NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Examples of recurrence relations

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Now that the professor has introduced you to recurrence relation and he gave the definition of it, let us see an example for clear understanding. The example goes like this, let AN be a sequence that satisfies the recurrence relation AN = AN-1 -2AN-2, so this is the relation for N = 2, 3, 4 and so on.

Suppose A naught is 3, and A1 is 5 then what are A2 and A3?
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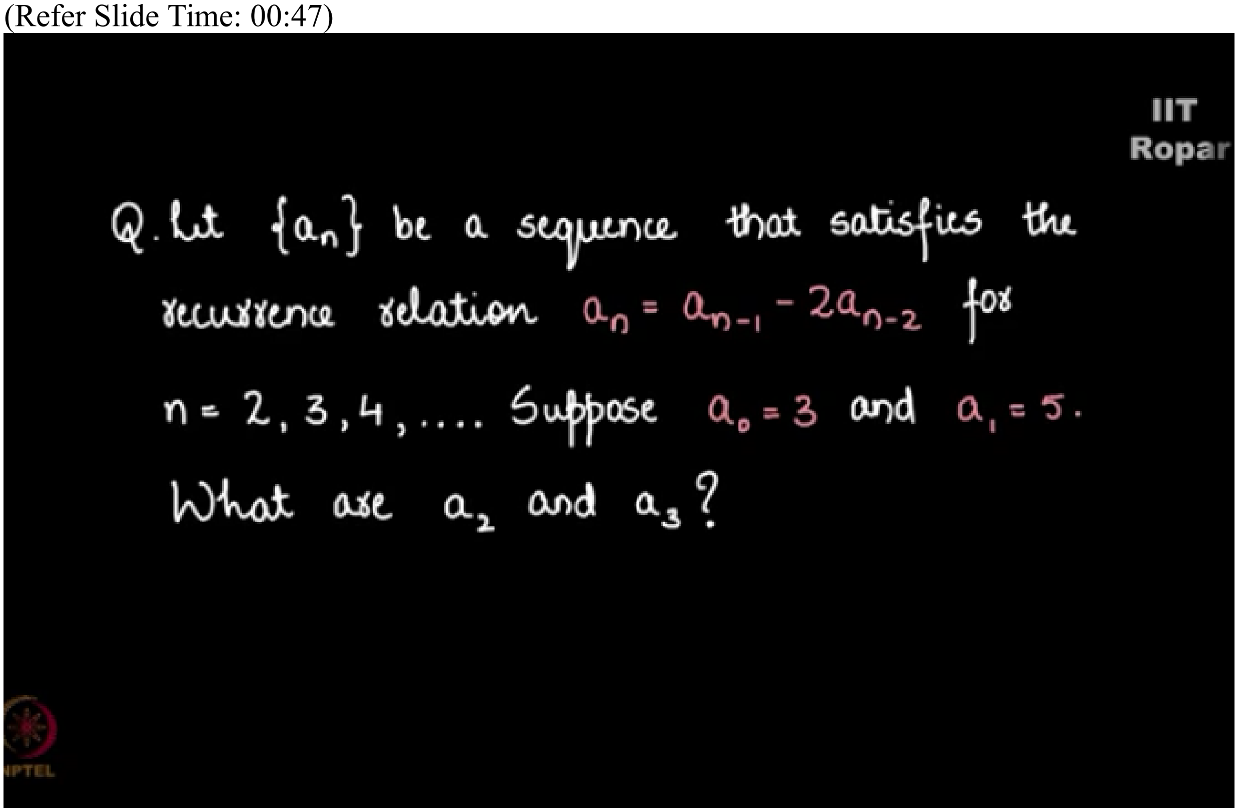

So the recurrence relation is given to us and it is given that A naught is 3, and A1 is 5, we have to find out the terms A2 and A3, so AN is AN-1 - 2AN-2, and A naught is 3, A1 is 5.

Now what do we have to find out? A2 and A3, let me start with A2, how do we obtain A2? A2 can be obtained by substituting 2 for N, right, so for N = 2, now you see if I substitute N as 2, the recurrence relation becomes A2 = A 2-1 which is A1 - 2 into N-2 which is 2 - 2 which becomes -2 A naught, you see to obtain A2 we must first find out A1 and A naught, and to find out A3 we must do something else that is we must have A2 and then A1, do you see the relation to find out a bigger term you need the previous terms, so A2 is given to be, we have to find out A2, A1 is given to be 5, and A naught is 3 so let me substitute that 5 – 2(3) which is 5 - 6 and this is -1.

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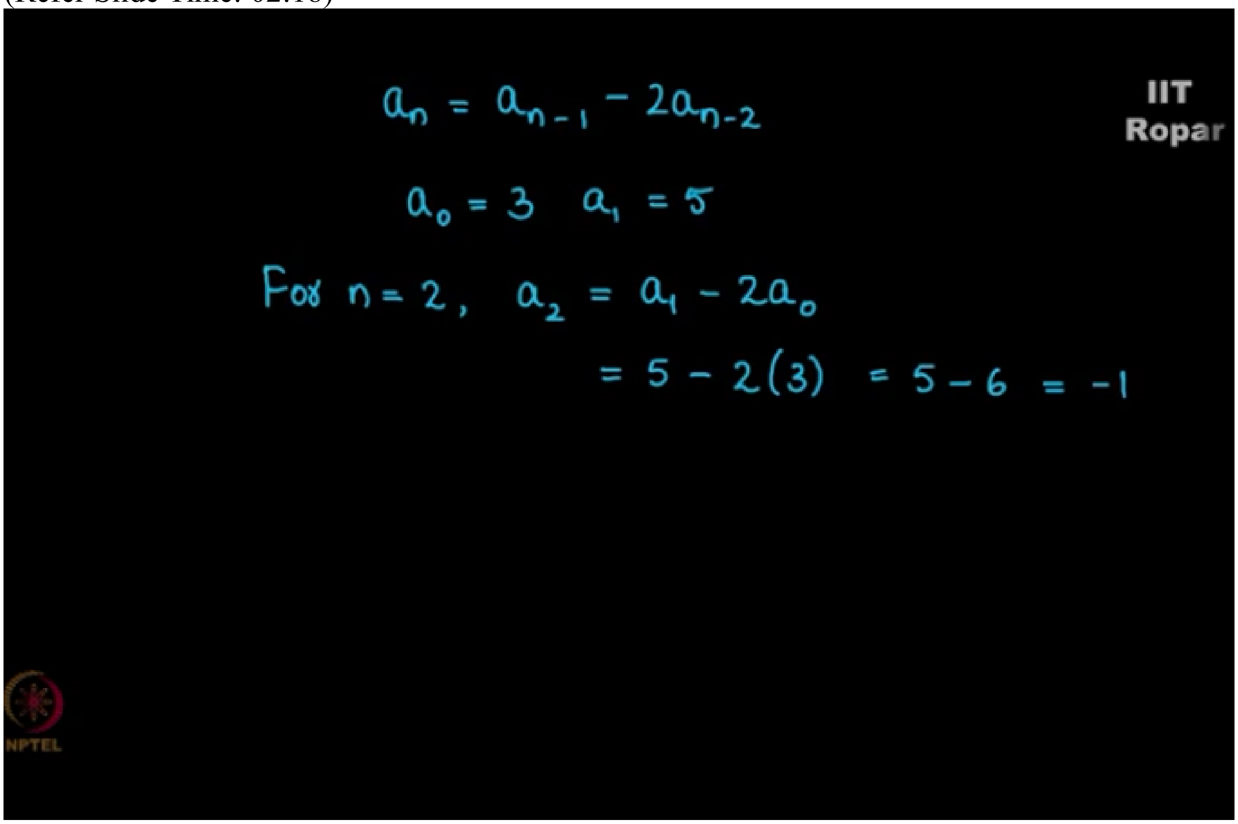

Now when I substitute N as 3 I get A3 equals, so 3-1 is 2, A2 - 2 into 3 - 2 is 1, so -2A1, do you see earlier to calculate A2 we wanted A naught and A1, now for A3 we want A1 and A2, so the relation is you need the preceding terms to find the next terms A2 is -1, we just found out that A1 is 5, so it is -1 -2(5) which is -1 -10 = -11, and hence the question has been answered A2 is -1, and A3 is -11.

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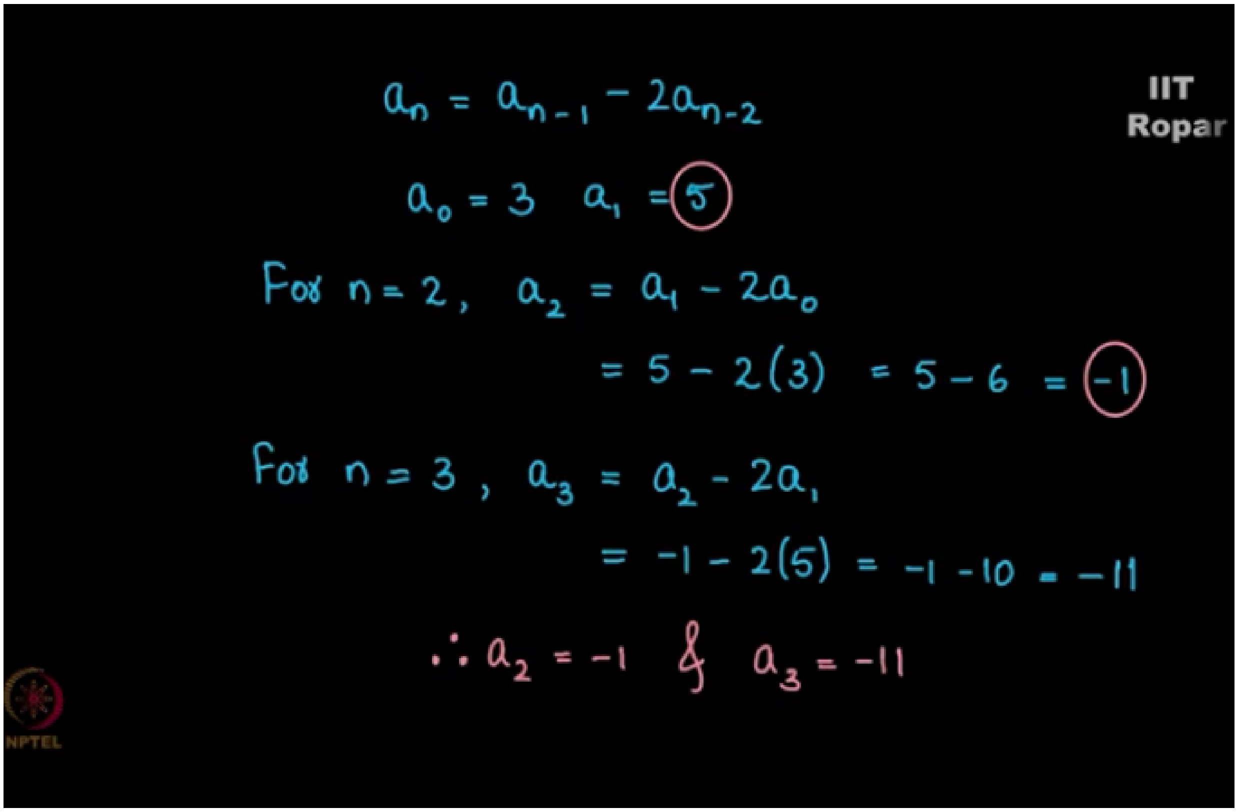

Let us see another problem now, consider the sequence 0, 2, 6, 12, 20, 30, 42 and so on, now the question is you have to write a recurrence relation for this sequence, so earlier question was you were given the recurrence relation and you were given two terms you had to find out some other terms, but in this question you have been given the sequence and you have to find out the recurrence relation, how will you do that? So the given sequence is 0, 2, 6, 12, 20, 30, 42 so on.

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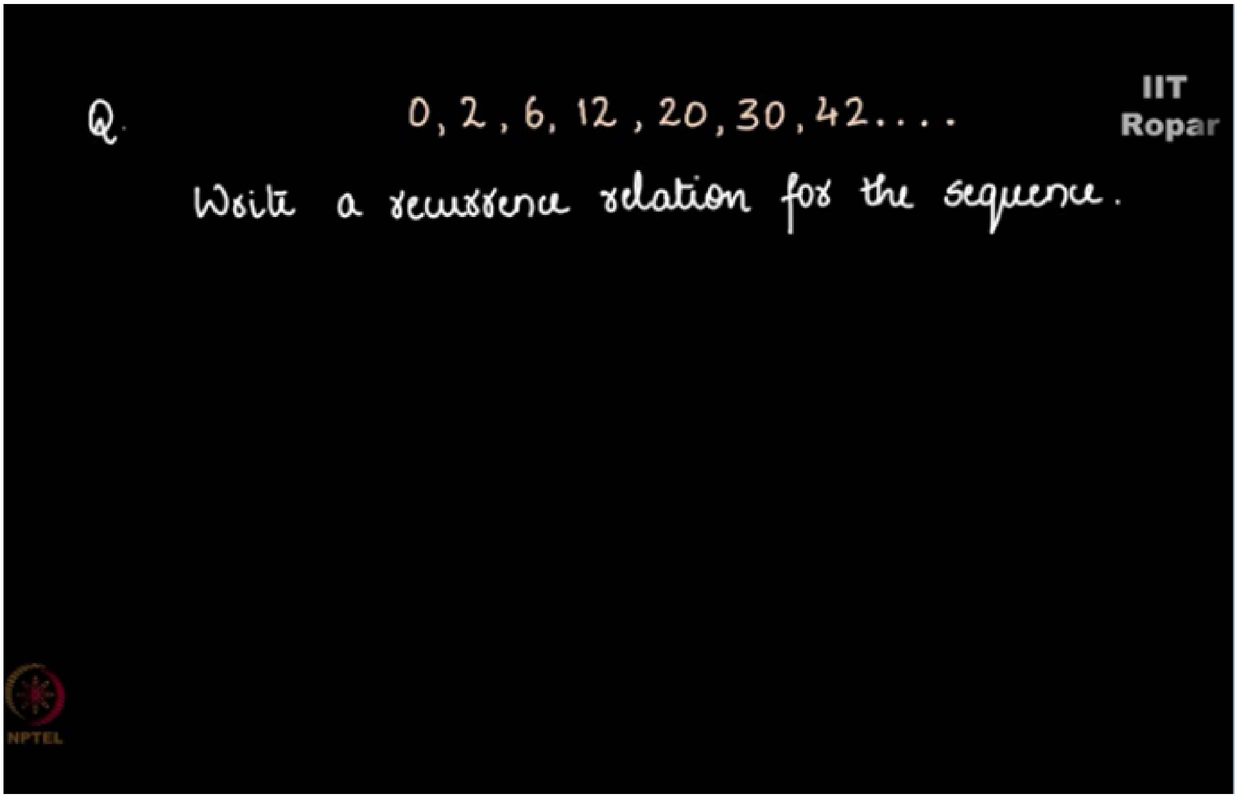

Now the first step would be to find out the difference between corresponding or consecutive terms, let us see how to do that, A naught is given to be 0, A1 is given to be 2, A2 is given to be 6 and so on, let me calculate the differences A1 - A naught is given to be 2, A2 - A1 is given to be 4, A3 – A2 is given to be 6, A4 - A3 is 8, A5 - A4 is 10, A6 - A5 is 12 and so on, now take a minute here,

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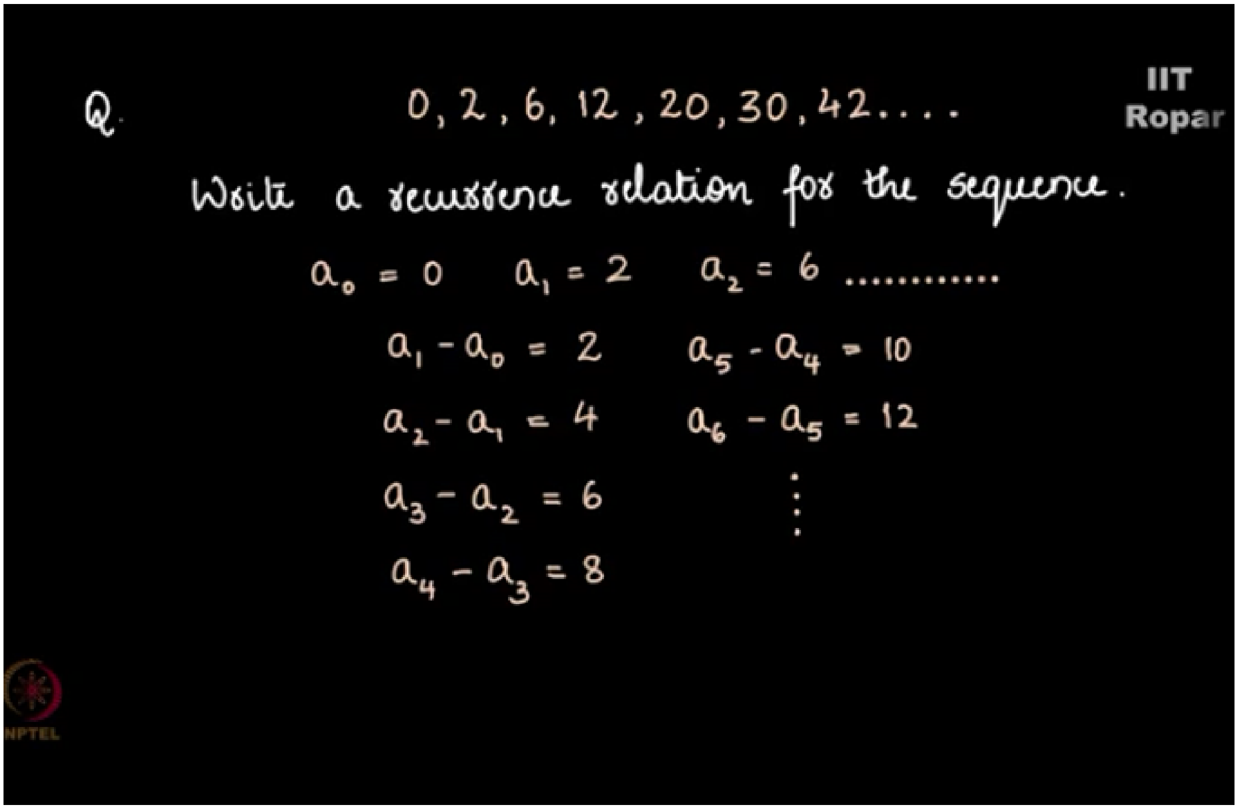

take a pause and observe the differences, what do you see? You see that the differences can be written as AN – AN-1, why? Because each time I am writing A1 - A naught, A2-A1 so I am taking the difference of successive terms, so in general I can write it as AN - AN-1 = N,

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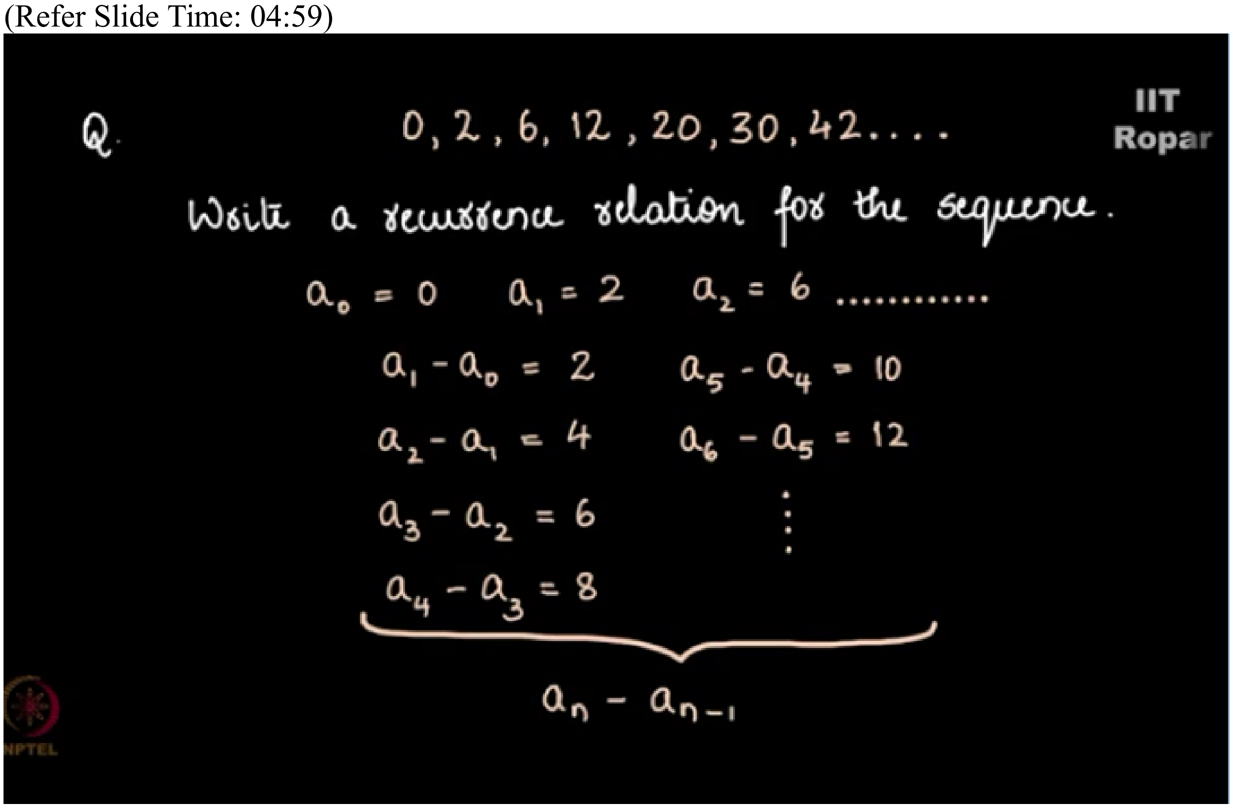

did you observe that? Let me just take one difference and explain it A1 - A naught is 2, so if I write N = 1 it becomes, in the general form if I write N = 1 we get it as A1 - 1-1 is 0 and hence 0, A naught = 2(1) which is 2, and this holds true for every difference.

So in general these differences can be written as AN – N-1 = 2N, so this is the recurrence relation for the sequence which is given 0, 2, 6, 12, 20 and so on.
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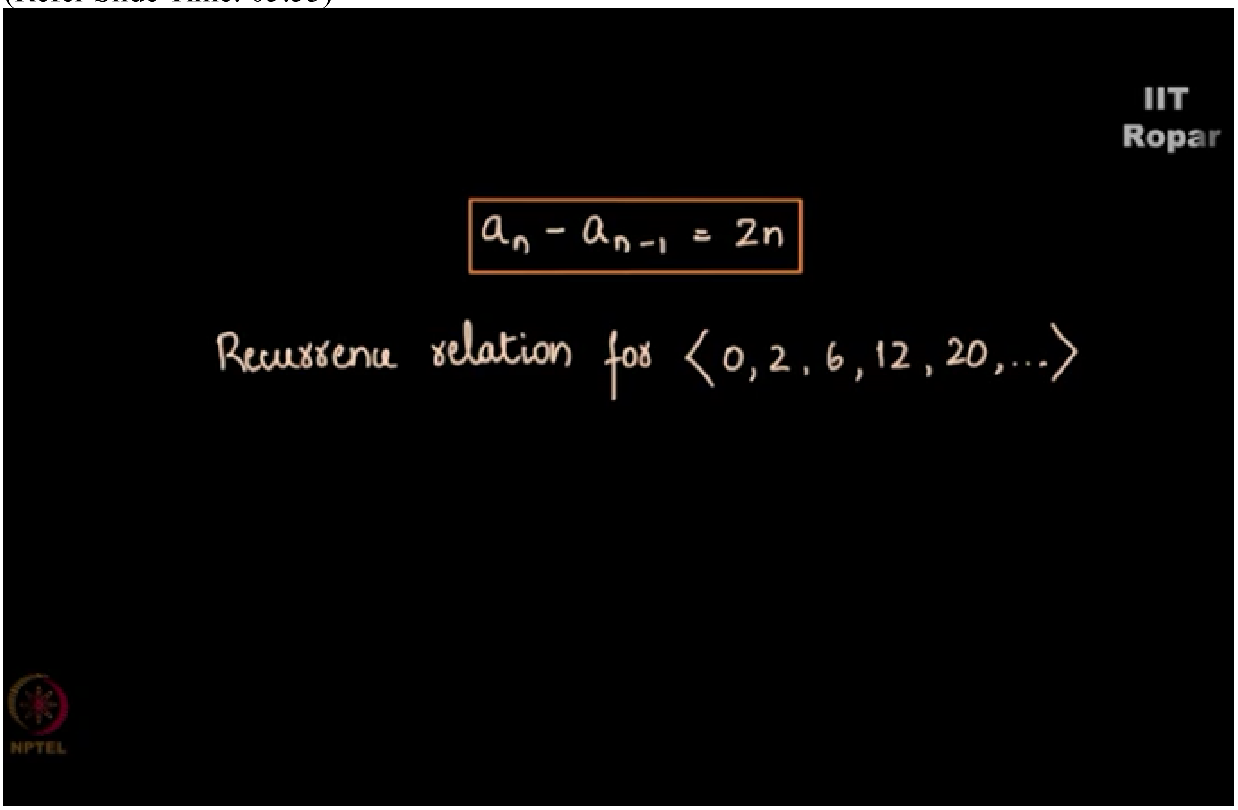

Now we have found out the recurrence relation for the sequence, but wait a minute we can do something more here, what can we do? Observe I take all these differences, so A3-A2, 4-3, 5-A so on, so I have taken all the differences up to the nth term, AN - N-1 = 2N right,
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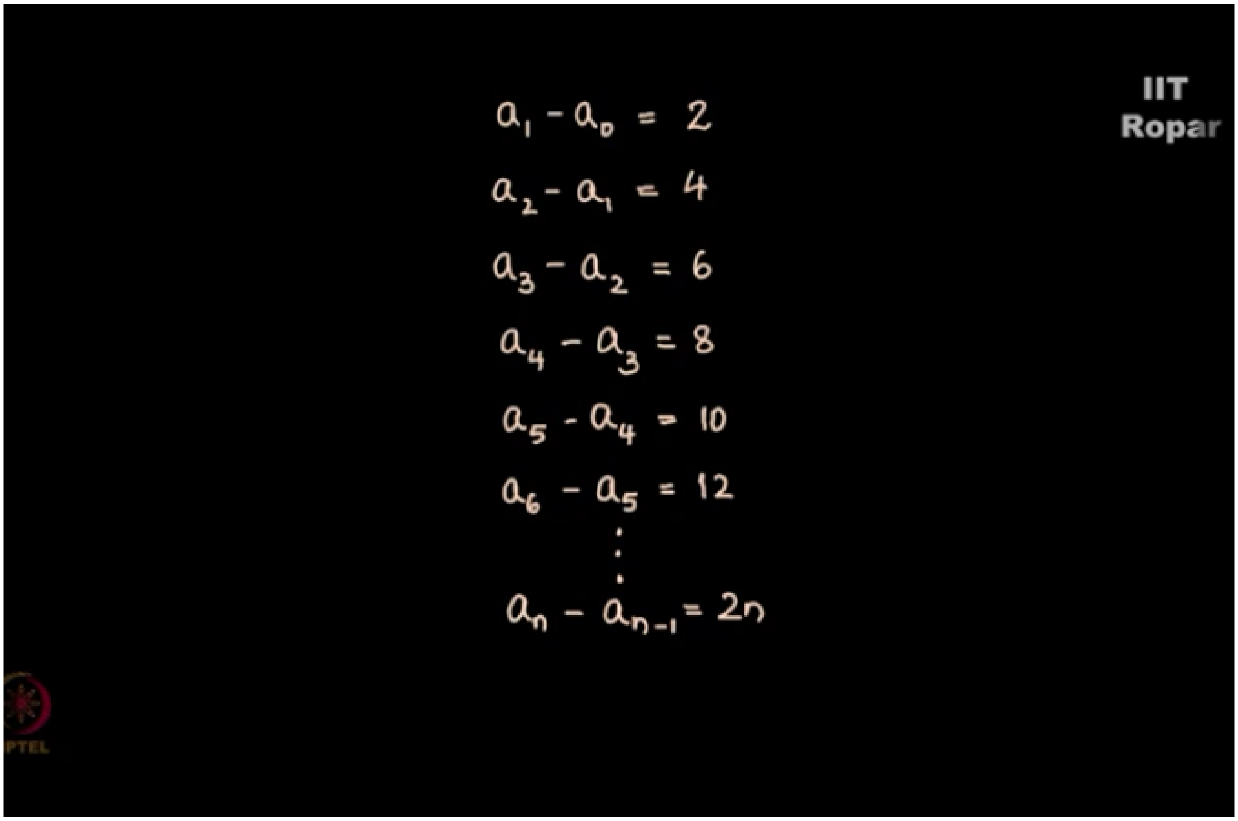

so these are the differences.

Now what do I do is I add up, I sum up all these equations okay, now some basic math tells me that I can calculate or I can rather sum up these by canceling some terms, what are those terms? A1 - A1, A3 - A3, A4 - A4, A5 – A5, all these terms get cancelled, what remains is you see AN-1 – AN-2 = 2(n-1) this is the last but one term. Now N-1 – AN-1 this also gets cancelled so what remains at the end is AN-A naught = 2 + 4 + 6 + 8 + so on up to 2n,

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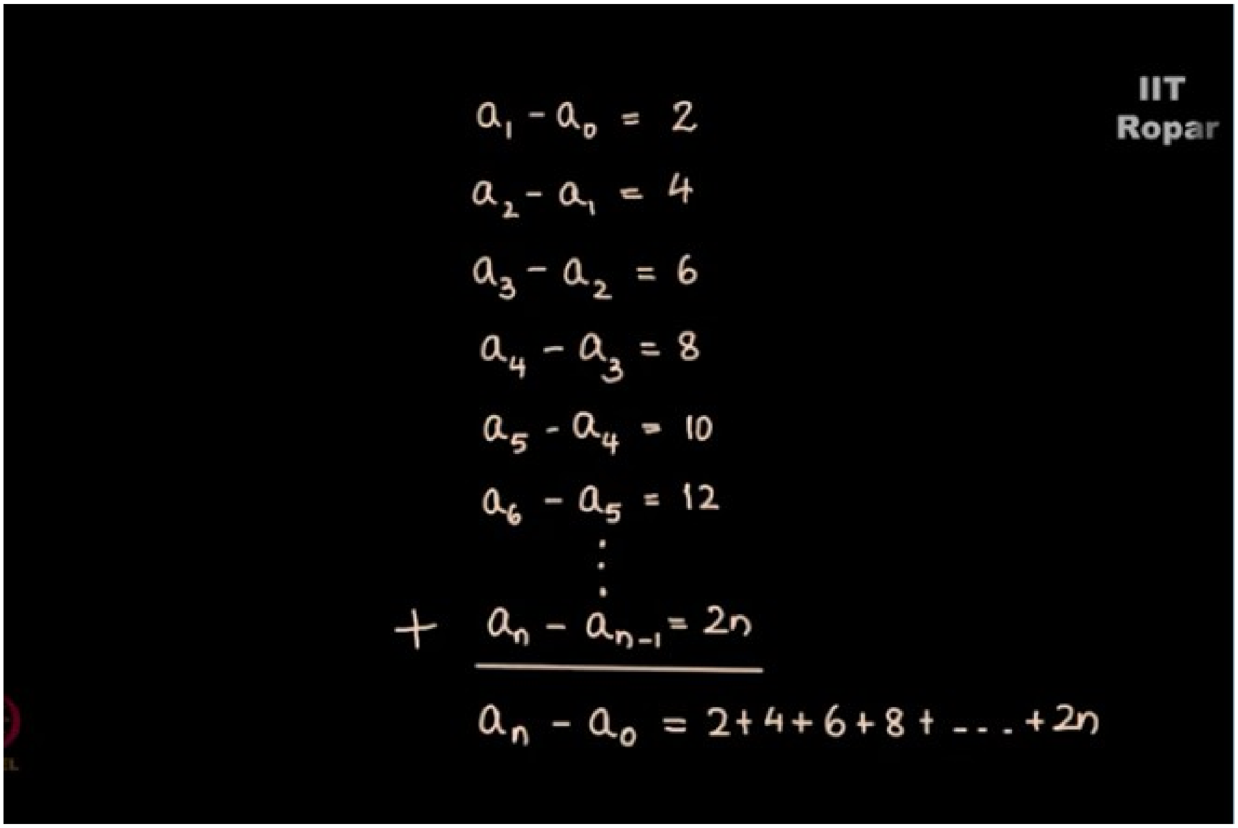

so how did we get this? Only AN terms remains, and A naught term remains at the end after summing up, and the sum on the right hand side becomes 2 + 4 + 6 + 8 so on up to 2n, right.

Now given this equation I can do some jugglery here, what is that? I'll take out 2 common, so when I take out 2 common what remains is 1 + 2 + 3 + 4 + so on up to N, and this you must be remembering from induction chapter, mathematical induction chapter we had seen that sum of N terms happens to be N(n+1/2) so this can be written as 2 into N(n+1/2), now canceling 2 on numerator and denominator I get it as N(n+1),
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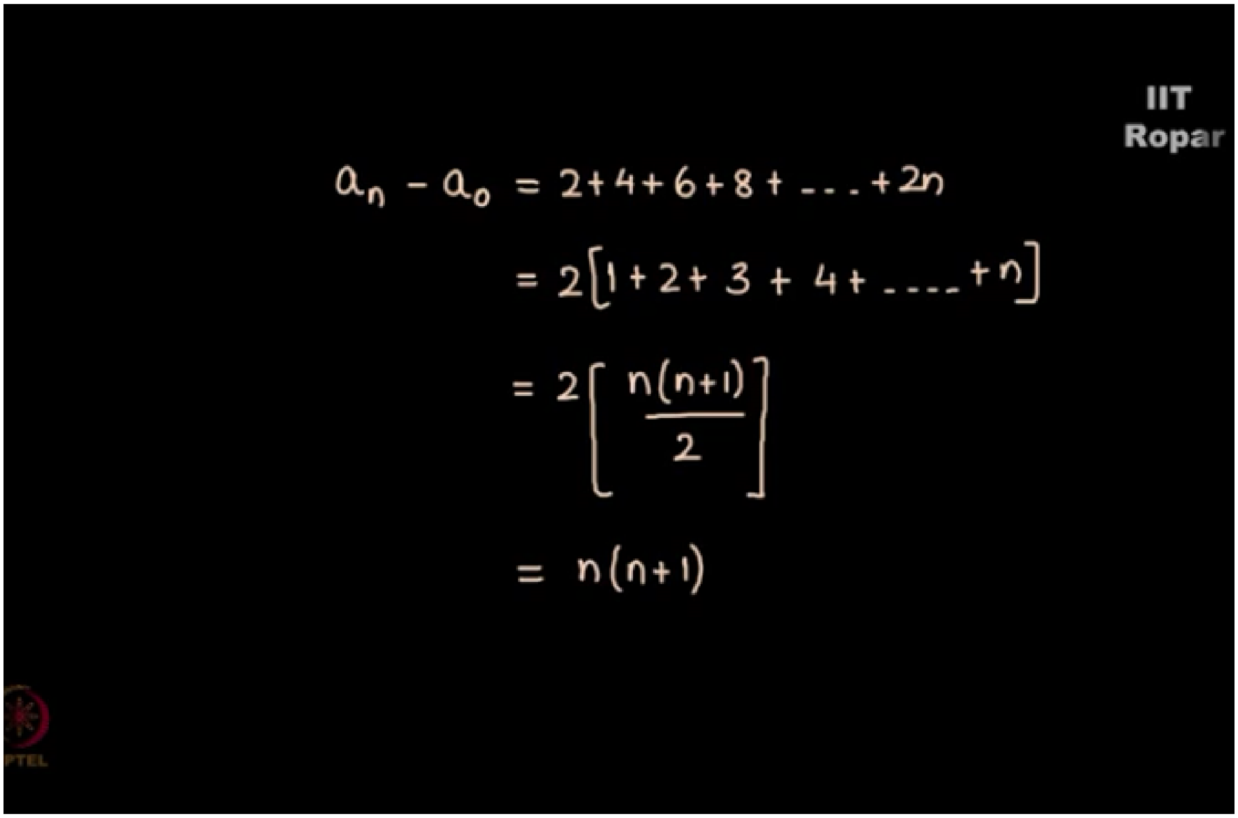

now do you see that AN - A naught is N(n+1), well A naught I can substitute as 0 because A naught is given to be 0 and hence AN = N(n+1) now this is the closed form for the sequence.

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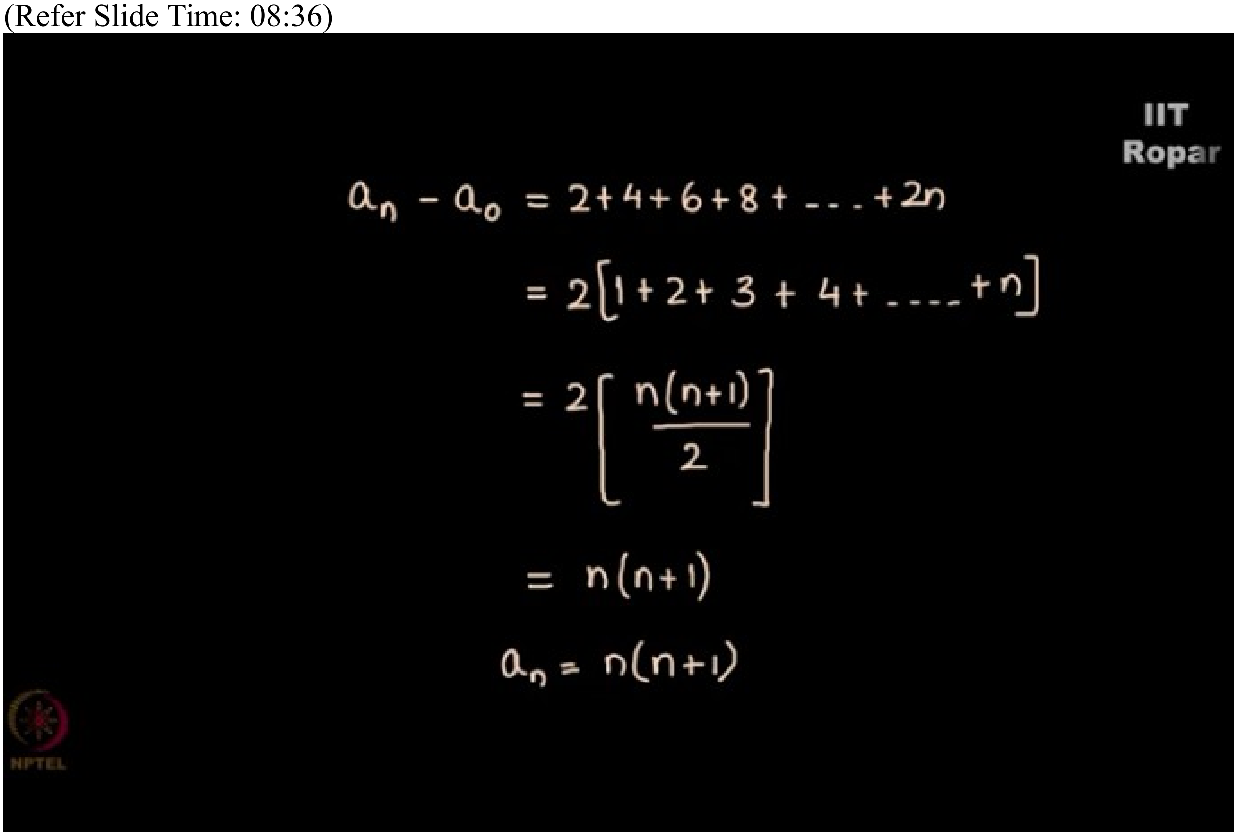

Did you observe that? We first found out what is the recurrence relation for the sequence and now we are finding out the closed form for it, so representing AN in terms of N this becomes a closed form, so this is also called as the solution for the recurrence relation, so AN = N(n+1) is the solution for the given recurrence relation.

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