NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Solving Linear Recurrence Relations - A theorem

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Look at this recurrence relation AN = AN-1 + 2 times AN-2, my question is assume A naught is 2, A1 is 7, okay, you start with 2 and 7, you can compute what is A2? What is A3? What is A4? But my question in general is do you have a closed form for this, what is the word closed form mean?
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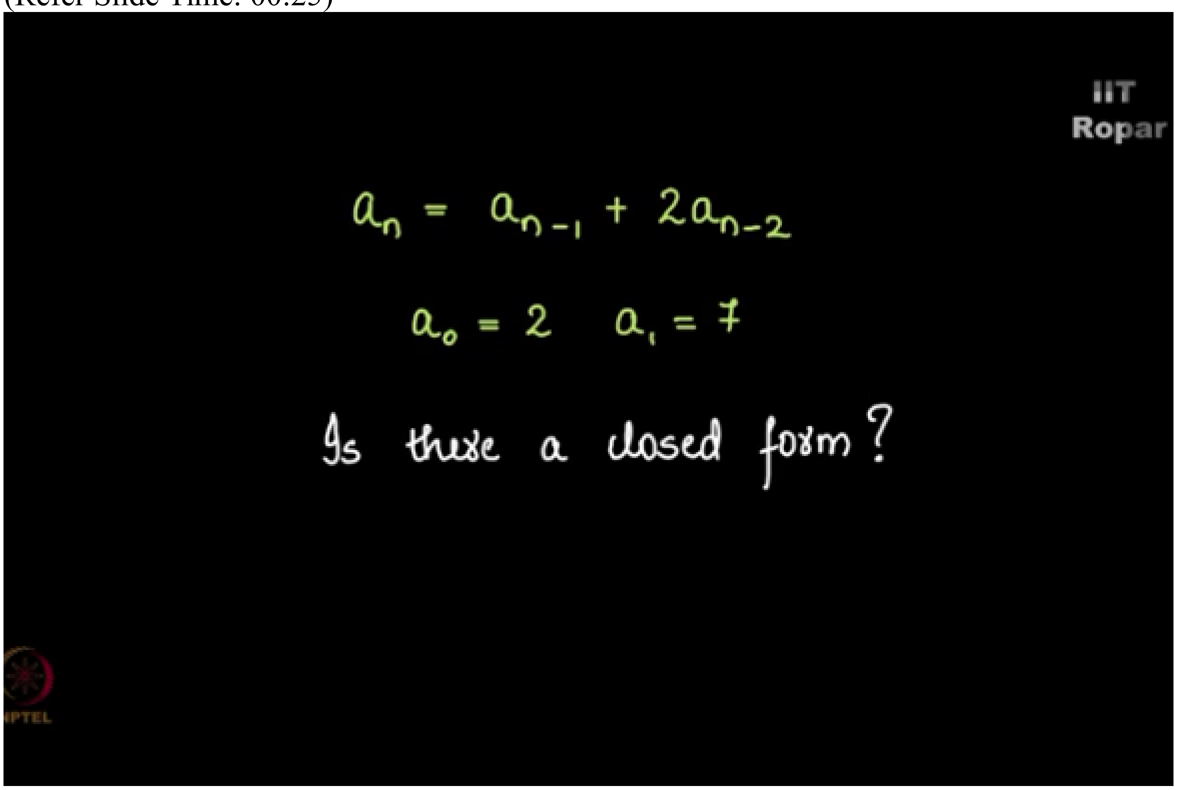

You saw this right? If AN is 1.1 times AN-1 give the initial condition that A naught = 1000, if you remember right we could find a closed form for AN, either example that we discussed a while before, right.

Similarly when AN is AN-1 + 2 times AN-2 with A naught being 2 and A1 being 7,
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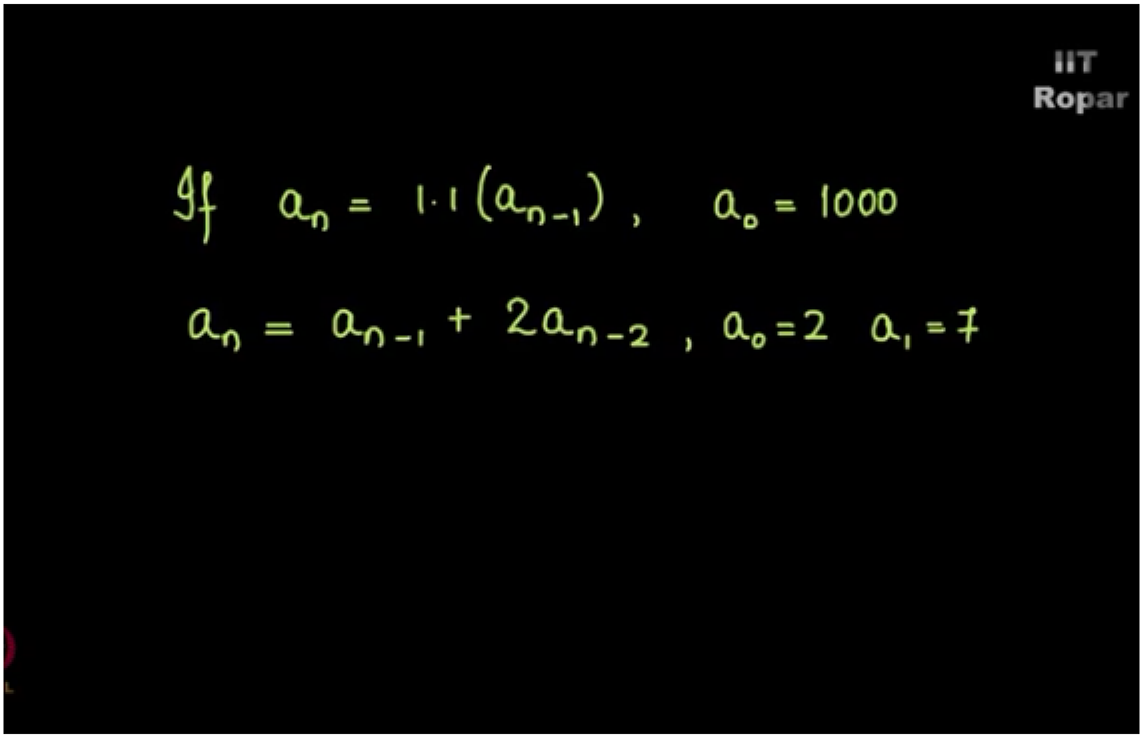

can you give a closed form a formula like thing for AN, the question can actually be generalized, what do I mean by that? By that I mean if AN is given to be some constant times AN-1 + another constant times AN-2 this constraints were 1 and 2 here in the problems case, but in general if I ask this question if AN is C1 times AN-1 some constant C1 + some constant C2 times AN-2 then what can you say about AN? There's a very neat way to solve such problems, and the method goes like this, look at this C1 and C2 and construct a quadratic equation with C1 and C2 like this,

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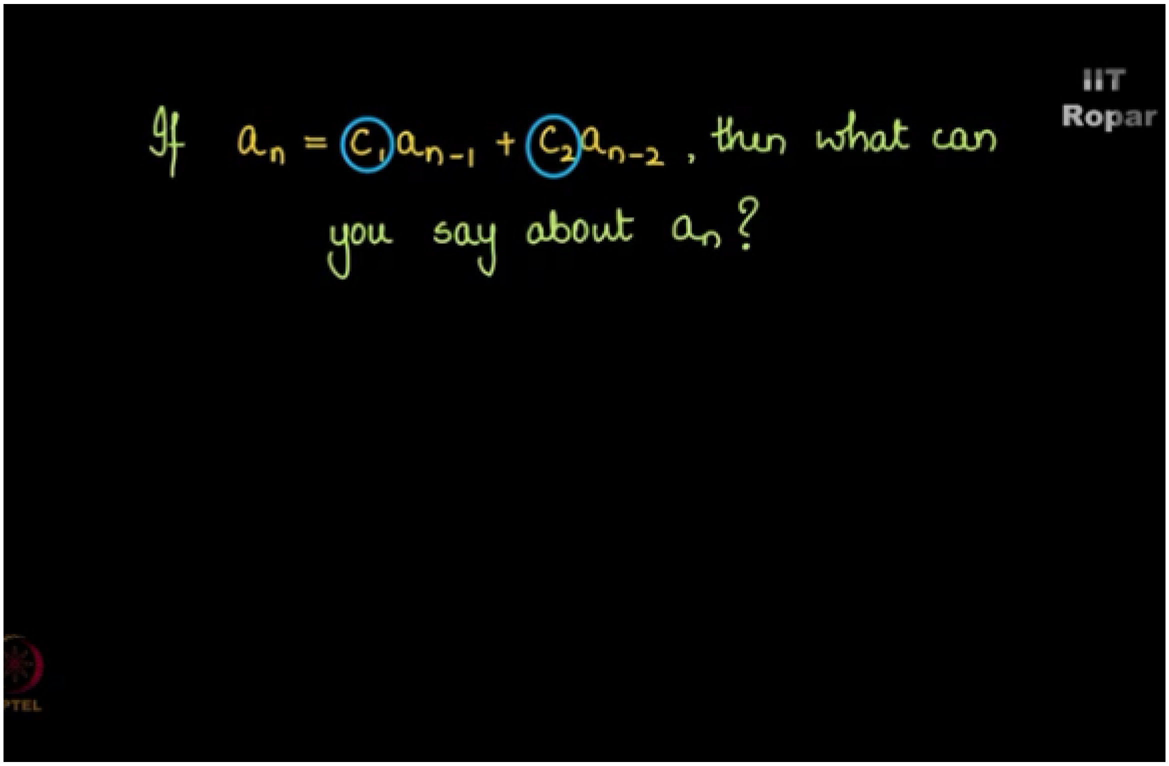

write down this correlation X square - C1X - C2 = 0, and solve this quadratic equation, okay, and you will get let's say 2 roots X1 and X2, and assuming that these two roots are real and are distinct, that things are getting very complicated, but don't worry you will understand once we write an example and then see it,

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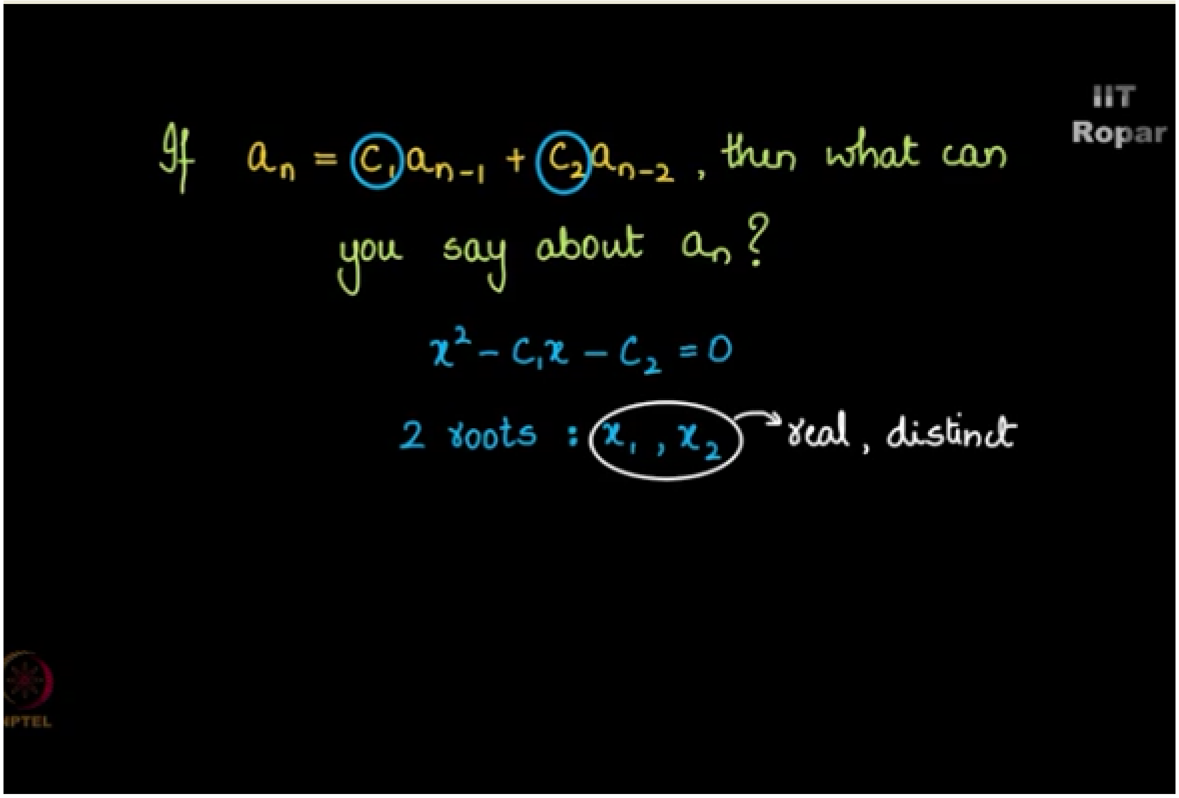

the story so far is if AN is C1, AN-1 + C2 AN-2, simply pluck out that C1 and C2 and create this quadratic equation X square - C1X - C2 = 0 and find the roots of this quadratic equation, and call these roots X1 and X2.

The solution will always be of the form AN= alpha 1 some constant alpha 1 times X1 to the N, X1 being one of the roots + alpha 2 times X2 to the N, this is the closed form,
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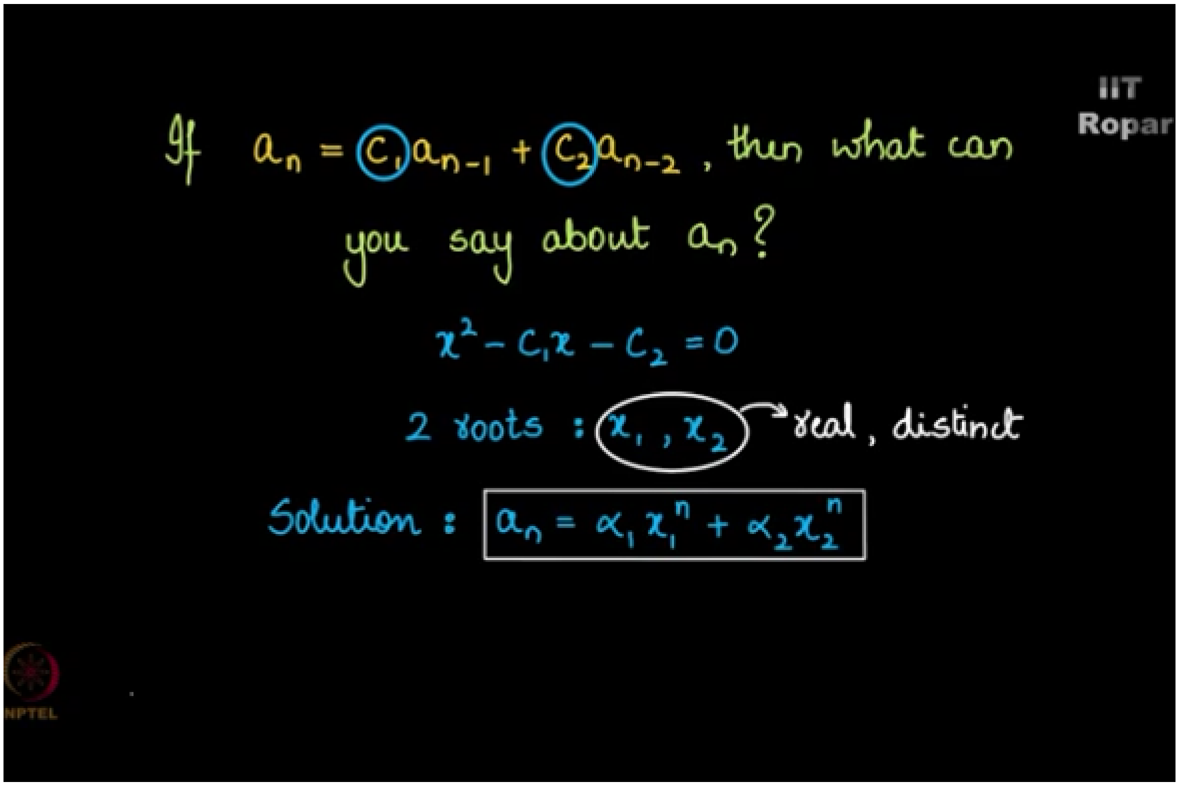

this is the formula for AN, okay, alpha 1, alpha 2 are constants and X1, X2 are the roots of this quadratic equation, this is always true.

Let us try to see this in action for this question, I don't know whether this theorem is really true or not, let us try applying this theorem on this small problem see how it works and then go ahead and prove the theorem, let's recollect the problem AN = AN-1 + 2 times AN-2 with A naught being 2 and A1 being 7, let us write down the quadratic equation, so pluck out the 1 and 2 here the constants that are sitting in front of AN-1 and AN-2 you get the quadratic equation X square - 1 times X-2 is 0, solve this plug in the quadratic equation formula -B + or - square root of B square -4AC divided by 2A, putting all the values you will get X = 1 + or - square root of 1 - 4 times - 2 whole by 2, 2 times 1 rather, which is equal to 1 + or – 3/2 which will give you 4 and -1 as the roots.
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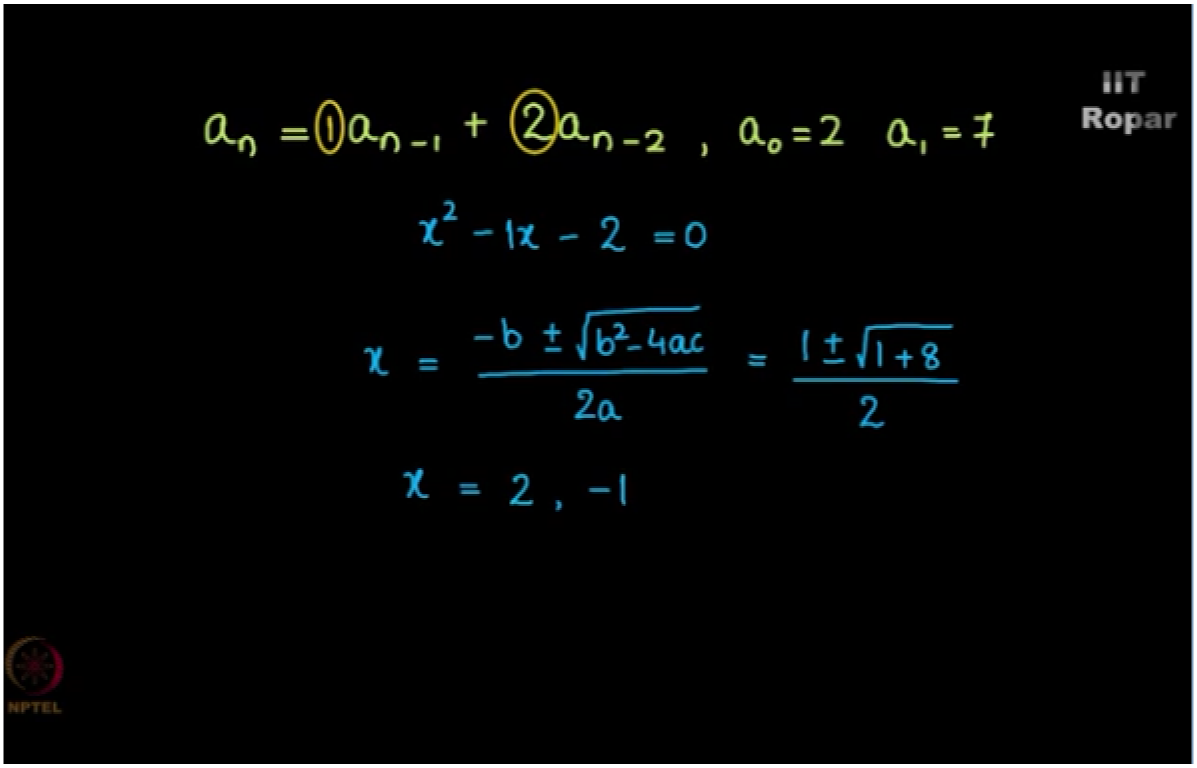

Now going by the theorem that we stated the answer would be, the general solution, I mean the closed formula would be AN is equal to some constant alpha 1 times the first root 2 to the power of N + alpha 2 times the second root -1 to the power of N, the roots are 2 and -1, correct.
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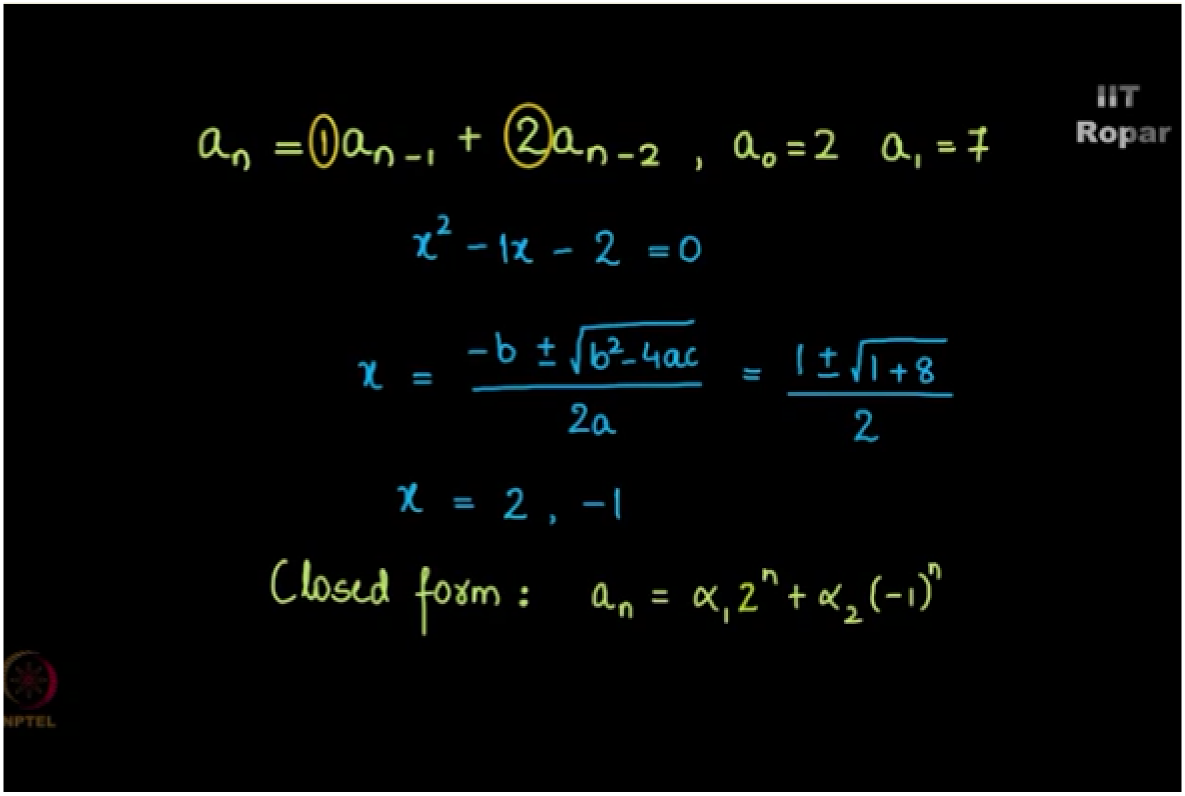

Now what are these alpha ones doing here what are those constants, now look at this we have A naught = 2 and A1 = 7 plugging that in you can find out what is alpha, how do we do that? A naught is 2 and 2 will be equal to alpha 1 + alpha 2 whenever you plug in N = 0, correct, 2 = alpha 1 + alpha 2, similarly A1 = 7 which is equal to alpha 1 times 2 + alpha 2 times -1, so solving this you will get alpha 1 = 3, and alpha 2 = -1, so you now know what is alpha 1 and alpha 2 plug that in, you have the solution that you want, so AN is going to be equal to in place of alpha 1 plug in 3, 3 times 2 to the N-1 to the power of N.

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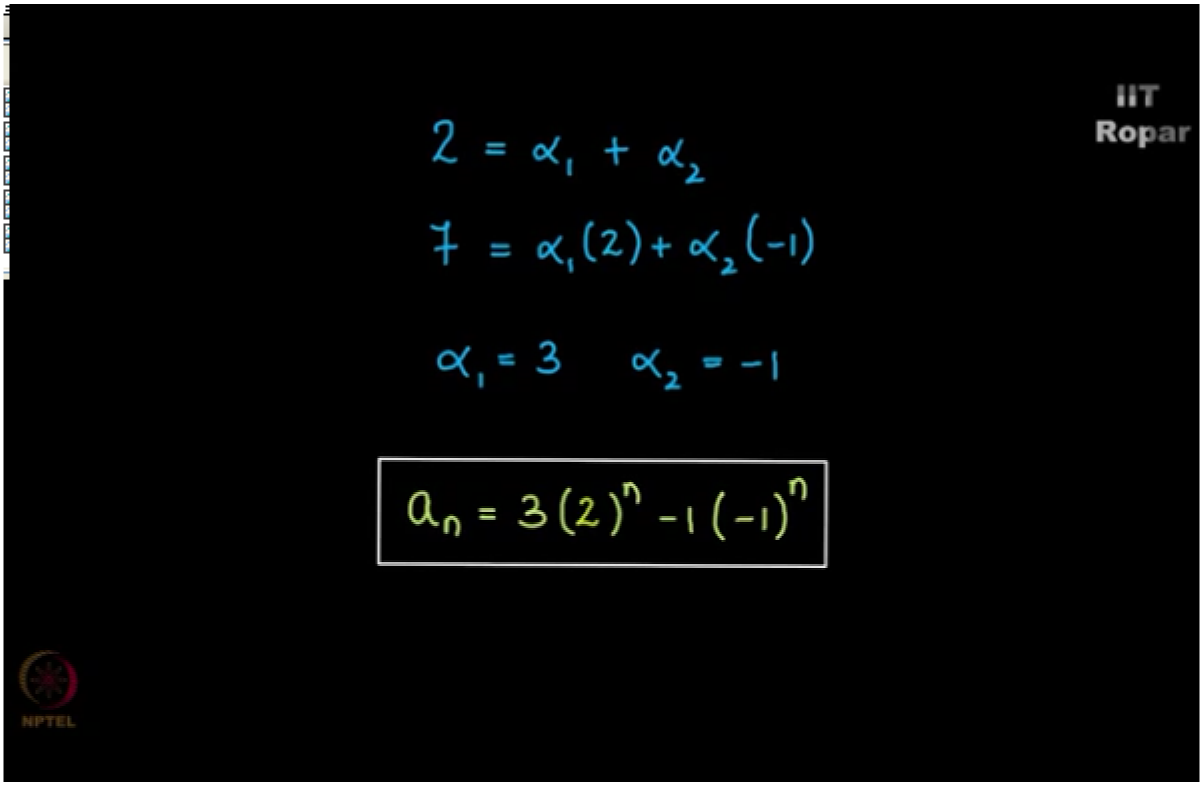

Now look at the power of this theorem it helped you solve the given recurrence relation and it helped you find a closed formula AN = 3 times 2 to the N-1 to the N is such that you plug in N equals whatever value you will get the answer immediately, it is not in the recurrence relation form you have solved the recurrence relation, how did you solve it? You employed the technique that the theorem suggested, but how do you know that the theorem is true, let us now go ahead and prove the theorem and let us understand how this theorem is in general true always.

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