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NPTEL ONLINE COURSE

Discrete Mathematics

Functions

Advanced Topics

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Chromatic polynomial of complete graphs

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Chromatic polynomial of complete graphs


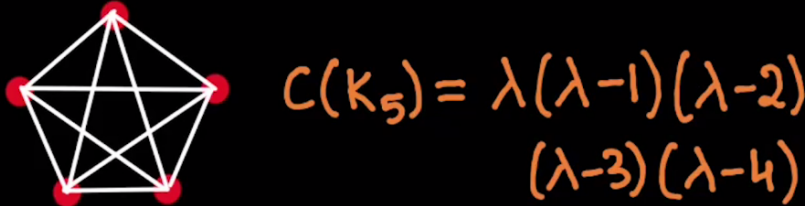
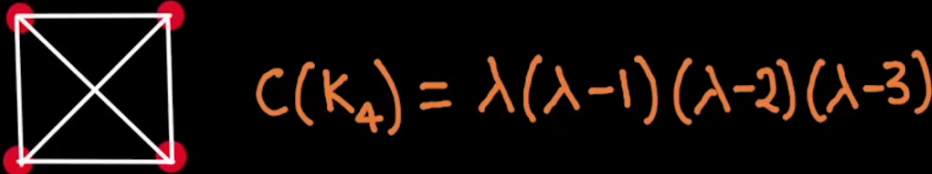

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Let us look at the chromatic polynomial of a complete graph. Look at a triangle. λ color is here, $\lambda - 1$ color is here. Now finally for the third node you are left with two different colors consumed so you have $\lambda - 2$ colors left. So it is λ into $\lambda - 1$ into $\lambda - 2$ for a complete graph with three vertices also called K_3 .

So what is it for K_4 ? Think about it. Let me be fast here because you all know graph theory well. It's going to be λ into $\lambda - 1$ into $\lambda - 2$ into $\lambda - 3$ for K_4 and for K_5 , it's going to be λ into $\lambda - 1$ all the five vertices right. $\lambda - 2$ into $\lambda - 3$ into $\lambda - 4$. In general for K_n it's going to be λ into $\lambda - 1$ so on $\lambda - 2$ so on into $\lambda - n + 1$ it will become $\lambda - n + 1$. correct. That is the chromatic polynomial for K_n .



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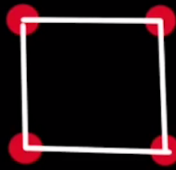
$$C(K_4) = \lambda(\lambda-1)(\lambda-2)(\lambda-3)$$
$$C(K_5) = \lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4)$$

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Now let us ask this question, what is the chromatic polynomial of this C_4 a cycle with four vertices? Think about it.

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$$K_n \quad C(K_n) = \lambda(\lambda-1)(\lambda-2) \dots (\lambda-n+1)$$



C_4