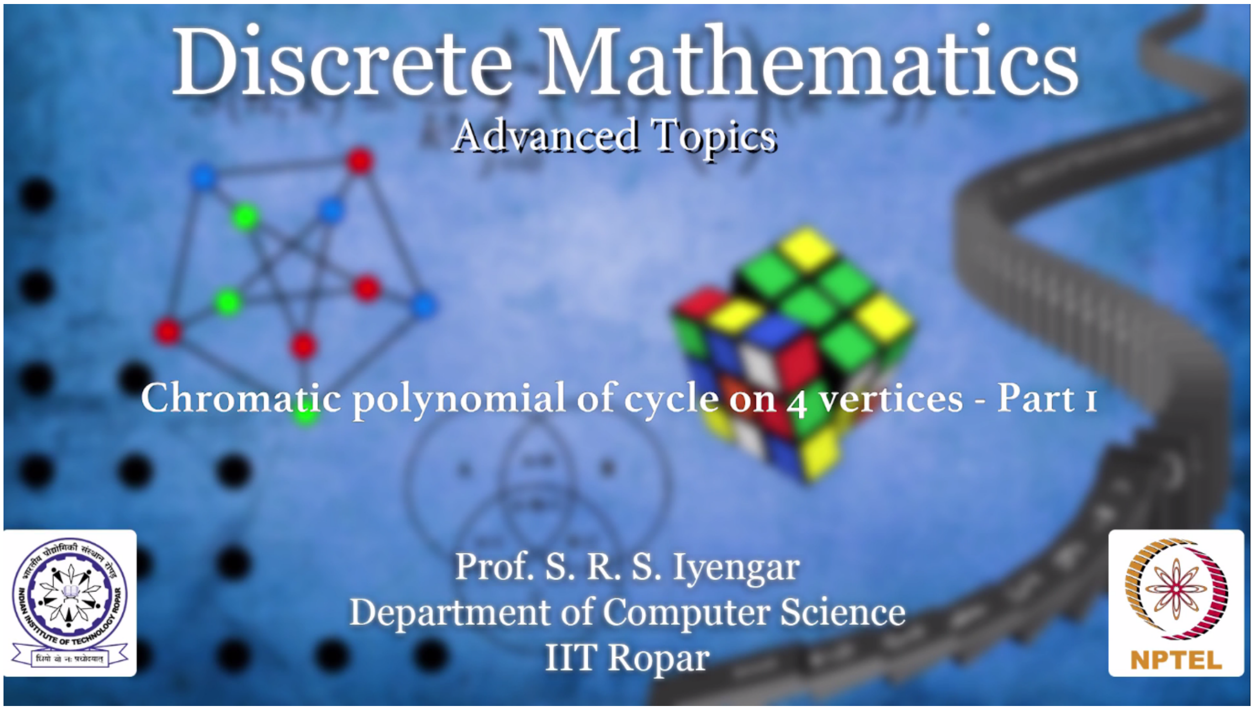
Discrete Mathematics

Functions

Advanced Topics

Chromatic polynomial of cycle on 4 vertices   Part 1



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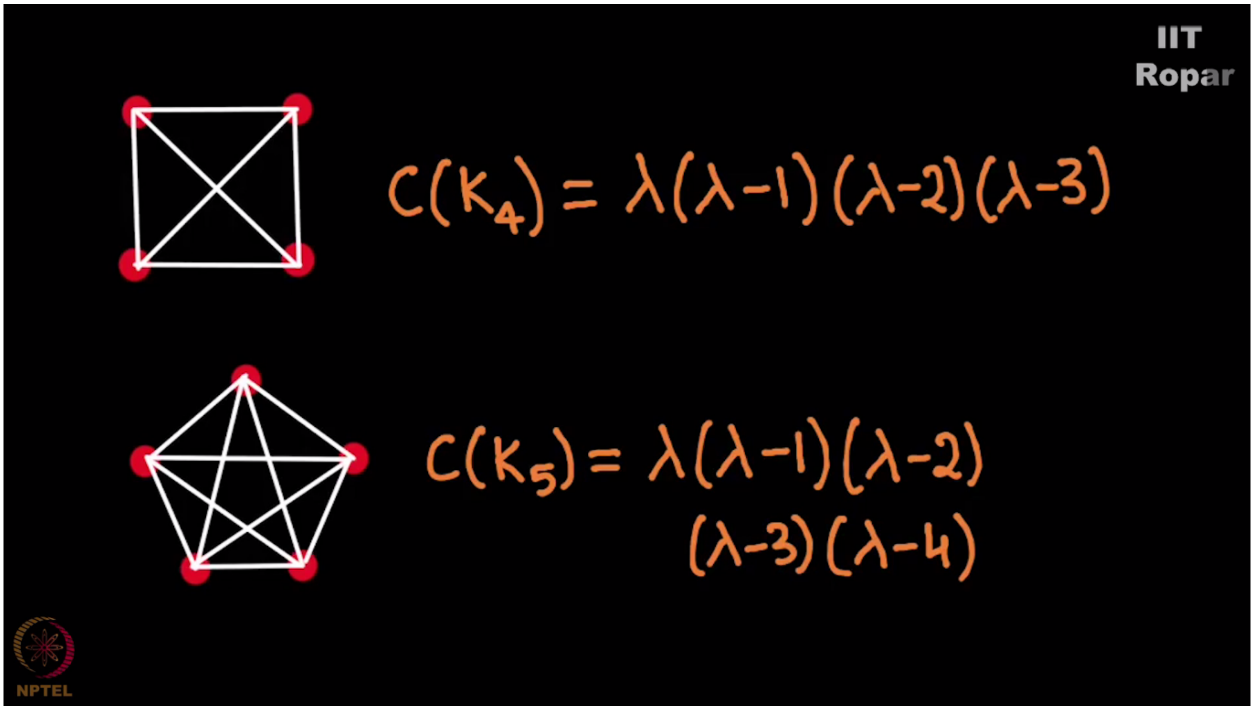
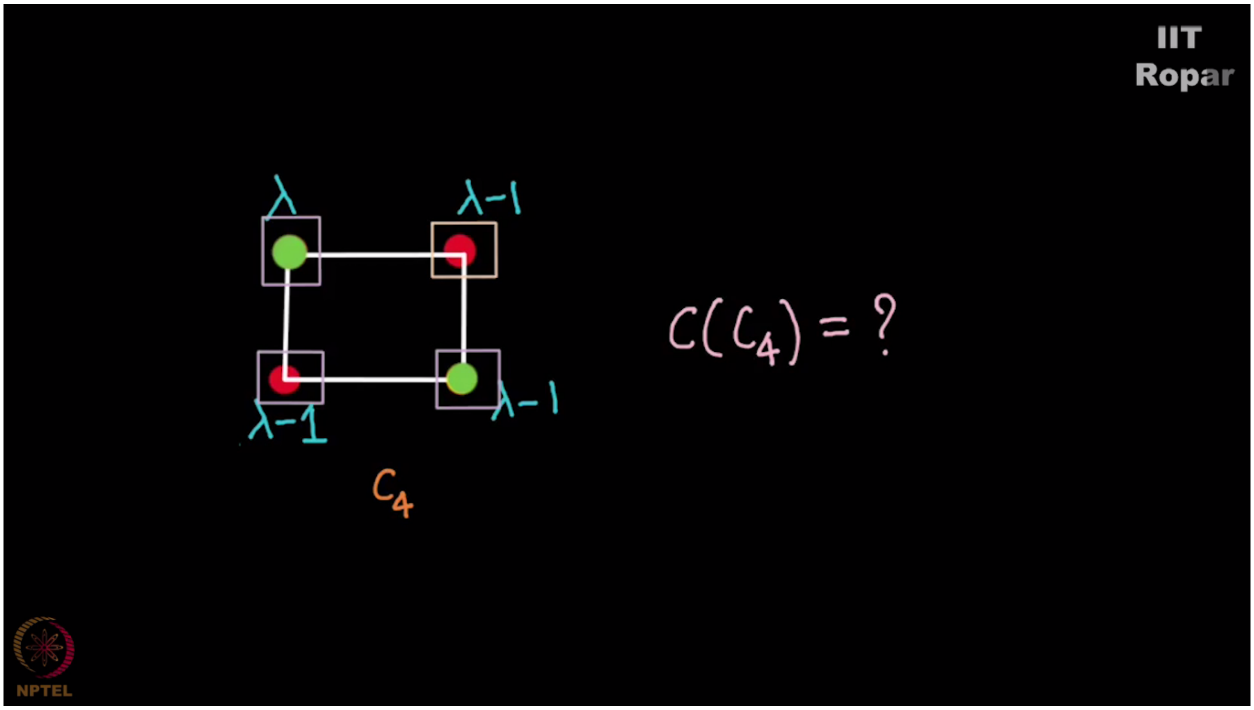
Okay. so if you have solved the problem I didn't mean to discourage you, but you are mostly wrong. Because this problem is not as straight forward as the problem for a path or the chromatic polynomial that we found out in the previous video for a complete graph.



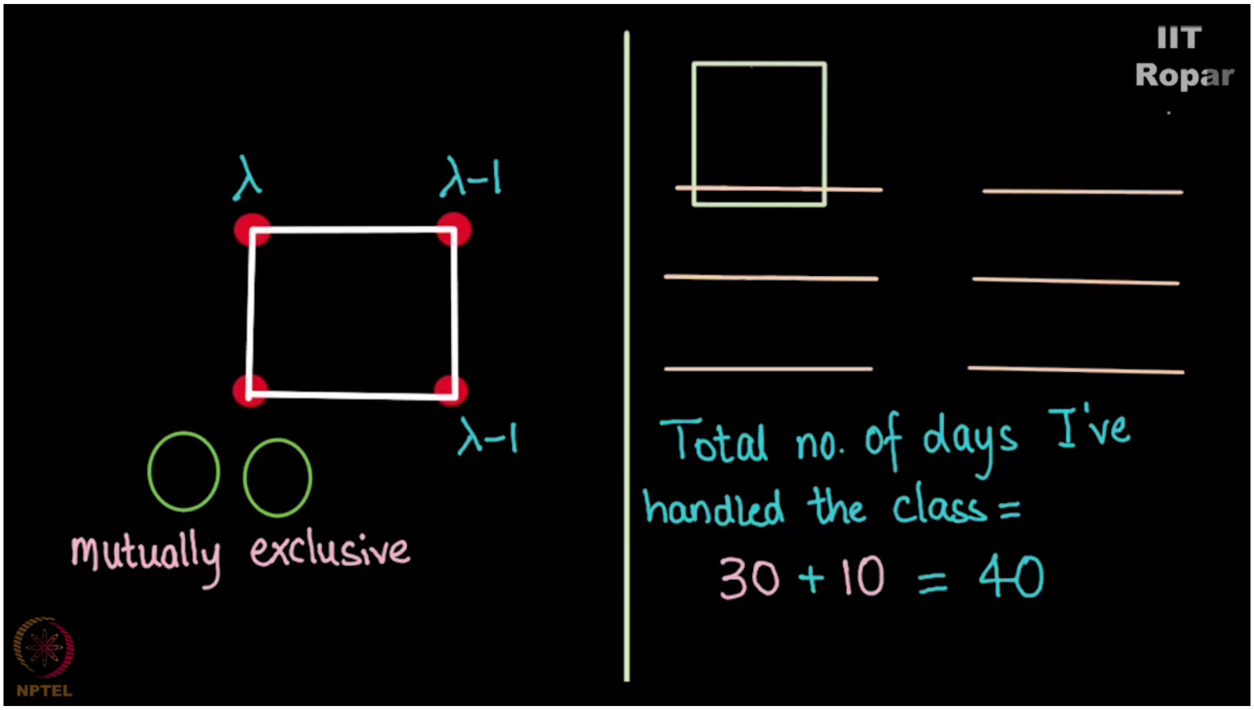
This problem is slightly different. Not only is the problem slightly different it will confuse you to a state that you will end up solving it but your answer will be incorrect. I will tell you what I am trying to point out. So look at this first node. In lamda ways you can color it. Look at the second node you can color it in lamda minus one ways. Look at third node you can color it in lamda minus one ways because this fellow is colored with lamda minus one the previous vertex.

Now the last vertex can also be colored with lamda minus one ways is what you would think but that is wrong. Why? Think about this. This person here gets a color. This person here gets a color. The first vertex and the third vertex. If they both are different then you have lamda minus two choices for the fourth vertex. If they both are the same then you have lamda minus one choices. You see what I am saying.

The number of choices for this last vertex depends on what was the color of the first and third vertex. Now you cannot solve this problem simply by putting lamda minus one or lamda minus two here. You should do something else here. What is that something else? Let us think about it patiently.

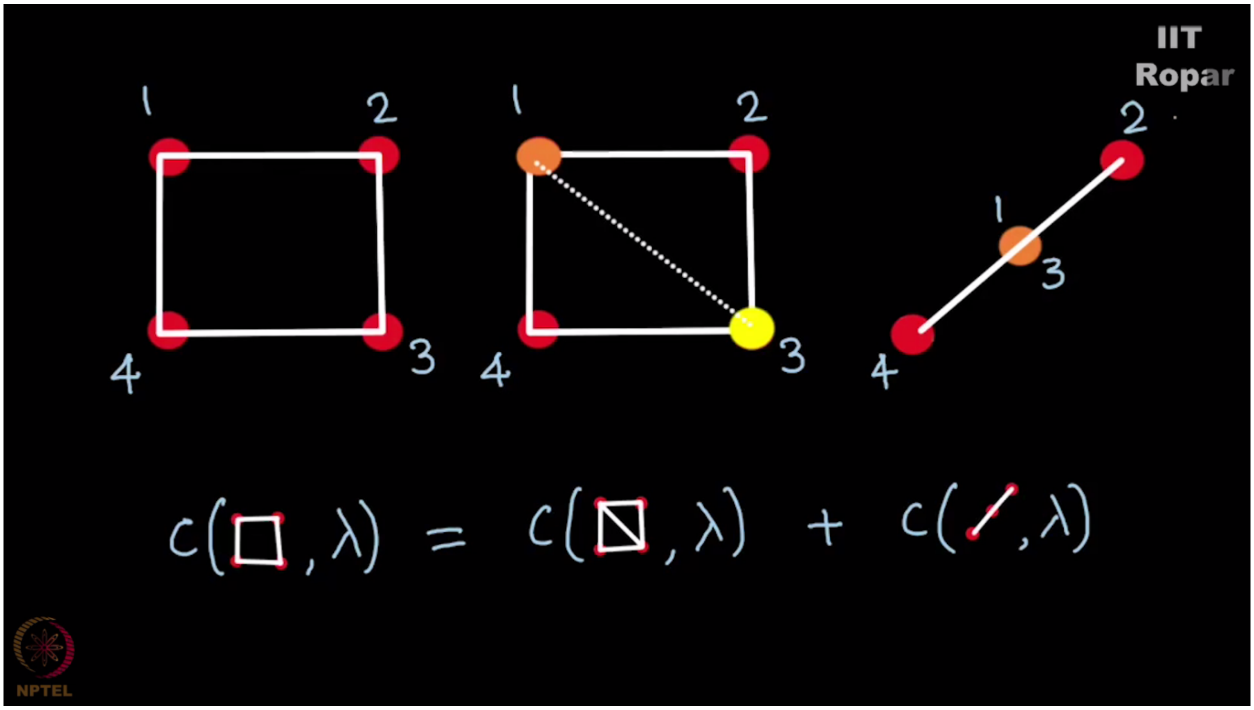


First node, second node, third node, fourth node. First is definitely lamda, second is definitely lamda minus one, third is again lamda minus one but fourth we have a problem but then let me use a technical term. There are two mutually exclusive possibilities here. What do I mean by that? So assume I am taking attendence in my classes. And you are my student. And I now I want to compute the total number of days I have handled my class which means total attendence. So what I can do is I will look at total days I have handled the class in your presence in my class. Assume you the front bencher. I see you in my class. 30 classes you are there in my class. Ten classes you are not there in my class. So I have handled a total of 40 classes which is 30+10, 30 with you, 10 without you. Is there a third possibility? None at all. These two are mutually exclusive possibilities and the sum total gives me the total possibilities.



Similarly analogously speaking think about it the total number of ways in which you can color this C4 here is the total number of ways in which you can color with the first and the third vertex. These two vertices having the same color, case one. And case two, is this two vertices having different color. Correct. If we look at the case one here. Case one is let me write this down the same graph and I will say these two vertex one and vertex three have different colors. Now isn't that equal to simply you putting edge here. I will write dotted edge because dotted edge denotes that there is no edge here but I am considering an edge because I am considering the case when these two vertices have different colors. So what is the chromatic polynomial of this? In how many ways can you color this graph with lamda colors? That will give you some possibilities but then there is also case two. Case two would be case one is the number of classes I have taken with you in my class. Similarly the case one is number of possibilities where these two nodes are of different colors. That is case one.

Case two would be number of possibilities with these two having the same color with you not being in the classes. Correct. So what is that? You put one, two, three and four vertices join one, two, three and four and finally one and three should be identified as the same vertex only then you are forced to put the same color think this is the place where you might get confused. One and three should be of same color. So what I will do is instead of writing a square like this, C4 like this I will identify one and three as the same vertex and have on either sides two and four. In how many ways can you color this with lamda colors will be the question then. What is the chromatic polynomial of this will be the question. This captures the possibility of one and three having the same color. The answer for this is the same as answer for C of this path with three vertices. So the question of in how many ways can you color a C4 with lamda colors is equal to two mutually exclusive possibilities. Total number of ways in which you can color this C4 with a diagonal representing these two vertices will have different colors plus the total number of ways in which you can color this simple path with three vertices.



So what's the answer for this? What is the answer for this gives us the answer for this.