NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Tower of Hanoi

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We are now going to talk about an interesting puzzle called the Towers of Hanoi, it is also called the Towers of Brahma
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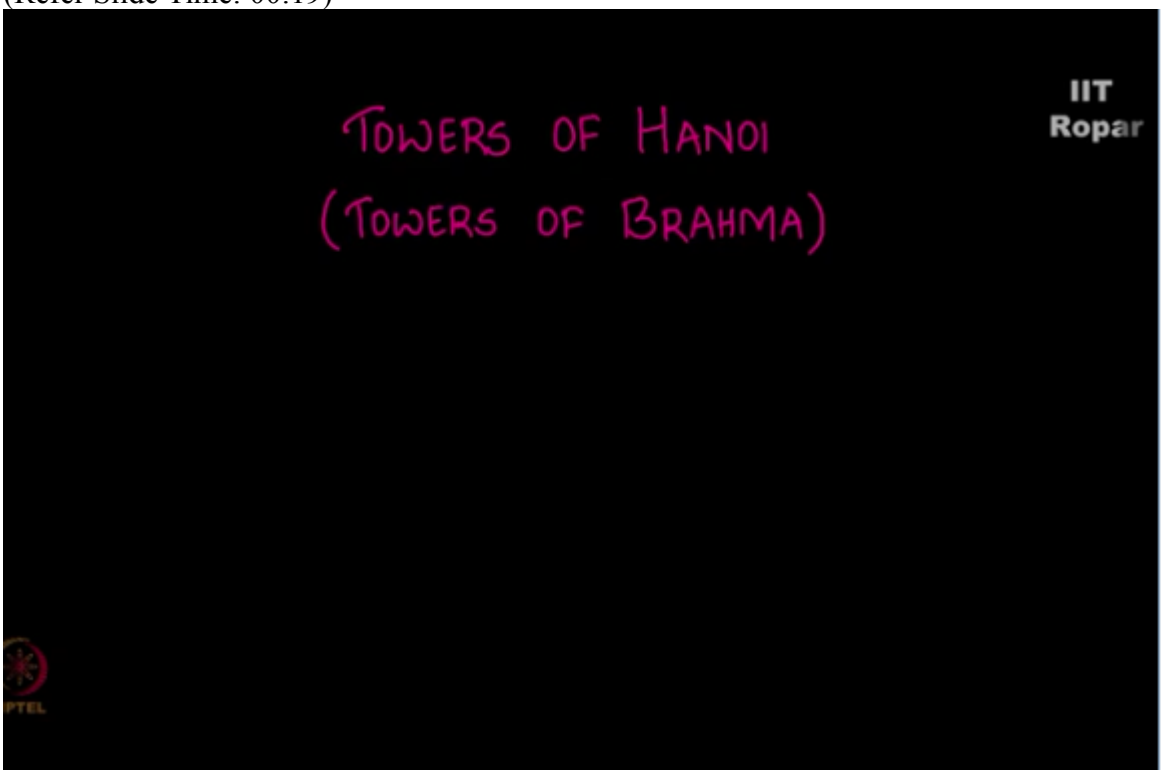

where it is believed that long, long ago there were this bunch of people who were trying to solve this puzzle in this temple of Kashi Vishwanatha in India, so the puzzle goes like this, you have 3 sticks and some 5 discs like this, as you can see in the first needle,
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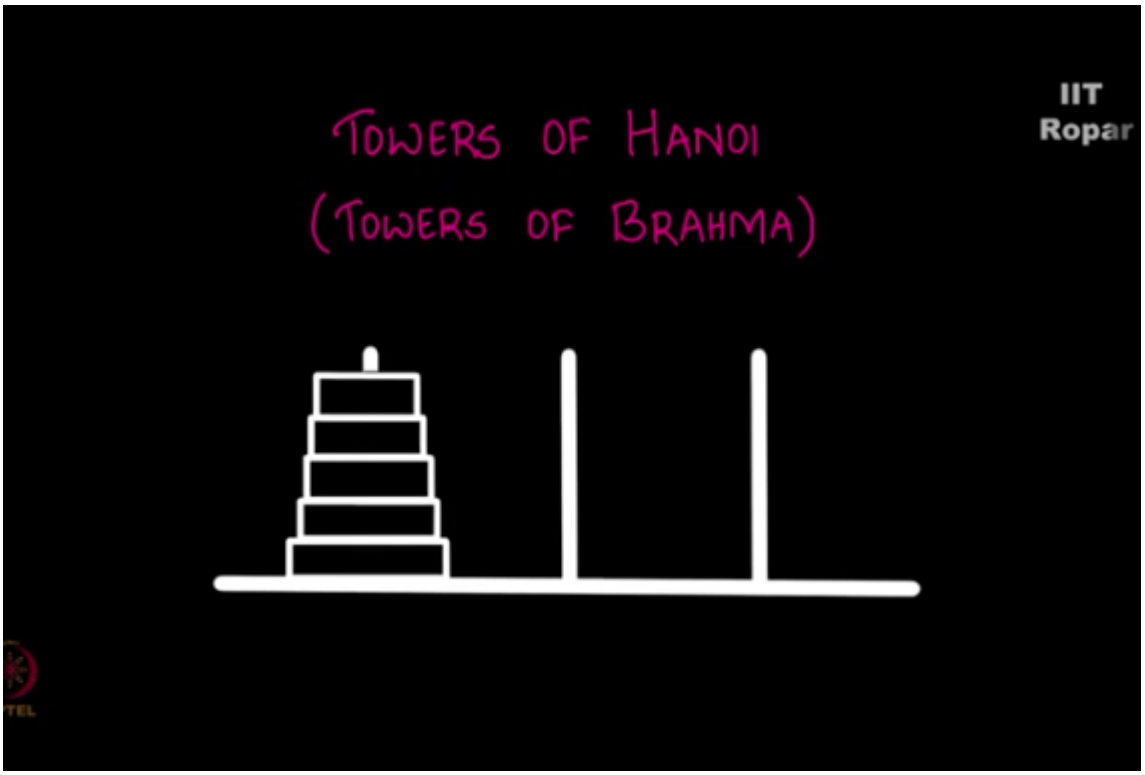

first stick you have 5 discs in the increasing order from top to bottom, and these are circular discs, one on top of the other, the biggest one in the bottom and the smallest one on the top, now what you must do is shift these discs from first stick to the last stick, and it must be shifted in such a way that in the last disk it's again in the ascending order from top to bottom, but you can only move one disk at a time.

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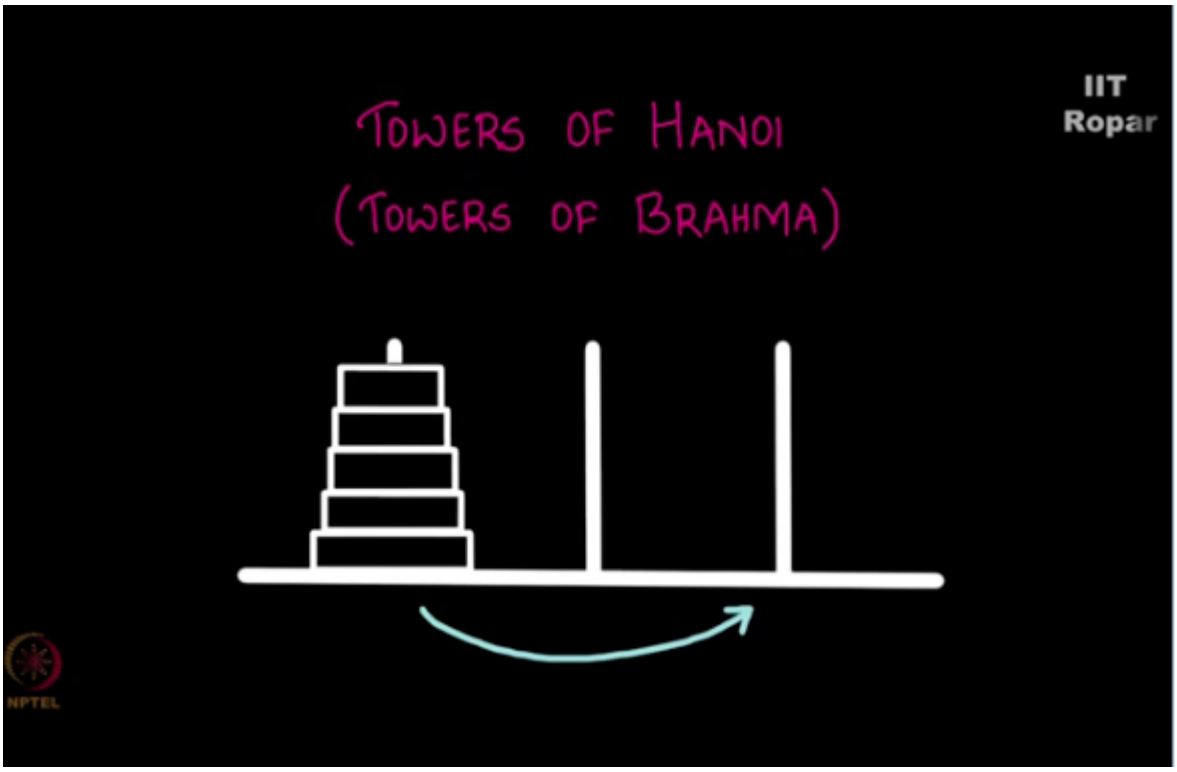

There is one important constraint you cannot put a bigger disk on top of a smaller disk like this,
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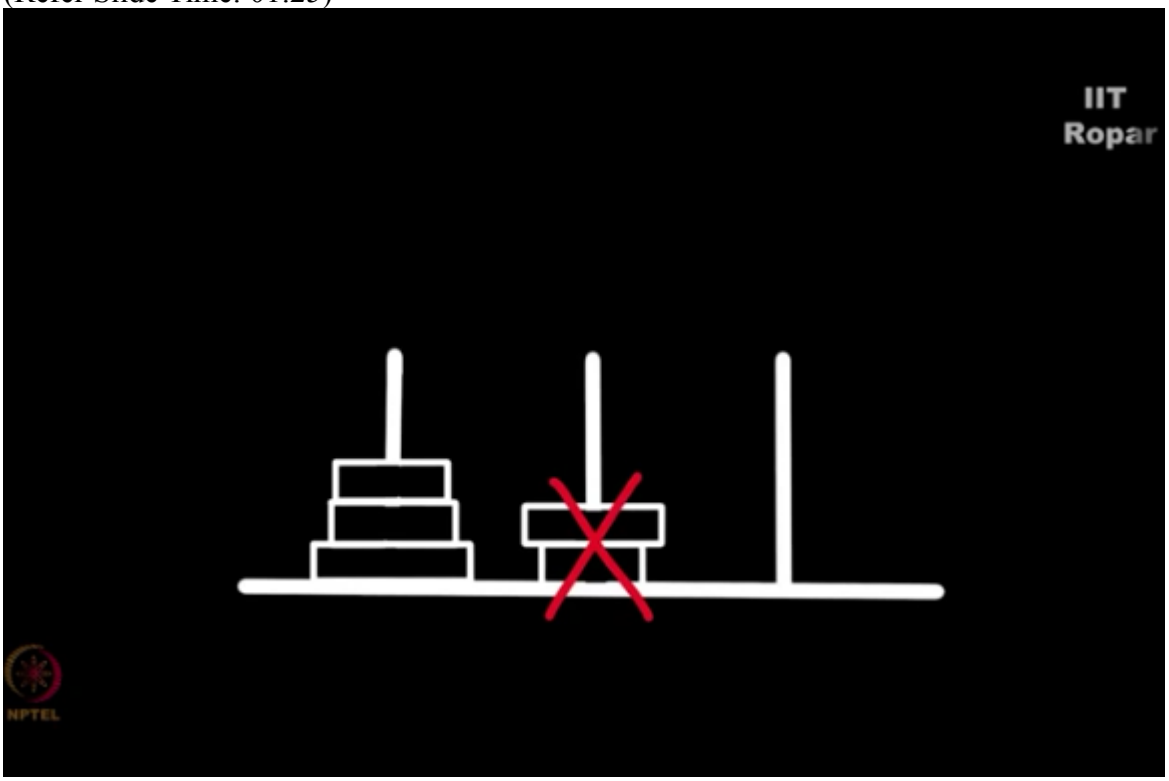

this is not allowed, you can however put a smaller disk on top of a bigger disk like this,
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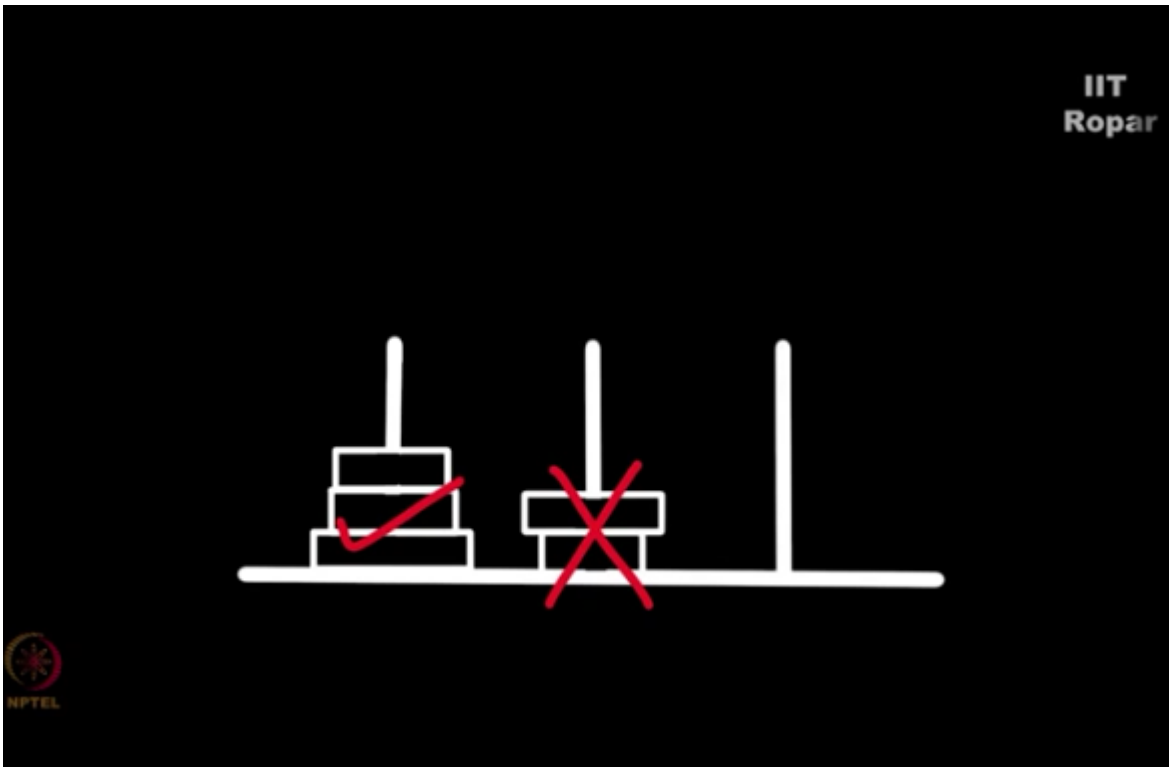

so do whatever you want and you must shift these 5 discs from here to here, this can be done by hook or crook if you can think this takes some time,

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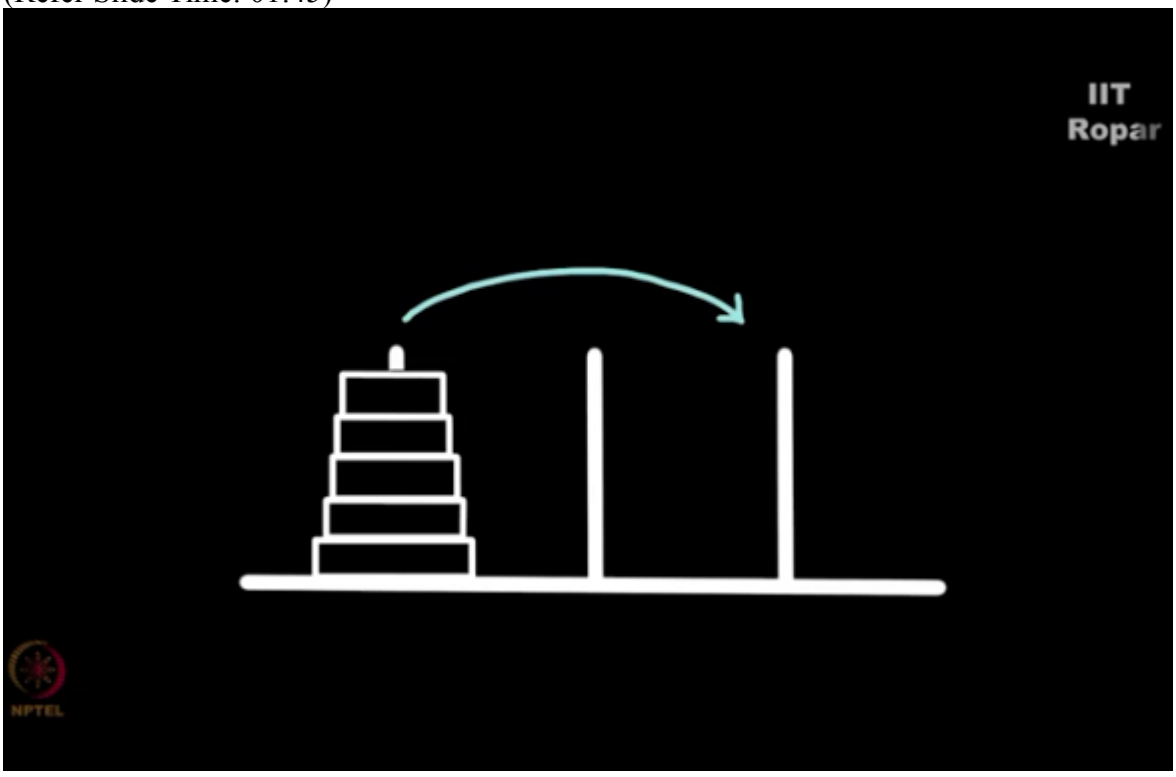

in fact we are going to show that the sticks roughly 2 to the power of 5 operations, by operations I mean you remove one disk put it here, that is called one operation, okay, remove the second disk and put it in the last stick this is the another operation,
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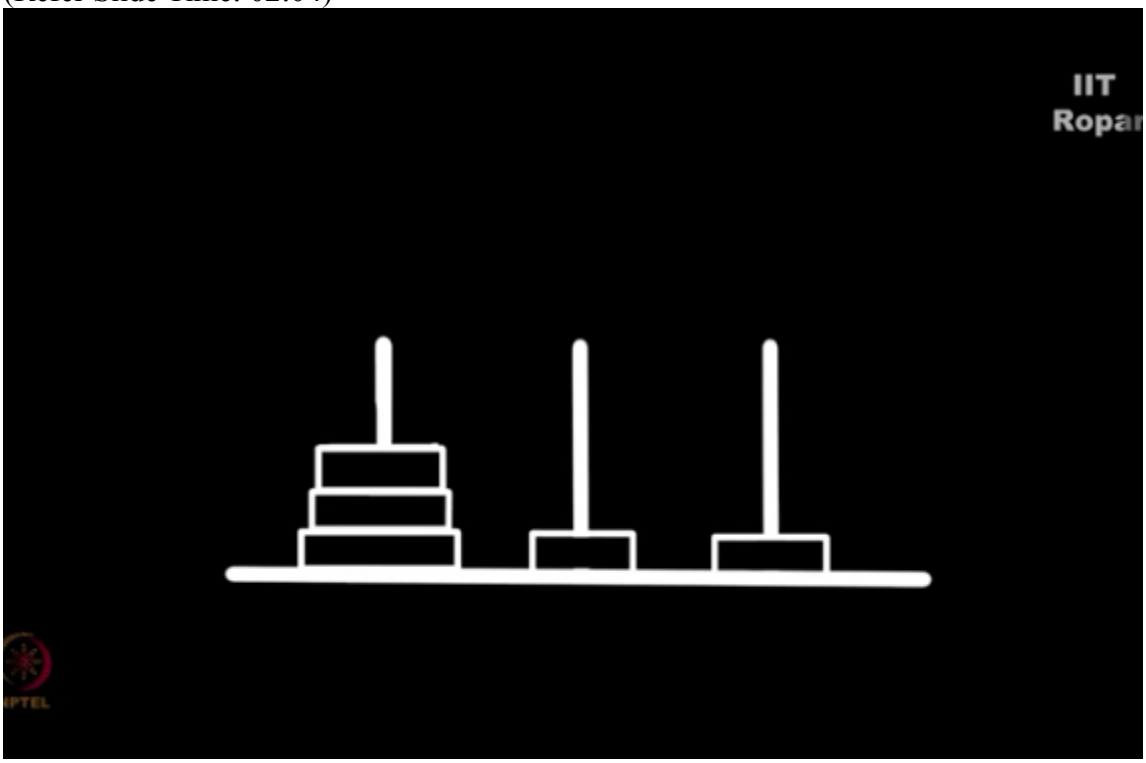

and then put this smallest disk back to the last disk, last stick now you have the first two disk shifted to the last stick,
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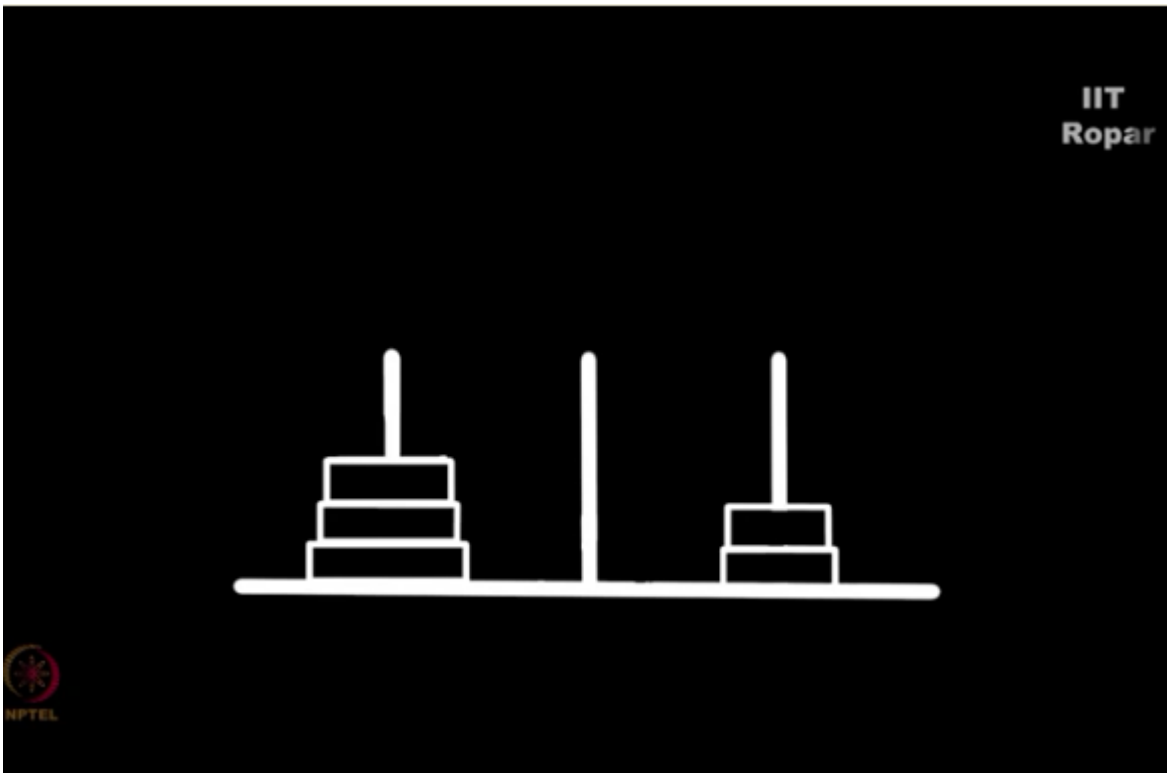

right, and you did this in how many operations? 1, 2, and 3 operations, correct, for you to shift all these discs we are going to show very soon that it's going to take roughly 2 to the power of 5 operations, assuming that you are really fast and you can do it in one second per operation, shifting these discs from the first disk, first stick to the last stick it's going to take you 2 to the power of 5 seconds, which is 32 seconds which is the small time,

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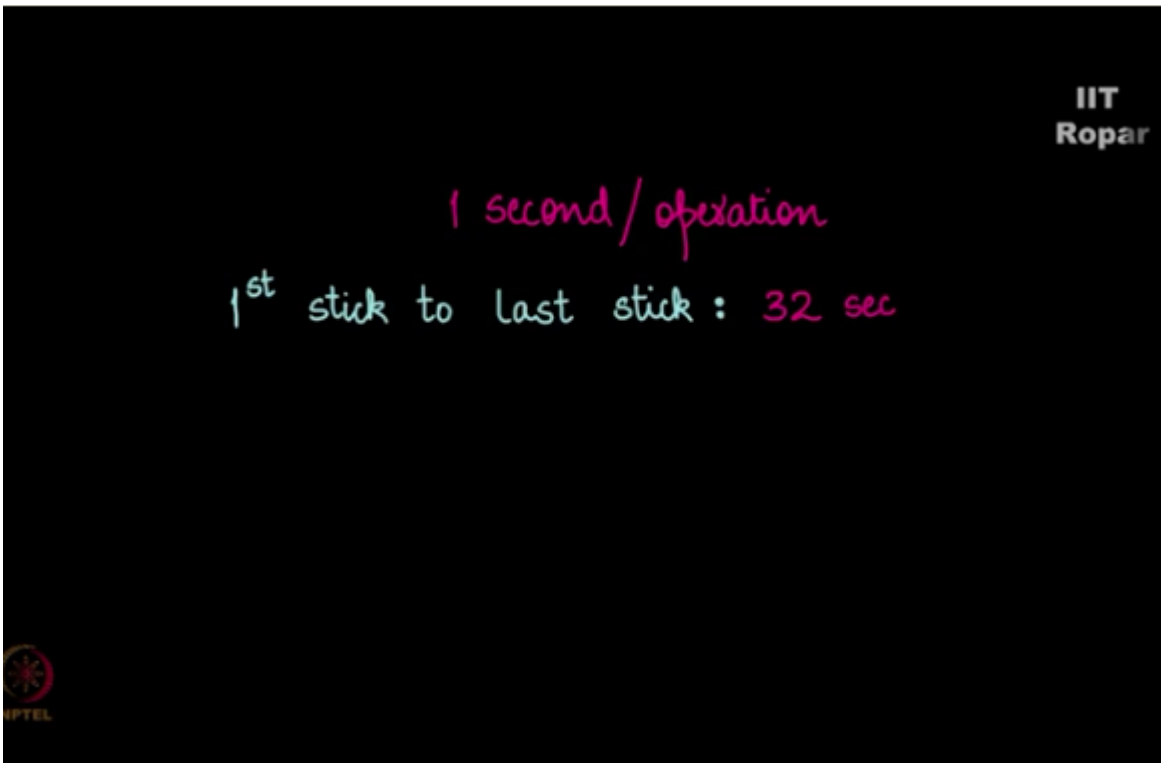

roughly half a minute, but then as I was telling you it is believed that in this Kashi Vishwanatha temple there was this puzzle and people were asked to shift roughly some 64 discs from the first stick to the last stick, which means it would take them 2 to the power of 64 seconds,

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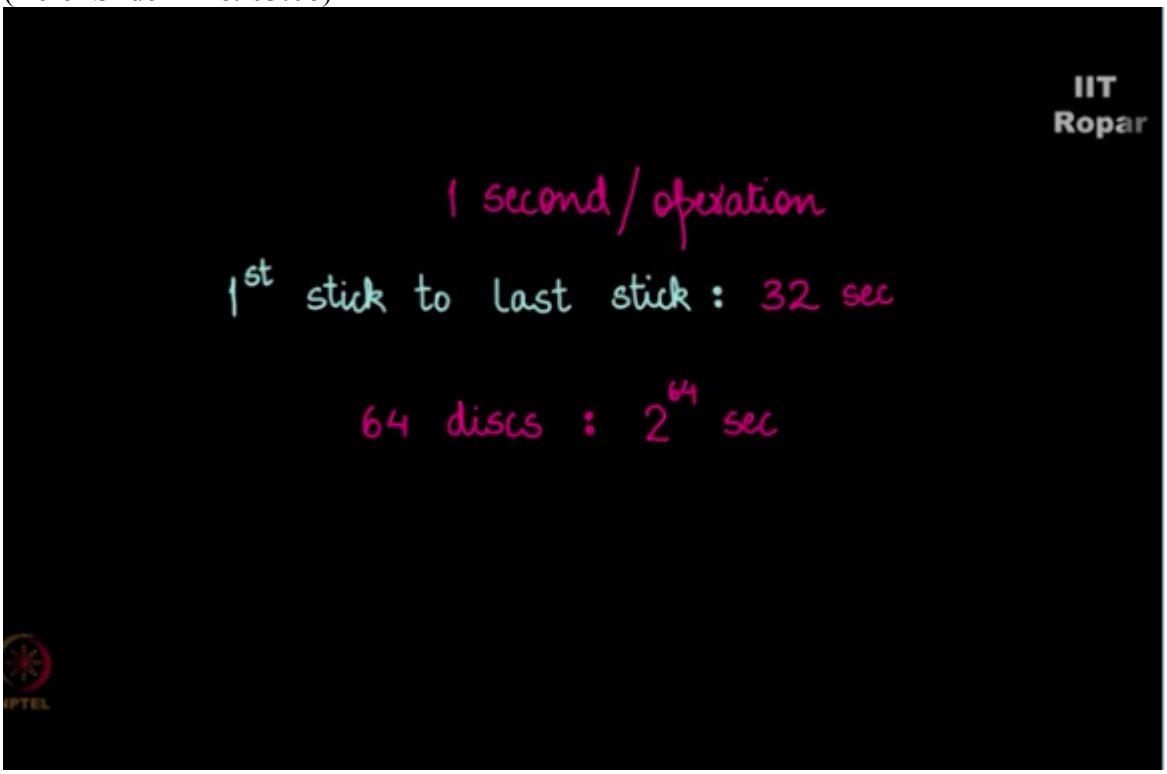

if you remember the chapter counting we did discussed that 2 to the power of 60 is the age of the universe, 2 to the power of 60 seconds that is, now this is going to take more than the age of the universe for you to shift 64 discs from first disk, first stick to the last stick.
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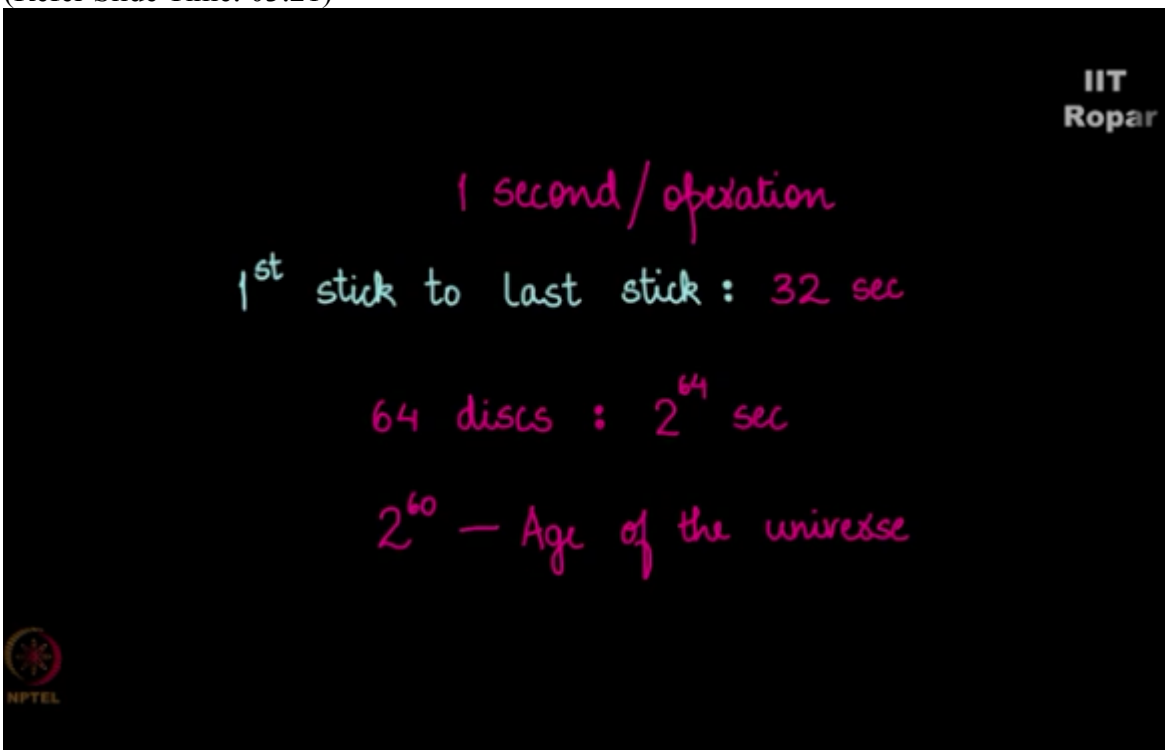

How, why, what? We will see in the fourth coming lessons, well, Towers of Hanoi is one of the best puzzles to explain what one means by a recurrence relation, you may want to wait and watch how this is corrected to this chapter, recurrence relations.

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