NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Solution for the recurrence relation of Tower of Hanoi

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We have seen that the recurrence relation for Towers of Hanoi goes like this T(n) = 2 into Tn-1 + 1, given the initial condition T1 is 1.

Now I can write Tn-1 as 2 times Tn-2 + 1 with the initial 2 as it is and the last one as +1 as it is, now I can further simplify this as 2 square Tn-2 + 2 I have just expanded the bracket and written multiply 2 throughout, now Tn-2 can be further written as 2 square 2 times Tn-3 + 1 and the rest of +2 +1 remains as it is, simplifying this we get T(n) as 2 cube, Tn-3 + 2 square + 2 + 1,
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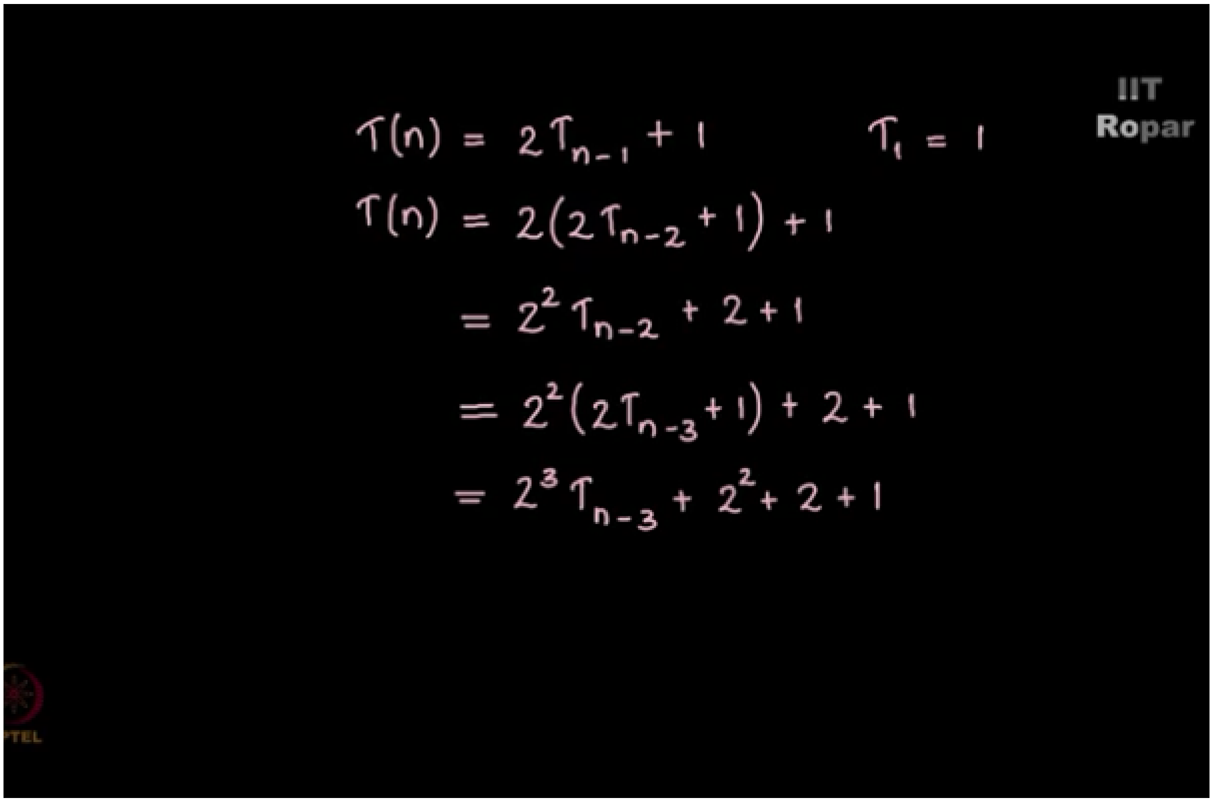

if you are probably not understanding you have to stop or pause the video, pen down things yourself and it will be much clearer.

The next is as usual Tn-3 can be written as 2 times Tn-4 + 1 + 2 square + 2 + 1, now this becomes 2 to the 4 Tn-4 + 2 cube + 2 square + 2 + 1 and the process continues,
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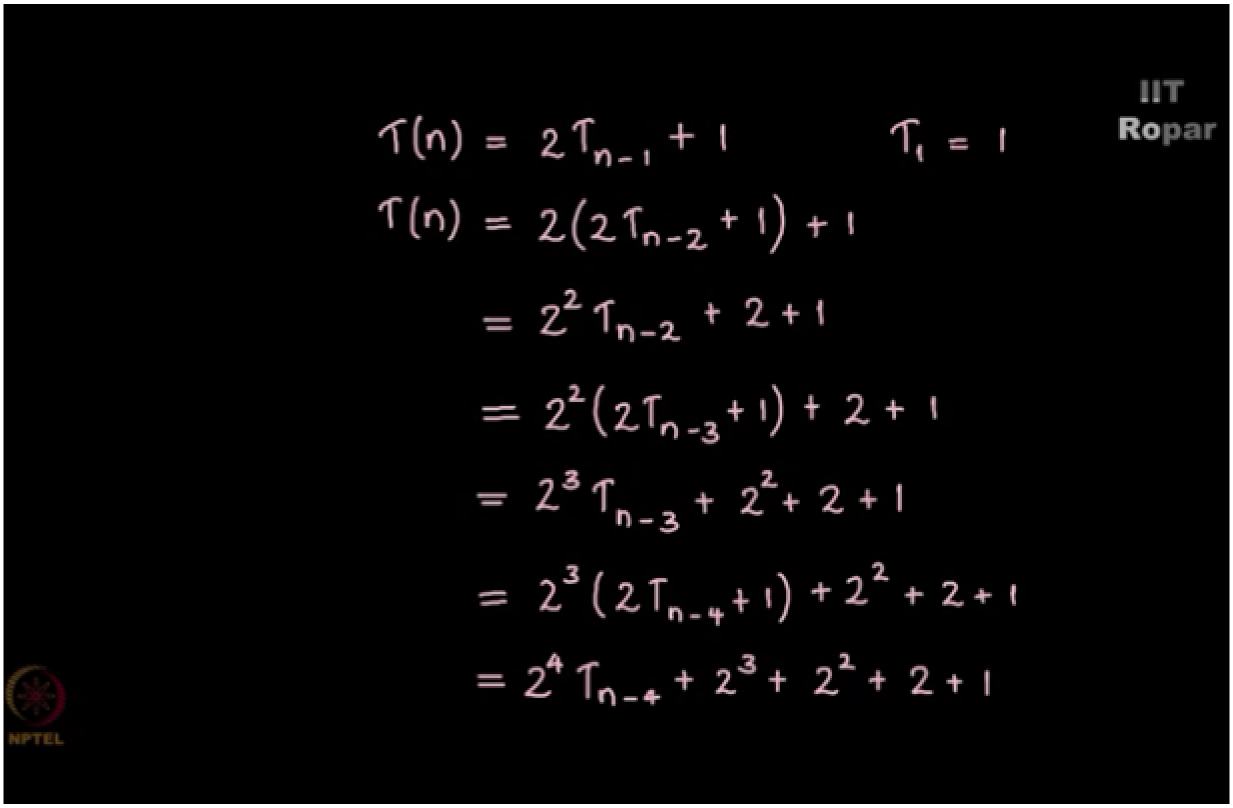

so what can we write in general? In general Tn can be written as 2 to the R Tn-R + 2 to the R - 1 + so on + 2 square + 2 + 1, now we know that T1 is 1, this was the initial condition and if I have to write here as T1, I write N-R = 1 and hence R is N-1, I've just taken R to the other side, so R becomes N-1, now I can write T(n) as 2 to the N-1 in place of R I'm going to substitute n-1 so it becomes 2 to the N-1, and N-R becomes 1, therefore it is 2 to the N-1 + 2 to the N-2 + 2 to the N-3 + so on + 2 square + 2 + 1, and for simplicity or for calculation I am going to just substitute T(n) as X, we are going to simplify this and I'll substitute back as T(n), so now for simplicity let me write this as X and the right hand side remains as it is, and now I'm going to multiply 2 on both the sides, when I multiply 2 on both the sides it becomes 2X = 2 to the N + 2 to the N-1 + 2 to the N-2 + so on +2 cube + 2 square + 2.

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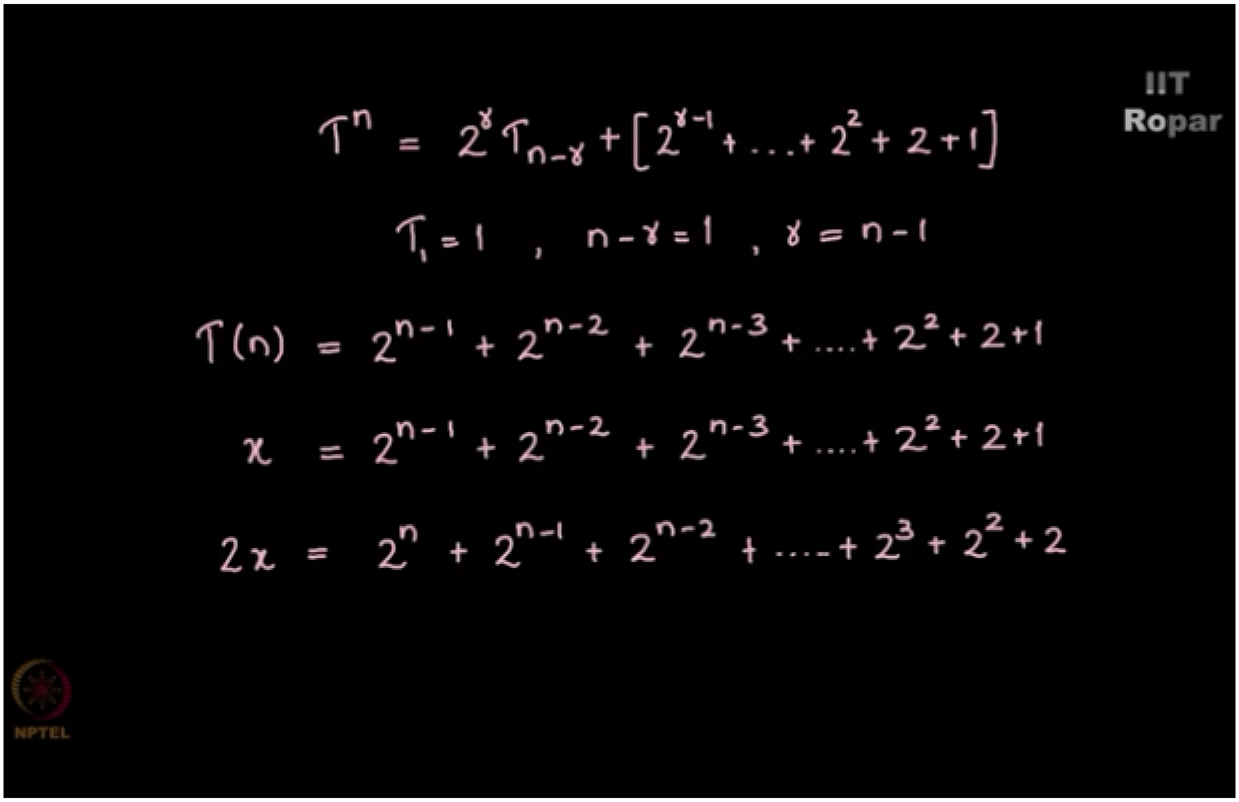

Now I am going to subtract this equation and this equation, and what do I get? On the left hand side 2X-X we get it as X, and that is equal to 2 to the N-1, how did I get 2 to the N-1? You see these terms 2 to the N-1, 2 to the N-2 so on, 2 cube, 2 square and 2, all of these get cancelled and what remains is 2 to the N and 1, so it becomes X = 2 to the N-1, and this was initially T(n) and hence T(n) is 2 to the N-1,

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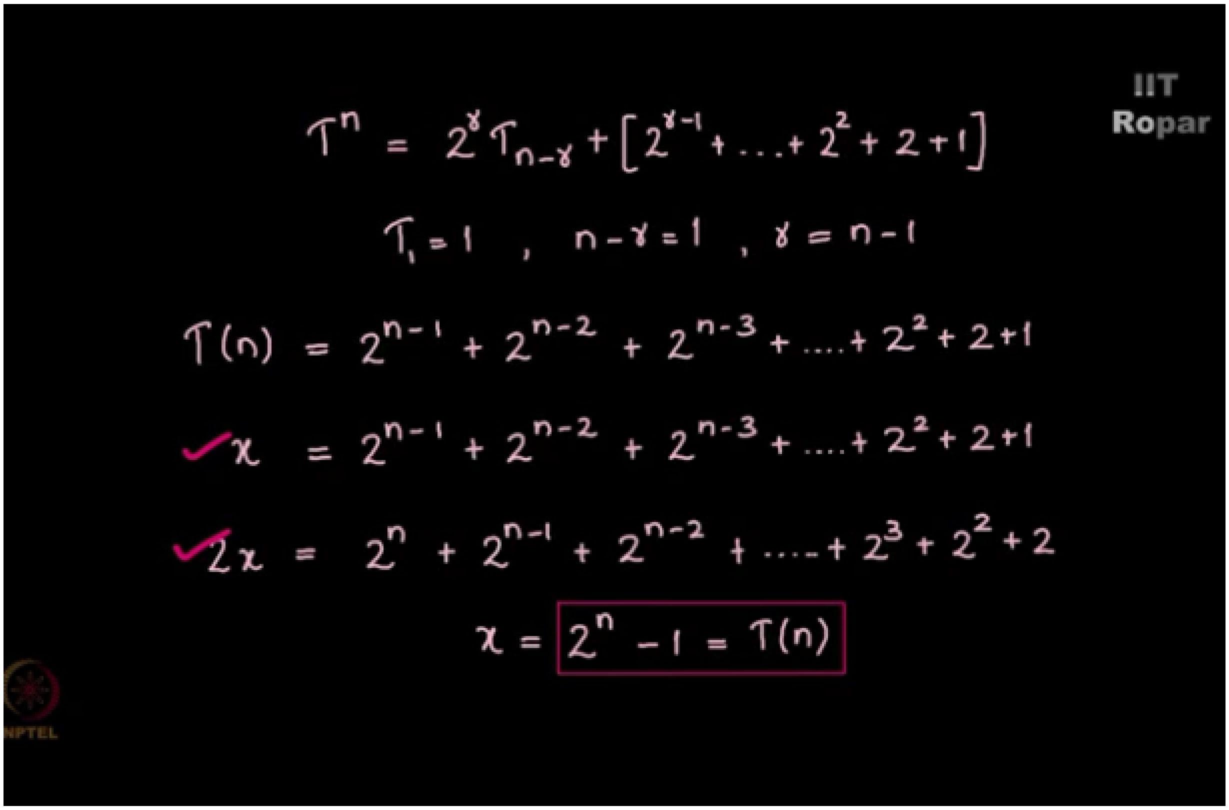

this is the solution for the problem of Towers of Hanoi that we have studied in the previous video.

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