NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Example - Rabbits on an island

By
Prof. S.R.S Iyengar
Department of Computer Science
IIT Ropar

Consider this problem, a young of pair of rabbits are placed on an island, they do not breed until they are 2 months old, after they are 2 months old each pair of rabbits give rise to another pair of rabbits, and this happens every month.

Find the recurrence relation for the number of rabbits on the island after n months,
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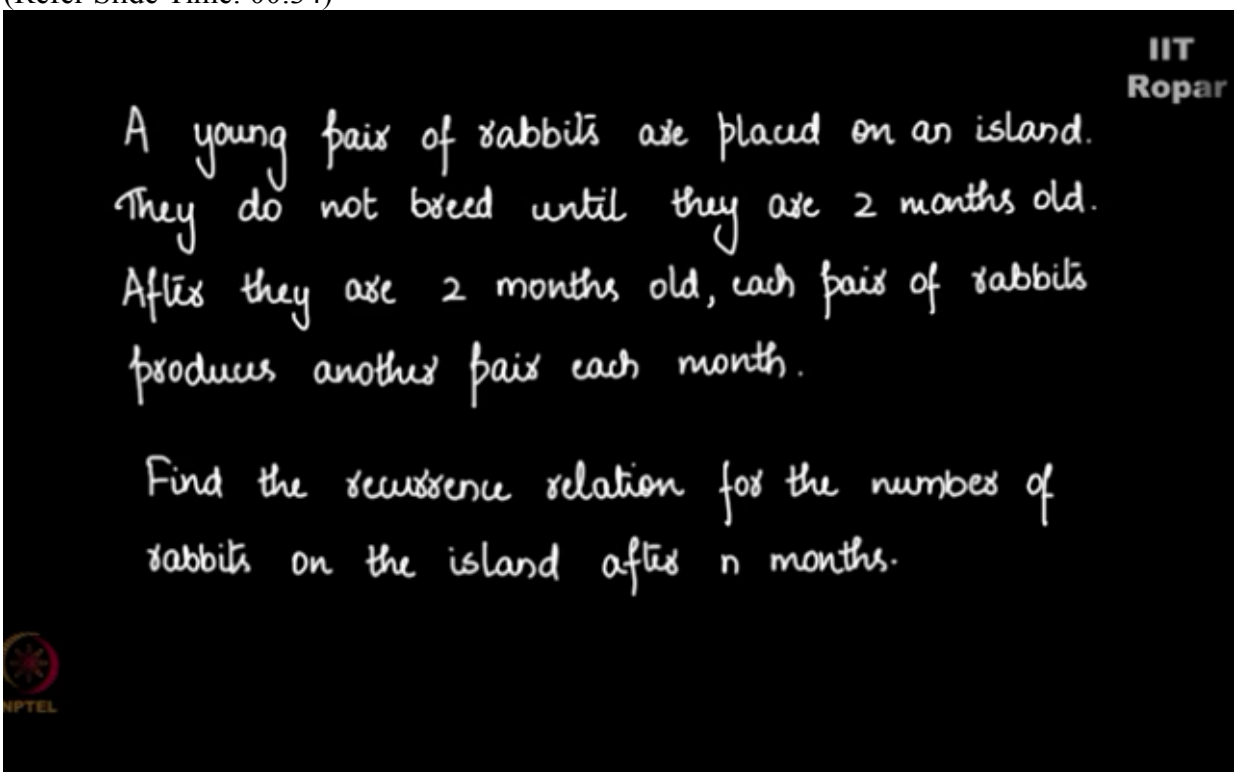

let us understand the question first, there are a young pair of rabbits and they do not reproduce until they are 2 months old, and once they are 2 months old they reproduce each month, they give rise to 2 rabbits that is each month they give, they reproduce a pair of rabbits.

Now we have to find the recurrence relation for the number of rabbits after n months, supposing R1 represents rabbits in the beginning, I'll write rabbits again R1 represents the pair of rabbits, please note it is pair not R1, it's not representing one single rabbit, it's a pair of rabbits, R1 again remains the same because it's just one month old and there are no new pair of rabbits, now one month is over, at the end of two months going to the second month R1 is as such and it gives rise toward produces another pair of rabbits and I'm going to write it as R2.

Now in the next month we have R1, R2, you see R1 will again give rise to another pair of rabbits, but R2 will not, because R2 is just one month old now, right, these rabbits are one month old and hence they do not reproduce another pair of rabbits while R1 will, therefore I'll write it as R3.

Now in the next month what happens is R1, R2, R3 they are existing as earlier, R1 as an always will reproduce R4 because it has crossed 2 months, now you see R2 has finished its second month, therefore it will reproduce to R5, but R3 will not, therefore at the end of 4 months we have R1, R2, R3, R4, R5 which is 5 pairs of rabbits, what will happen in the next month? At the end of next month we see that these pairs R1, R2, R3, R4, R5 will remain as such, right, they're already existing, and which are the new born, who will give birth to new born rabbits? R1 will definitely give birth that is we have R6, R2 will also give birth to another pair so we have R7 and did you observe that R3 has just completed 2 months and hence R3 will also give birth to new pair of rabbits and therefore I'll write it as R8.

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Now R4 and R5 have just finished one month each, and therefore they do not reproduce, so at the end of 5 months we have 1, 2, 3, 4, 5, 6, 7, 8 pairs of rabbits, now how can I write a recurrence relation of this model let me check, let FN represent the number of pairs of rabbits

after n months, so I am going to write the number of pairs of rabbits after n months as FN, now I can write the recurrence relation for this model as FN = FN-1 + FN-2, now how did I get this? Do you see that at the month n, at the nth month we have some pairs of rabbits, right, and where did they come from? From the previous month, right, so FN-1 contributes some pairs of rabbits to FN, we have some more in FN, where did they come from? They have come from those pairs which have at least cross 2 months, right, they're at least 2 months and the extra pairs come from them.

Now the initial condition is F(1) is 1 that is at the first month we have one pair of rabbit, F(2) is again 1 because they have just crossed 2 months and therefore there is just 1 rabbit, 1 pair, and therefore the last one is N greater than or equal to 3, right, we are considering for more than 3, N greater than or equal to 3.

Now FN – 1 this represents the number of pairs in the previous month, right, if you are considering the number of pairs of rabbits in the nth month, after nth month FN-1 will be number of pairs of rabbits in the previous month and FN-2 will be number of pair of, number of new born pairs, right, so combining these two we will be able to find out the number of pairs of rabbits after n months, so this is the recurrence relation FN = FN-1 + FN-2, you might want to watch the video once again by referring to the previous step where I have shown you at various months what are the number of pairs of rabbits.

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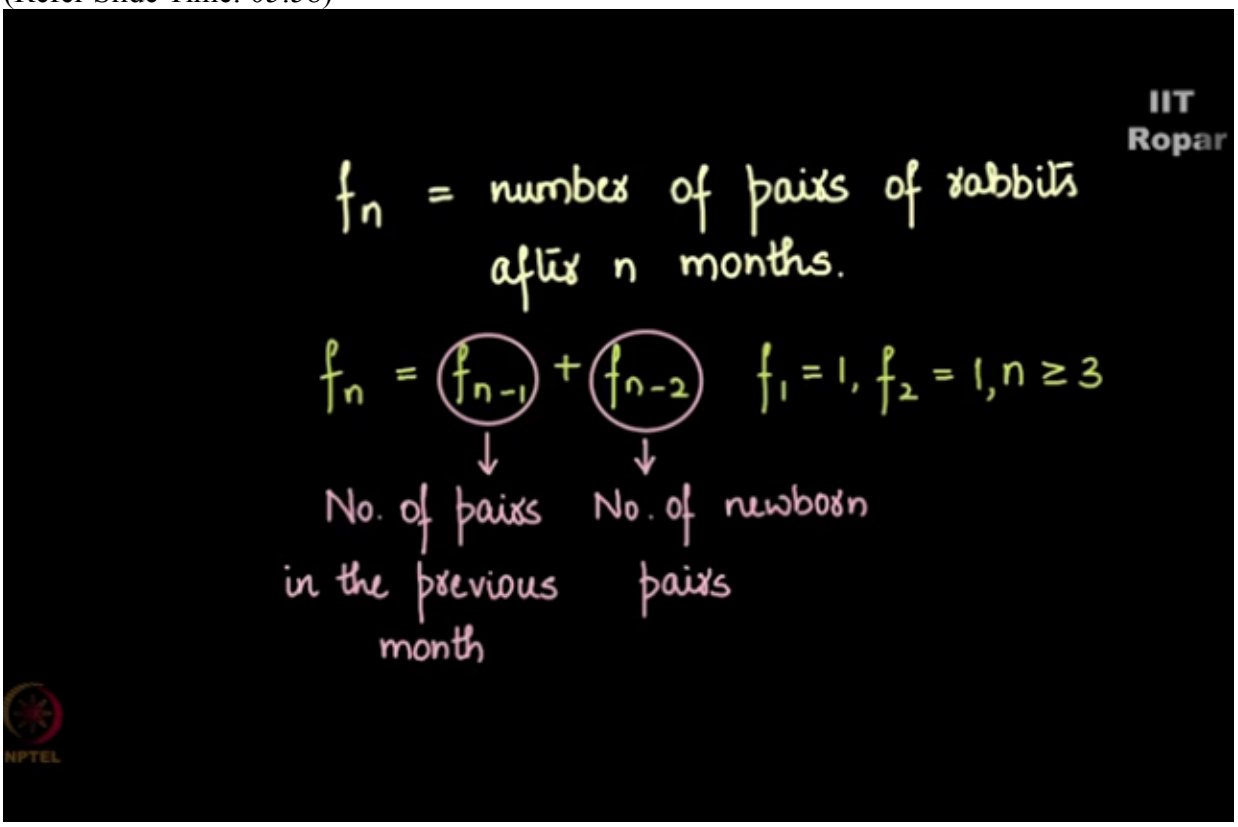

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