

NPTEL

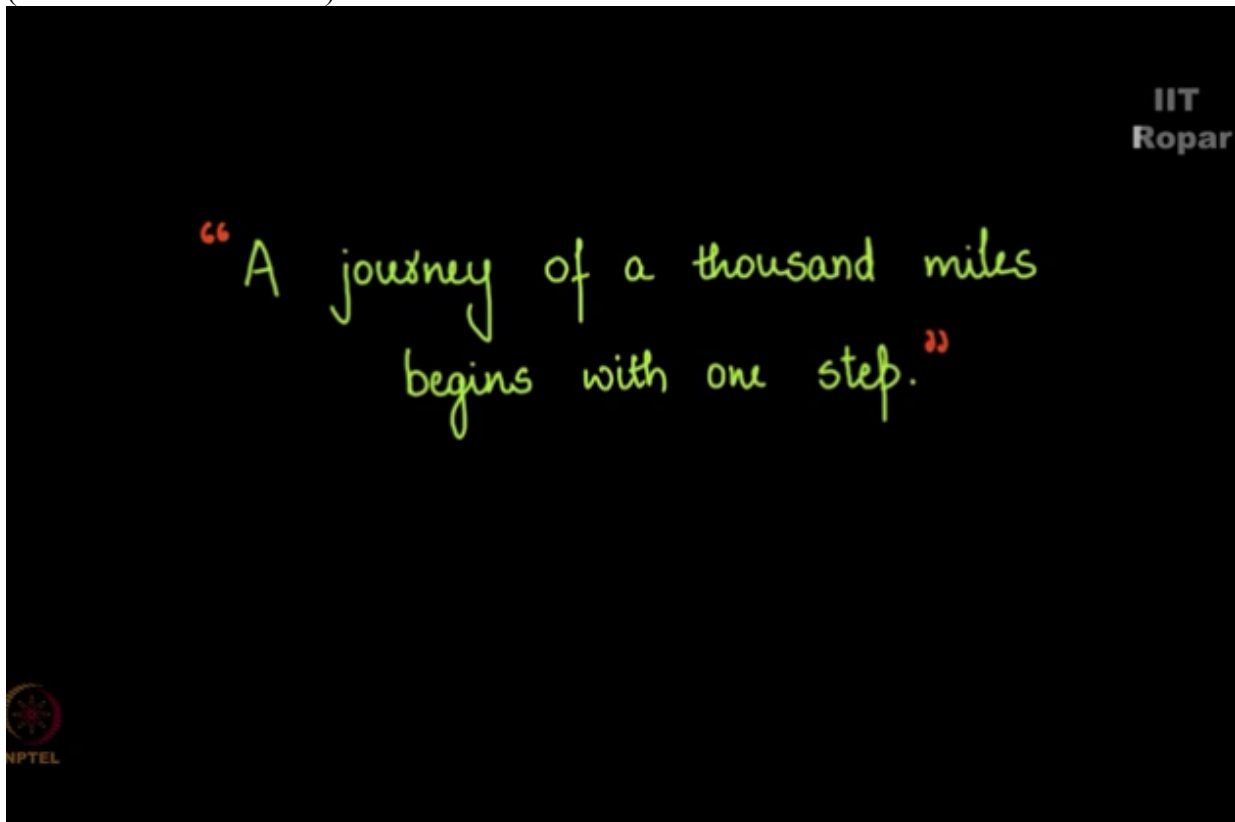
NPTEL ONLINE CERTIFICATION COURSE

**Discrete Mathematics
Recurrence Relation**

Recurrence relation of Tower of Hanoi

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Remember the code that we started from every journey starts with a single step, right,
(Refer Slide Time: 00:10)



so a long journey is basically let's say it's going to take you 10,000 steps to reach your destination you can see this problem as take one step and you must take 9999 more steps,
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10000 steps
1 step + 9999 steps

correct, now the problem of 10,000 boils down to 9999 steps when you have taken the first step, after that when you take another step the problem of going 9999 steps becomes one step less and hence it will become 9998 steps and so on, right, so what am I trying to say? Let the journey take $T(n)$ amount of effort where n is the number of steps that you are going to take, so you can write this as $T(n-1) + 1$,
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Journey : $T(n)$ [↖] number of steps
 $T(n) = T(n-1) + 1$

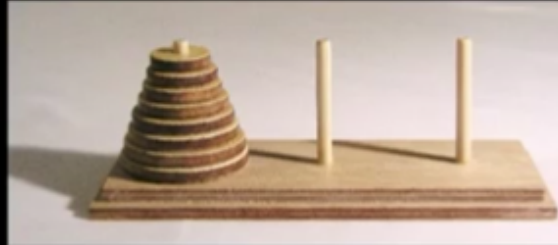


1 is the first step that you have taken and that makes it $n-1$ more steps, so here we write it as $T(n)$ to denote that $T(n)$ is simply n steps that you must take, correct.

Now look at this extended to Towers of Hanoi, so what are we supposed to do? We have n discs in the first stick,
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TOWERS OF HANOI

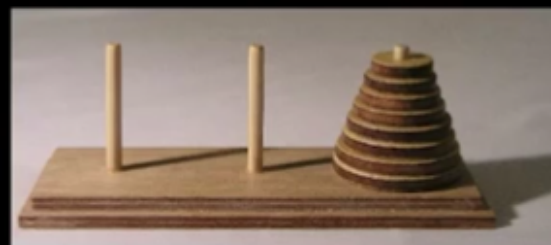
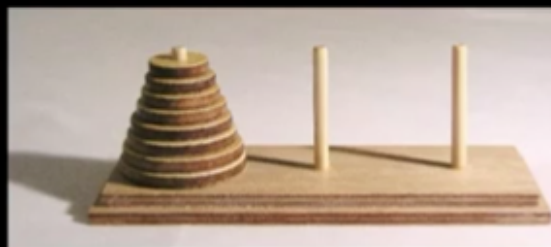
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and this n disc should go to the last stick like this
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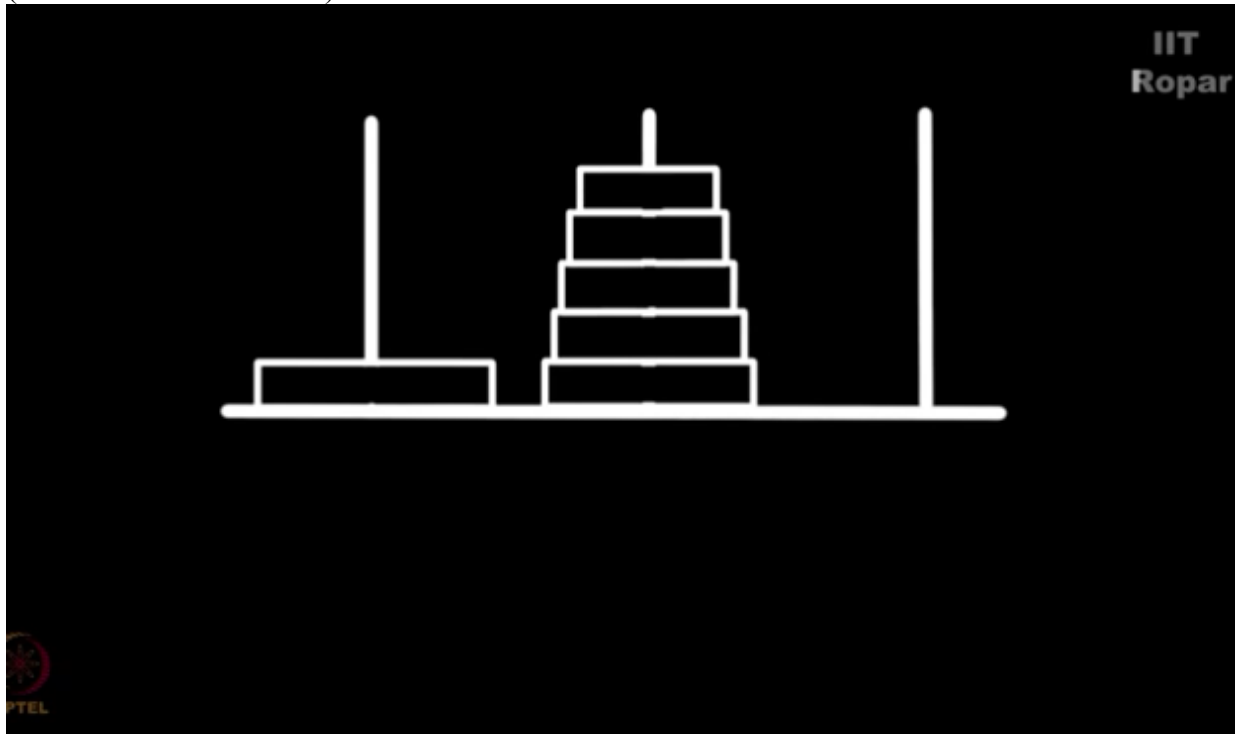
TOWERS OF HANOI

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what I do is I first take the top $n-1$ discs from the first stick and transfer them to the second stick,

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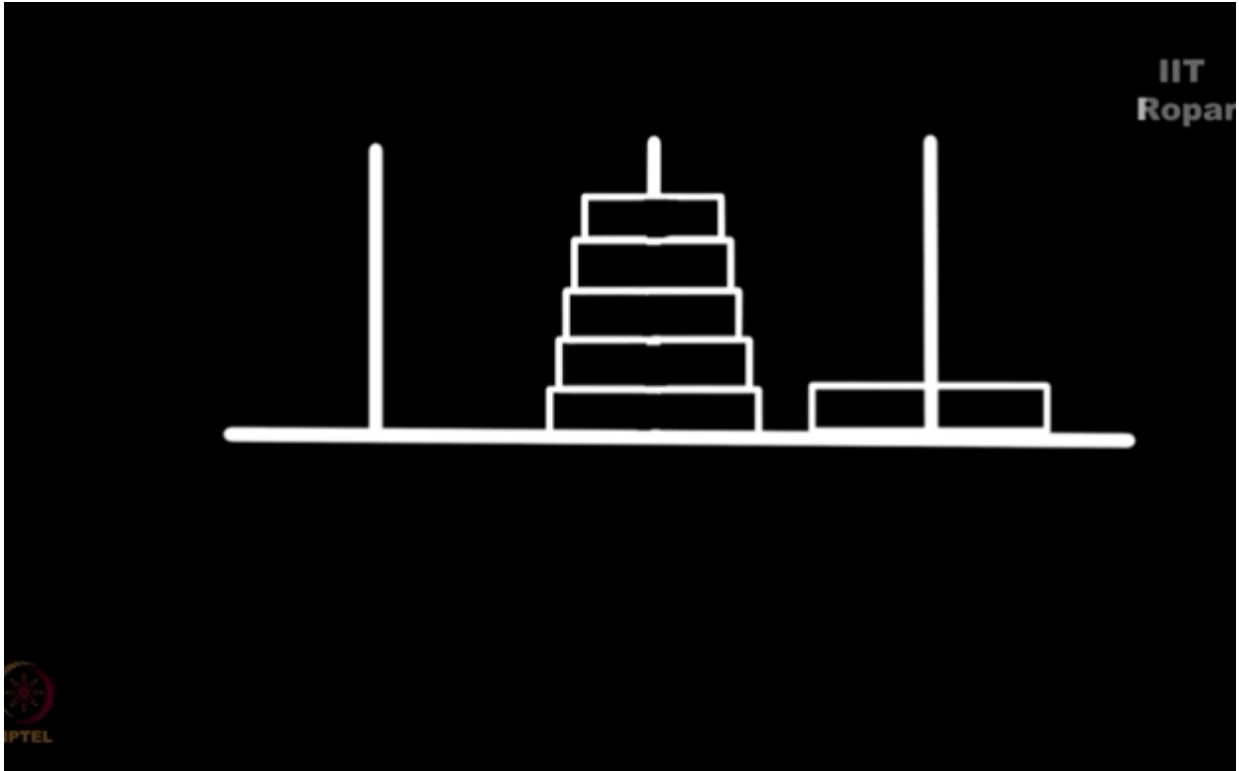
how do I do that? The routine process of however you are planning to do, so that's going to take some effort, that effort let's say is $T(n-1)$ so you see the effort that it takes for you to transfer n discs from stick 1 to stick 3 is equal to the amount of effort takes look at the figure here, (Refer Slide Time: 02:13)



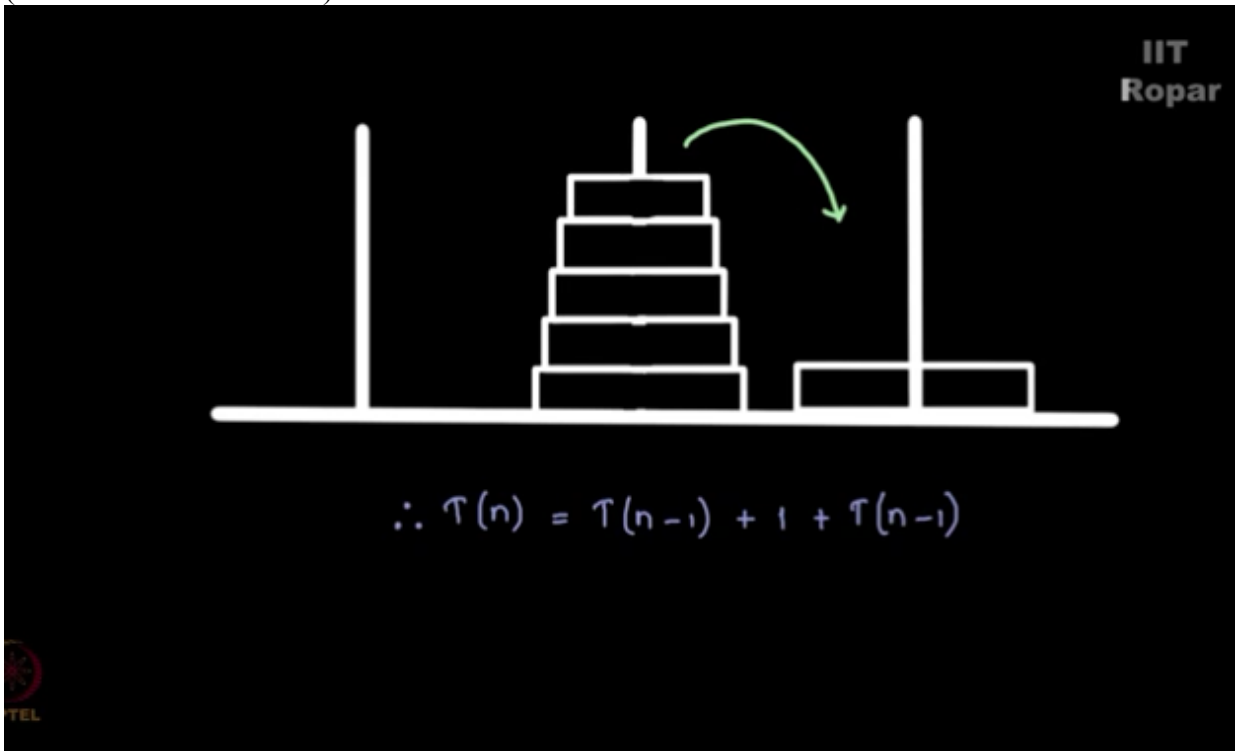
$T(n-1)$ = Amount of effort to transfer $n-1$ discs
to the second stick

the amount of effort it takes to transfer $n-1$ discs from the first stick to the second stick, and then transfer this biggest disc on the bottom in the first stick to the last stick that takes one transaction,

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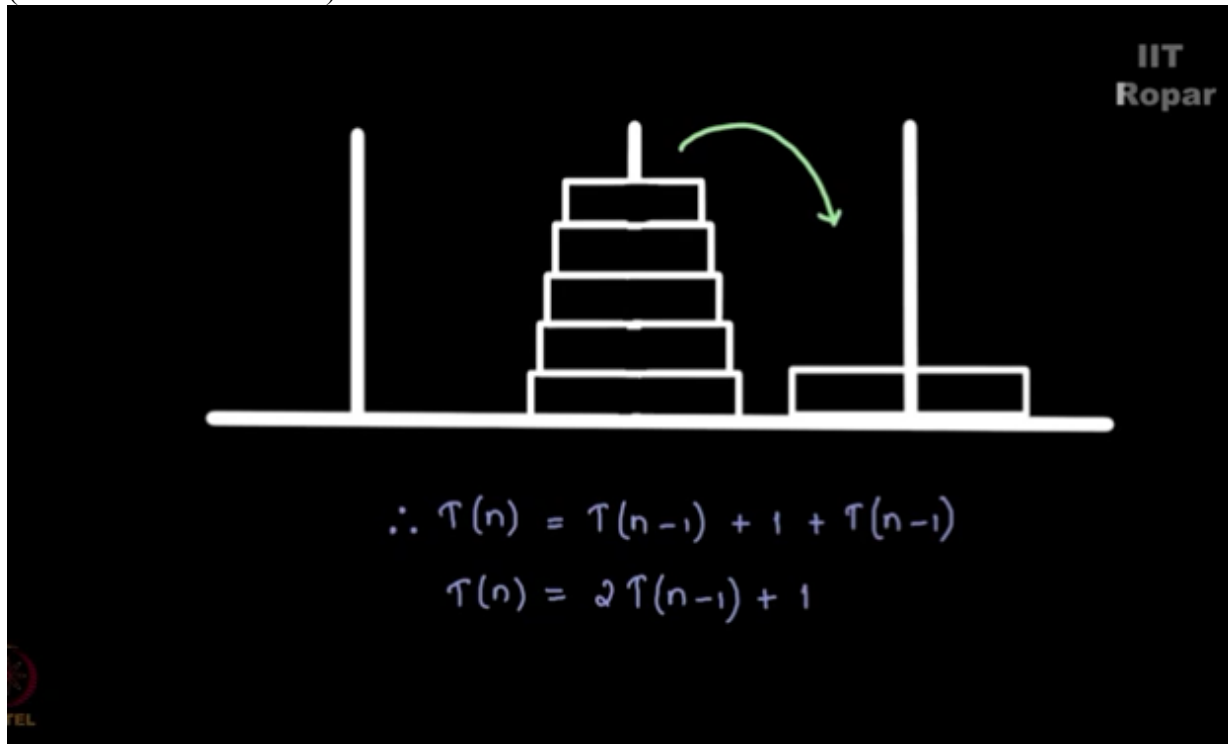


so $T(n) = T(n-1) + 1$ + now these $n-1$ discs in the second stick should go to the last stick, and that's going to take me $T(n-1)$ effort yet again, so $T(n)$ is $T(n-1) + 1 + T(n-1)$
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the last one being moving from the second stick all the n-1 discs to the last stick, so the amount of effort it takes for you to transfer n discs from the first stick to the last stick is 2 times the effort it takes for you to transfer n-1 discs + 1,

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think about it you may want to watch the video once more, this is not at all straight forward, certain things should not be learnt linearly, you may want to repeatedly watch the video, so there are some details here you would have probably missed it in your first attempt to understand this concept go through it, do a double coating you will understand it, I repeat the way in which you can transfer n discs from first stick to the last stick is equal to 2 times the effort it takes to transfer n-1 discs from one stick to the other + 1.

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