

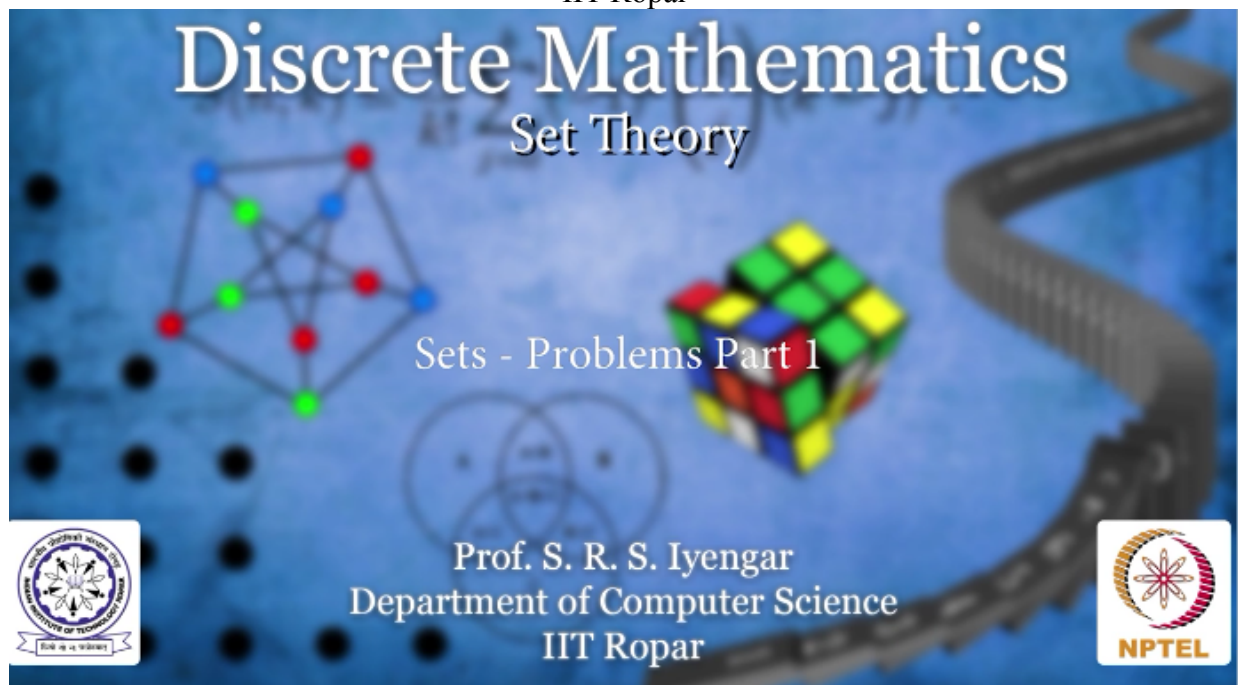
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NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Set Theory

Sets-Problems Part 1

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The notation which we just saw is called the set builder notation, why is it even important to study such notations, you might think that we are we completing things, well actually not, it's a pretty simple notation it will be very clear with the following examples, let us take the set 1, 2, 3, 4, if we write it in the set builder notation it will be nothing but X such that X is less than 5, and X is a positive integer within the curly brackets, how is it same as 1, 2, 3, 4? Let us decode this, this is read as set of all X, the vertical line is such that X is less than 5 and X is a positive integer, so we'll go step by step.

Set Builder Notation

WHY THIS? IIT Ropar

$$\{1, 2, 3, 4\} = \{x \mid x < 5 \text{ and } x \text{ is a positive integer}\}$$

5?

Set of all x Such that

$$5 \not< 5$$

$$6 > 5$$

$$7 > 5$$

The condition says that x is a positive integer, so we'll go from 1, is 1 less than 5? Yes, so 1 belongs to the set, so 1 is an element of the set. Next positive integer is 2, 2 is less than 5, so 2 is also an element of the set, then 3 also satisfies this condition, 4 also satisfies this condition. What about 5? 5 is the positive integer, but is 5 less than 5? Definitely not, because it's equal to 5, what about the remaining elements? What about 6? What about 7? Definitely those are greater than 5, so the set which we constructed using set builder notation is 1, 2, 3, 4 which is equal to this set, this notation is very simpler than writing this one.

$$\underline{\{1, 2, 3, 4\}} = \underline{\{x \mid x < 5 \text{ and } x \text{ is a positive integer}\}}$$

Simple



Well, let me give you another example, what if I ask you to write the set having elements 1 to 10, say its 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, instead of writing this what I'll do is I'll write X such that X is a positive integer and X is less than 11, or you can write X is less than or equal to 10, both are same. So now instead of writing 1 to 10 I'm just writing this one liner, in the curly brackets X such that X is a positive integer and X is less than 11.

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\{x \mid x \text{ is a positive integer and } x < 11\}$$



Now what if I ask you to write a set having elements 1 to 100, the left hand side looks pretty much bigger than the right hand side, because right hand side is just the one liner it's now the same X such that X is a positive integer and X is less than or equal to 100 that's it, this comes very handy now just in these cases, you might encounter a case where you cannot really enumerate elements of the set, give me a set of real numbers between 1 to 100, there are infinitely many elements so you can write this set like this, set X such that 1 is less than or

$$\{1, 2, 3, 4, \dots, 100\} = \{x \mid x \text{ is a positive integer and } x \leq 100\}$$

Set of real numbers from 1 to 100.

$$= \{x \mid 1 \leq x \leq 100, x \text{ belongs to set of all real numbers}\}$$



equal to X , less than or equal to 100, where X belongs to the set of all real numbers, you might have observe something and even might have concluded that a set is a collection of objects having some properties in common between them like in this example, these are all planets, these is a list of prime ministers, and these are days, and these are months, or even here, but that need not be the case, see if I write set R with the elements A the alphabet, 2 the number, NPTEL, New Delhi, yellow, this is a set, there isn't any common property among them, elements in the set need not be connected with common properties, okay, let's move on.

A set is a collection of objects having some properties in common

{Mercury, Venus, Earth, Mars, Jupiter, Saturn,
Uranus, Neptune}

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

{Monday, Tuesday, Wednesday, Thursday,
Friday, Saturday, Sunday}

{Jawaharlal Nehru, Lal Bahadur Shastri, Indira
Gandhi, Morarji Desai, Charan Singh, Rajiv Gandhi,
V.P. Singh, Chandra Shekhar, P.V. Narasimha Rao,
Atal Bihari Vajpayee, H.D. Deve Gowda, I.K. Gujral,
Manmohan Singh, Narendra Modi}

{January, February, March, April, May,
June, July, August, September, October,
November, December}

$R = \{a, 2, \text{NPTEL}, \text{New Delhi}, \text{Yellow}\}$



Elements need not have common properties.



Consider the set B consisting of elements 1, 3, 5, and 7, this one peculiar property about the set that is the order in which you write the elements of the set doesn't really matter, what I mean by


this is if someone gives you a set say 1, 3, 5, 7 and if some other person gives you a set say 1, 7, 5, 3, both the sets are same, because in the set the order in which you write it doesn't matter, so

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$$B = \{1, 3, 5, 7\}$$

Order in which you write the elements of the set doesn't matter.

$$\{\checkmark 1, \checkmark 3, \checkmark 5, \checkmark 7\} \xleftrightarrow{\text{Same}} \{7, 3, 1, 5\}$$

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you can write 1 and then 3 and then 5 and then 7 or you can start from 7 and then write 3 then write 1 and then 5, two sets are equal if they have same elements, $A = B$ whenever the elements of A are same as the elements of B, so here 1, 3, 5, 7 is same as 1, 7, 5, 3, though they are not in the same order the elements in the set are all same.

Two sets are equal if they have the same elements.

$$A = B$$

Elements are same

$$\{1, 3, 5, 7\} \begin{array}{c} \xrightarrow{\text{Same}} \\ \xleftarrow{\text{as}} \end{array} \{1, 7, 5, 3\}$$



Let us see this example, let me write the set as 1, 2, 3, 4, FIVE and then 5 this is A, I have B as 1, 2, 3, 4, 5 are A and B equal? Of course not, because this five which is in word is not there in B though the number 5 is there in B, so this is an element of A which is not in B, A and B are not equal, so even if you get one element which is not there in the other set you can tell

$$A = \{1, 2, 3, 4, \text{five}, 5\} \quad B = \{1, 2, 3, 4, 5\}$$

Is $A = B$?

NO!

A and B are not equal.



immediately that those sets are not equal, so precisely a set means it's a collection of unordered objects called elements also in the set repeated elements are counted once, let us take the set 1,

A set is a collection of unordered objects
called elements.

Repeated elements are counted once.



1, 3, this is same as 1, 3 because in sets repeated elements are not allowed, so this 1, 3 is same as 1, 1, 3, 3, 3, 3, what about in this set? 1, 2, two there is no repetition as you can see this two is different from this 2, TWO 2 is different from the number 2, so there are 3 elements in this set, the number 3 is called as the cardinality of the set, this is nothing but the size of the set, so 1, 1, 3 is same as 1, 3, so the cardinality of this set is 2, while the cardinality of this set is 3.


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$$\{1, 1, 3\} \xrightarrow{\text{Same as}} \{1, 3\} \xleftarrow{\text{Same as}} \{1, 1, 3, 3, 3, 3\}$$

$$\{1, \textcircled{2}, \textcircled{\text{two}}\} \longrightarrow \textcircled{3} \text{ elements}$$

↓
Cardinality (Size)

$\{1, 1, 3\}$ is same as $\{1, 3\}$

$$|\{1, 1, 3\}| = 2 \quad |\{1, 2, \text{two}\}| = 3$$


So till now we have seen what is called as set, what's the element, this notation, the set builder

- *) Set
- *) Element
- *) Set - Builder Notation
- *) Equality of sets



notation, and the equality of sets. I hope till now is very clear, so if something is there which you didn't understand, feel free to ask in the discussion forum, will be there to help and guide you.

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