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NPTEL ONLINE CERTIFICATION COURSE

**Discrete Mathematics
Recurrence Relation**

Solution for the recurrence relation of Binary search

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We are now going to see the solution for the recurrence relation of binary search, so the recurrence relation goes like T_n is $T_{n/2} + 1$ and the initial condition is given by $T_1 = 0$, we know that $T_{n/2}$ can be written as $T_{n/4} + 1$, and $+1$ remains as it is.

Now 4 can be written as 2 square and hence this becomes $T_{n/2}$ square $+ 2$, I have opened the bracket and added the 2 once, $N/2$ square if I expand it as I had done earlier $T_{n/2}$ square can be written as $T_{n/2}$ cube $+ 1$, and $+2$ remains as it is, again opening the bracket I can write this as $T_{n/2}$ cube $+ 3$, now what happens at the R th step, when we write it, it becomes T_n is $T_{n/2}$ to the $R + R$,

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$$T_n = T_{n/2} + 1, T_1 = 0$$

$$T_n = (T_{n/4} + 1) + 1$$

$$T_n = T_{n/2^2} + 2$$

$$T_n = (T_{n/2^3} + 1) + 2 = T_{n/2^3} + 3$$

At δ^{th} step, $T_n = T_{n/2^\delta} + \delta$

you might be wondering here how did I get this, observe the previous step you might find the pattern and hence T_n is $T_{n/2}$ to the $R + R$.

Now we know that T_1 is 0, and we have here $T_{n/2}$ to the R , so $N/2$ to the $R = 1$, I am going to write it like this which implies N is 2 to the R , now if N is 2 to the R , what does R become? $R = \log N$, and T_n becomes $T_{n/2}$ to the $R + R$ substituting for $T_{n/2}$ to the R , and R we get T_n is $T_1 + \log n$, T_1 as we know is 0 and hence T_n becomes $\log n$, so this is the solution for the recurrence relation of binary search.

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$$n/2^x = 1$$

$$\rightarrow n = 2^x$$

$$\therefore x = \log n$$

$$T_n = T_{n/2^x} + x$$

$$T_n = T_1 + \log n$$

$$\therefore T_n = \log n$$



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