NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Recurrence Relation

Compound Interest as a recurrence relation

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Assume I give you a 1000 rupees, and I tell you, you must return this back with an interest of 10% at the end of this month, how much will I get in return? Till the straight forward simple interest question
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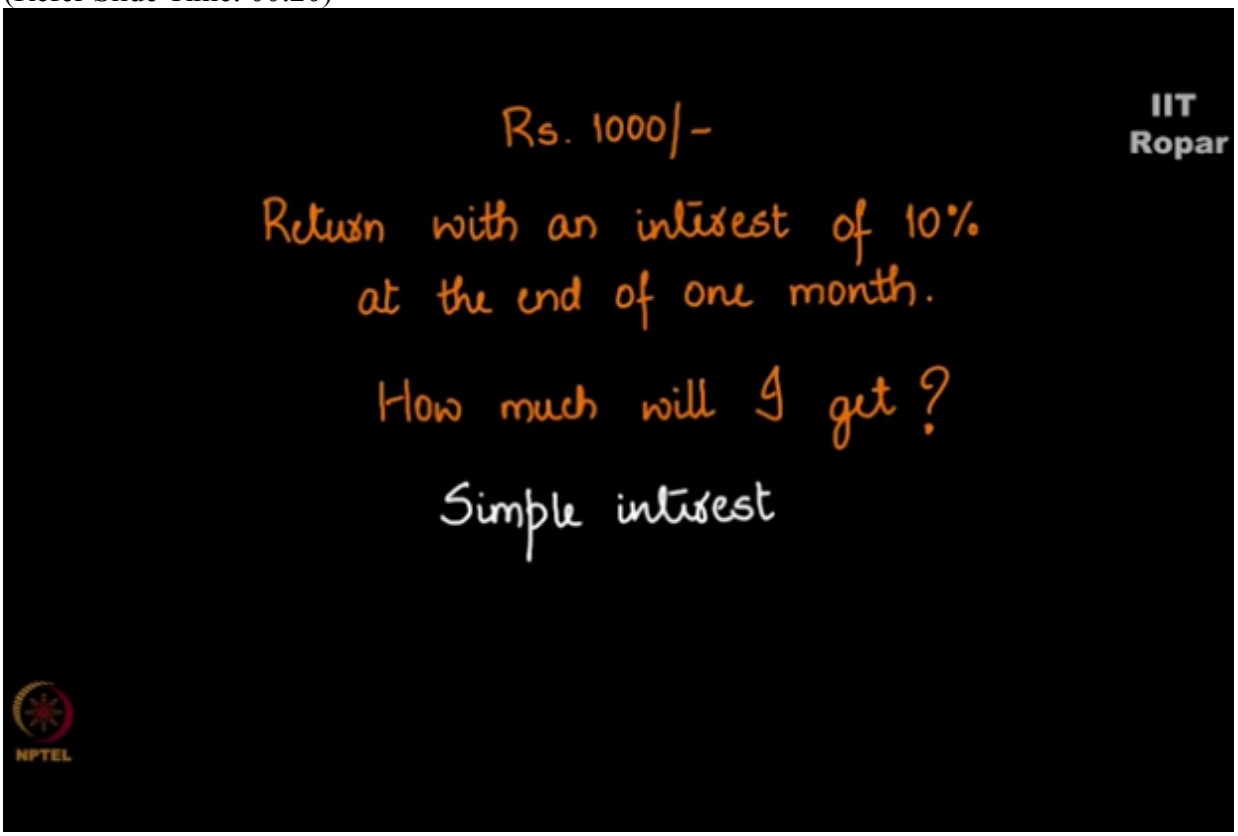

we all have solved many times and over in our school days, the answer is going to be 1000 + 10% of it, which is 100 so 1100, that's the money you will be giving me, right.
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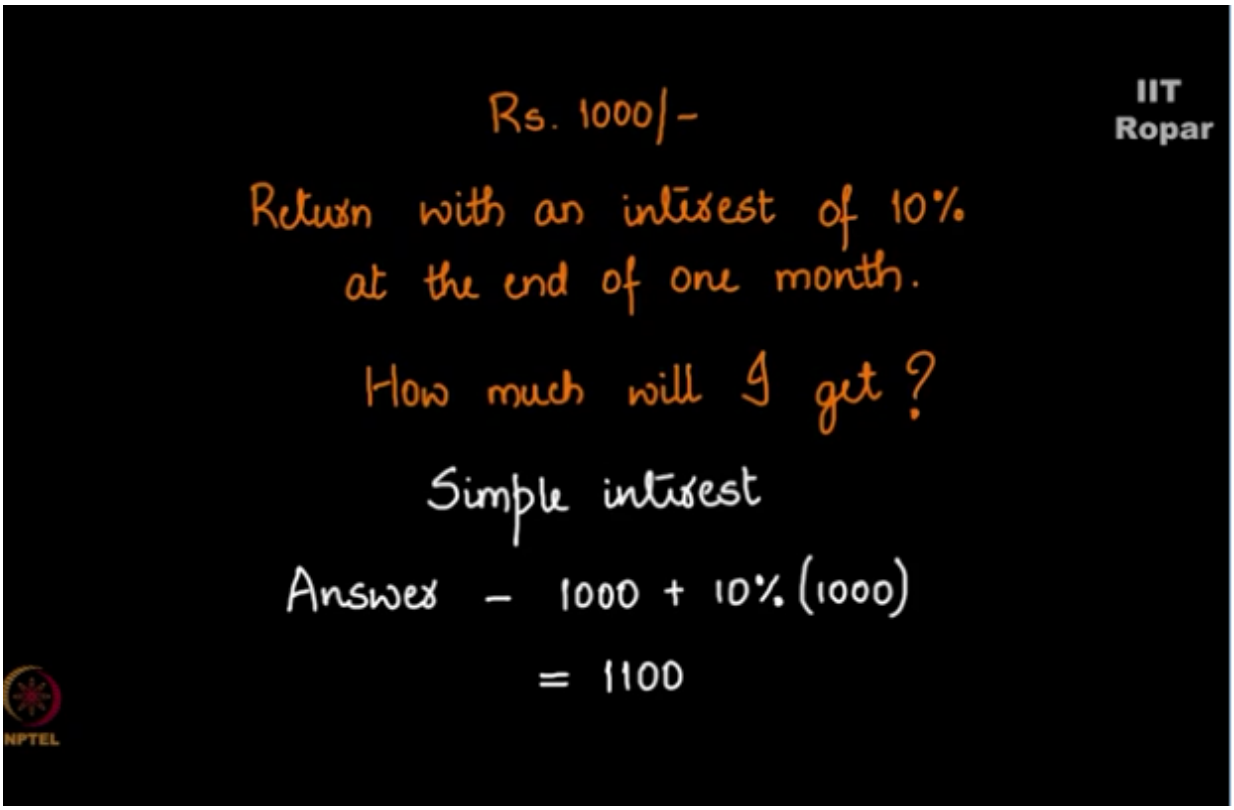

What if I say for 2 months you compute the interest and give me at the end of 2 months, if it's going to be 100 rupees per month 10% rate, for 2 months it will be 100 + 100, 200, so 1200 is what you will be giving me at the end of 2 months if it's for 2 months duration,
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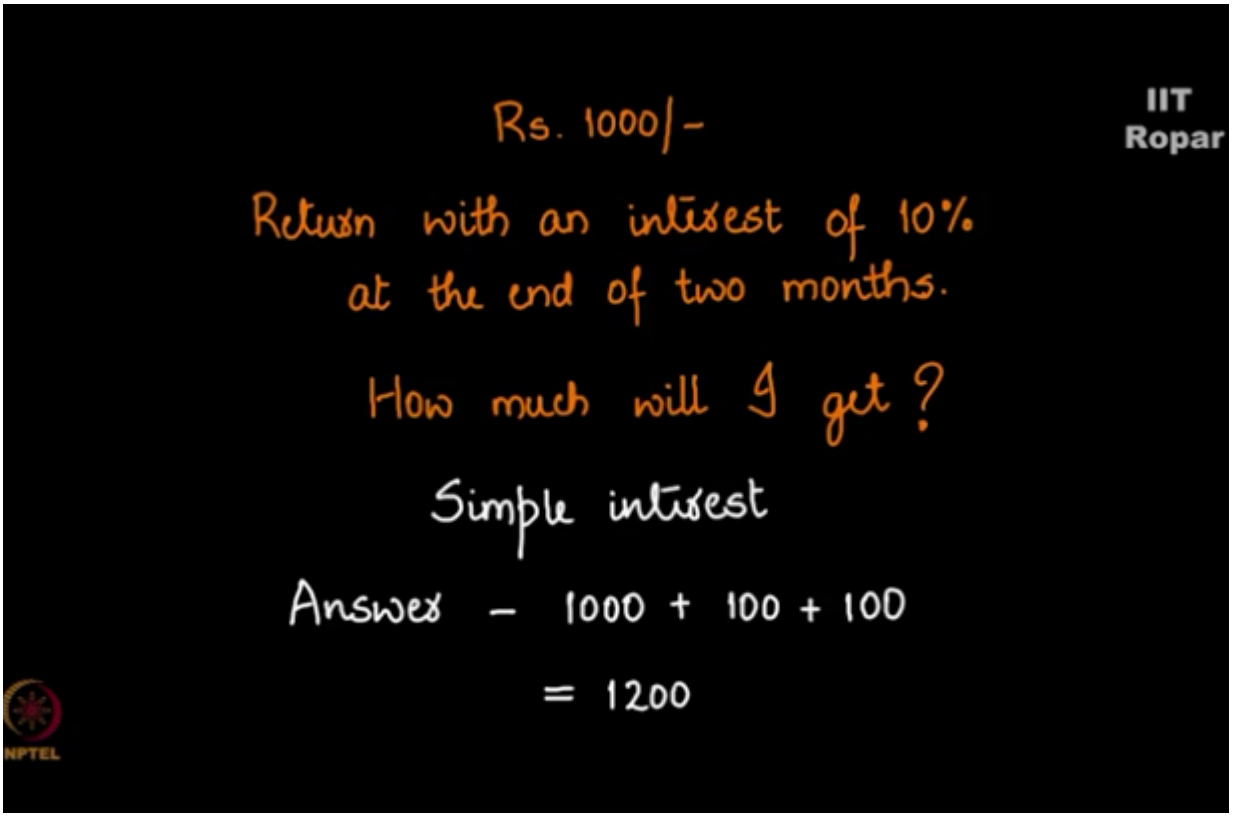

so but then the next step to complicate this very easy question is to bring in the notion of compound interest, sounds complicated but it's a very easy concept, the concept goes like this, 1000 rupees in the beginning at the end of one month you will give me the interest which is 10% of it which is 1100, what I say don't worry keep this 1100 with you, you give me next to next month rather next month from now with 10% interest on 1100, now what is that?

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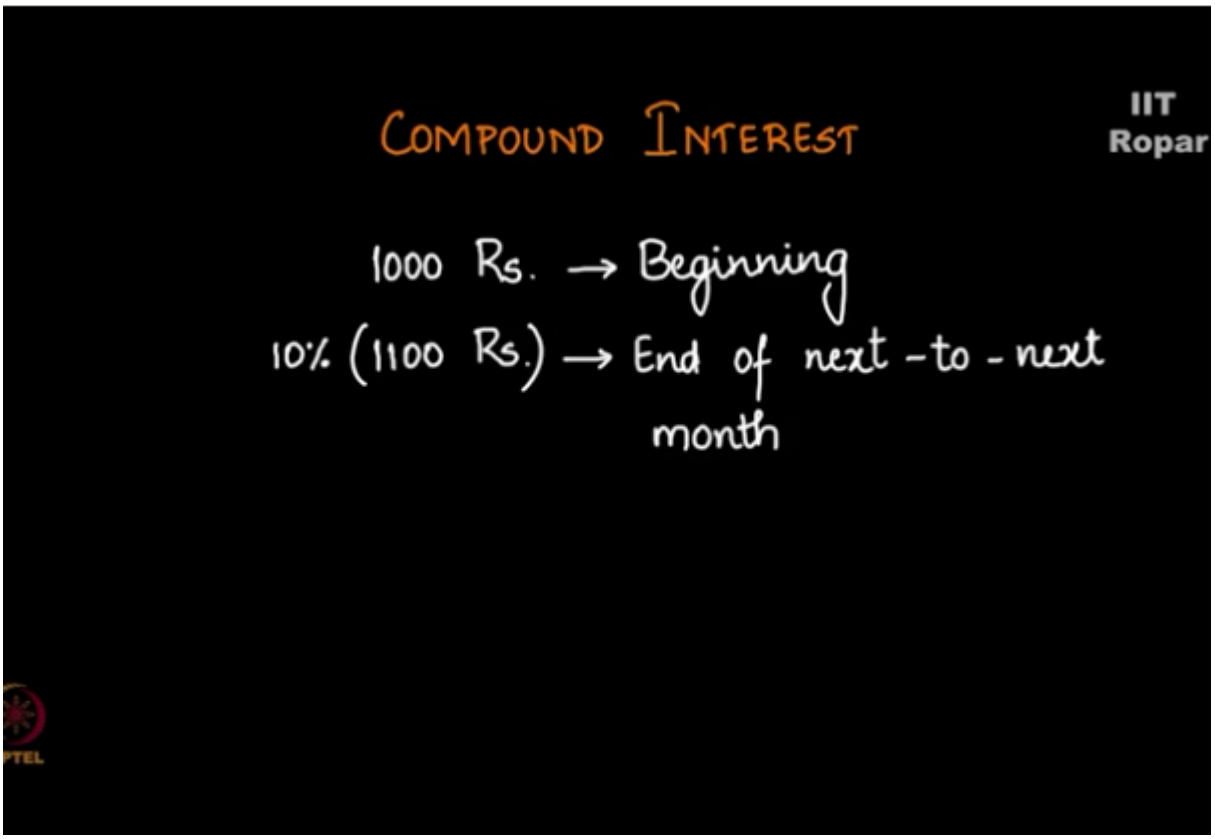

1100 times 10% which is 0.1 that gives me 1.21, right, so which is basically 1210 rupees at the end of 2 months,

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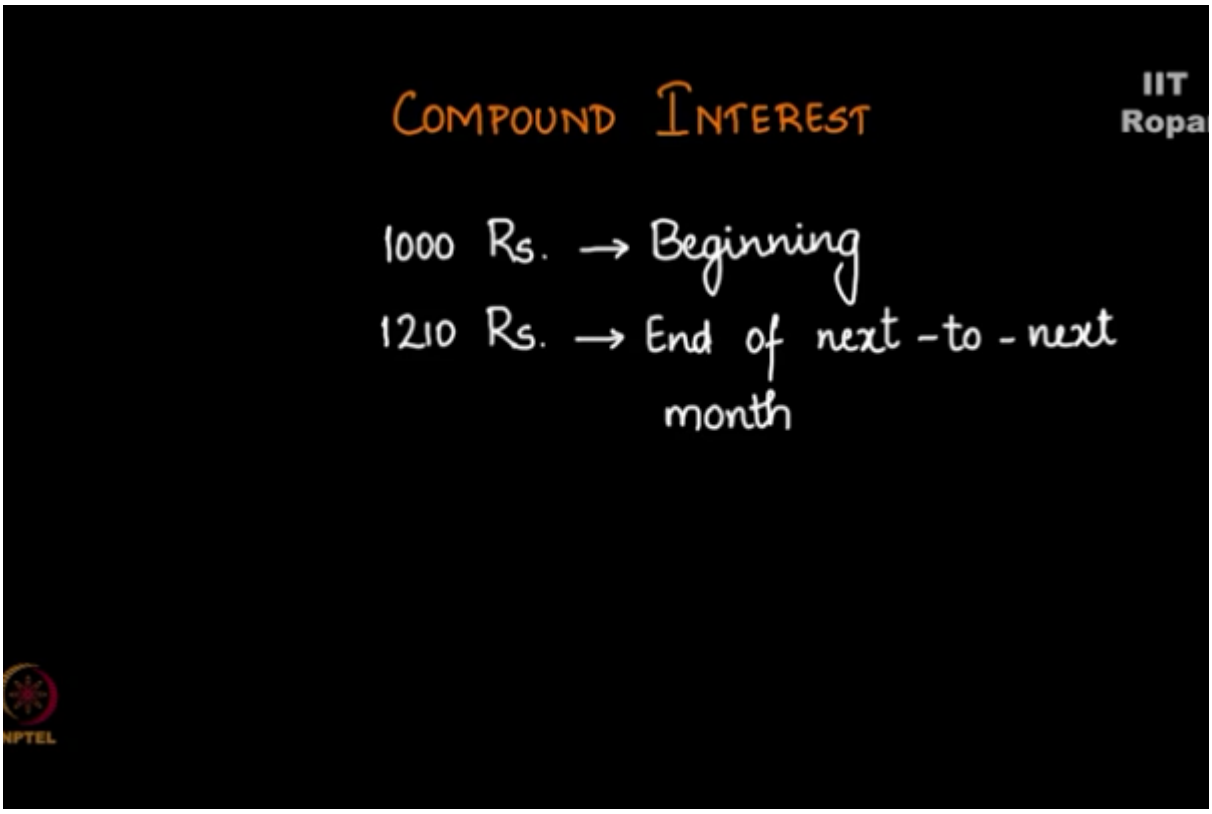

you see it is interesting to note that a simple interest doesn't give you a lot of as much money as compound interest, so simple interest give you 1200 but compound interest is giving you 1210, 10 rupees more, not a whole lot but still better than simple interest.

Now look at this, what if I asked you to compound this for 3 months, what is the formula?
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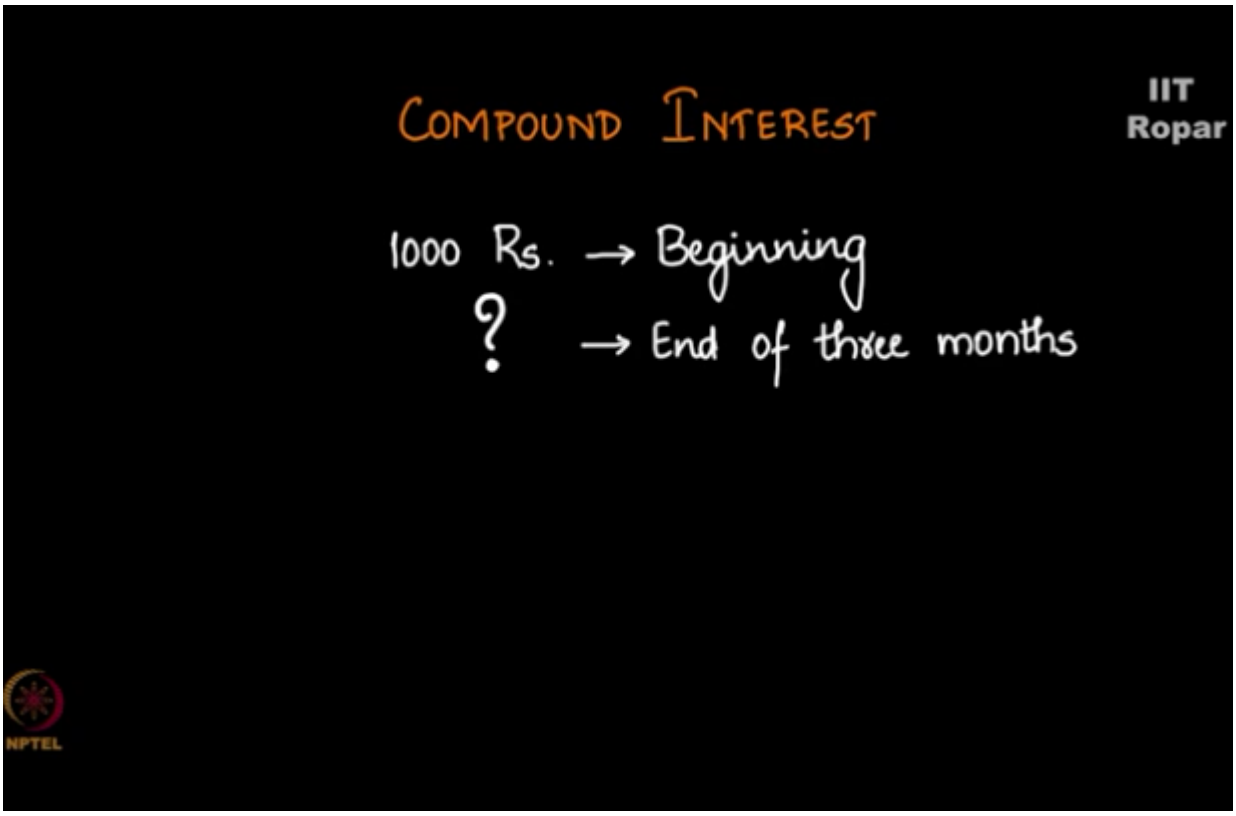

We all know there is a complicated formula for compound interest, so I'll tell you the version that my teachers taught me, the version goes like this it is P x 1+R/100 whole to the power of T, plug in T, plug in R, plug in P my friends you will get the answer, P for principal, R for interest, T for time,

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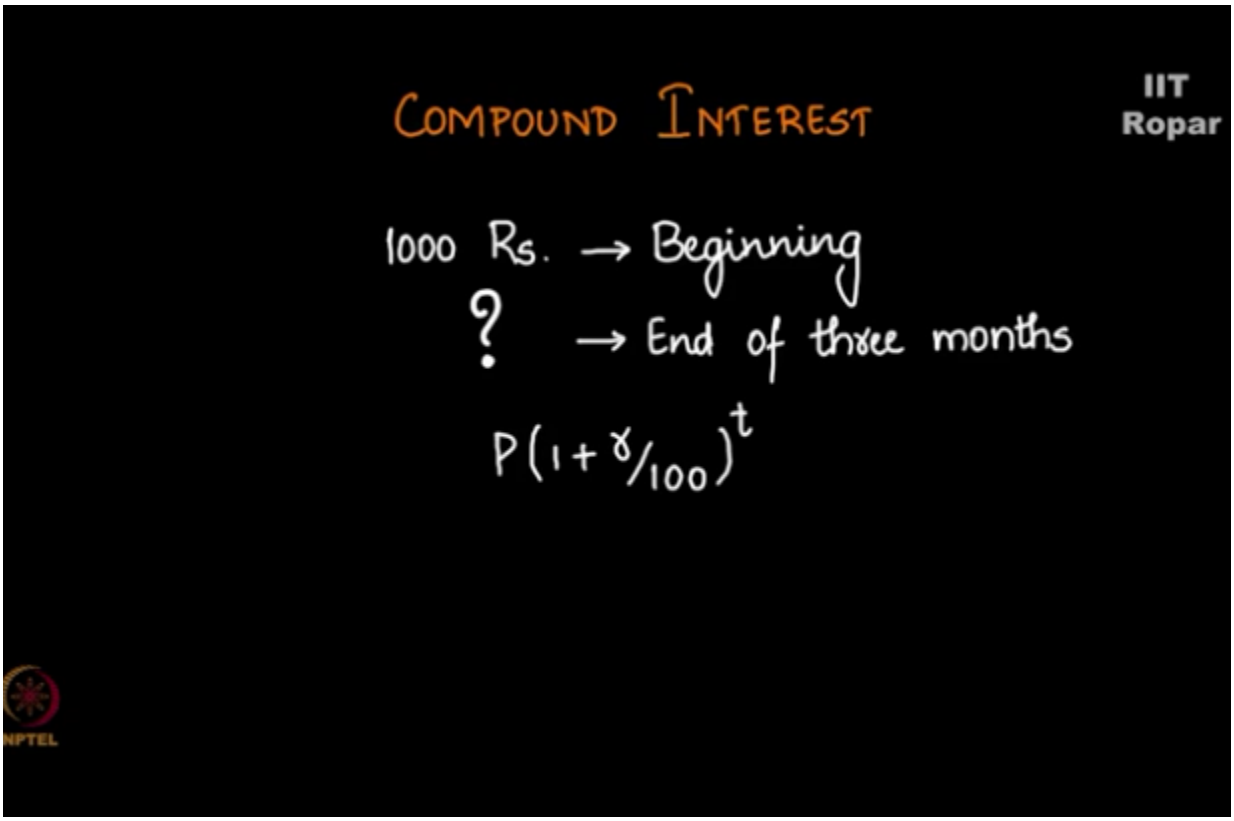

that's very complicated, but let's look at it in the language of recurrence relations, what are we saying here? All that we are saying is A2 at the end of 2 months will be whatever it is at the end of 1 month times, 1.1, why 1.1?

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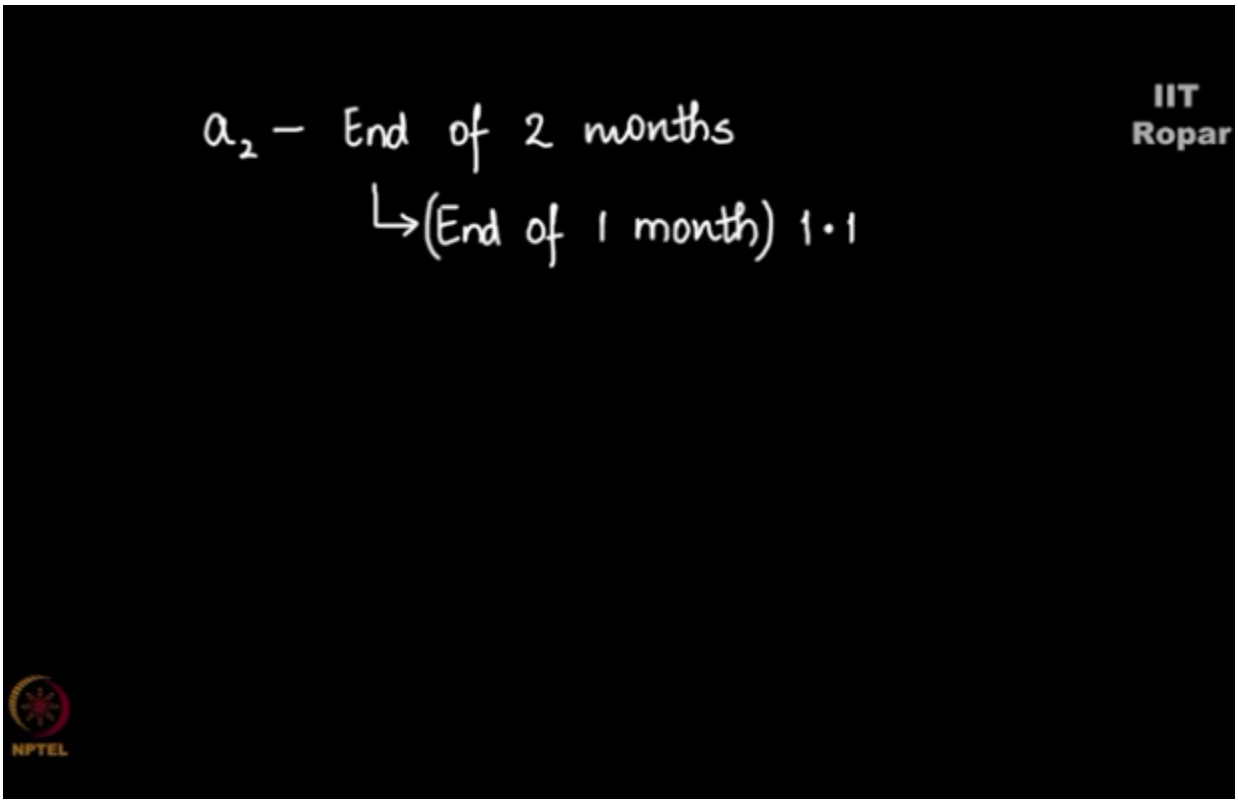

That's because it is basically A1 times 1 + 0.1, 0.1 represents 10% of A1, and 1 represents simply A1, so at the end of 2 months which is A2 you will be giving whatever you had it at the end of one month which is A1 times 1.1, but what is A1 here? A1 is at the end of one month you got to give me what you took from me which is the principal money let us call that A naught, which is 1000 by the way times 1.1 that is A1, so A1 is thousand times 1.1, calling thousand as let's say A naught, A1 will be A naught times 1.1, and A2 becomes A1 times 1.1, and A3 becomes A2 times 1.1, if you plug in and note you will observe that A3 is indeed 1.1 times A2,

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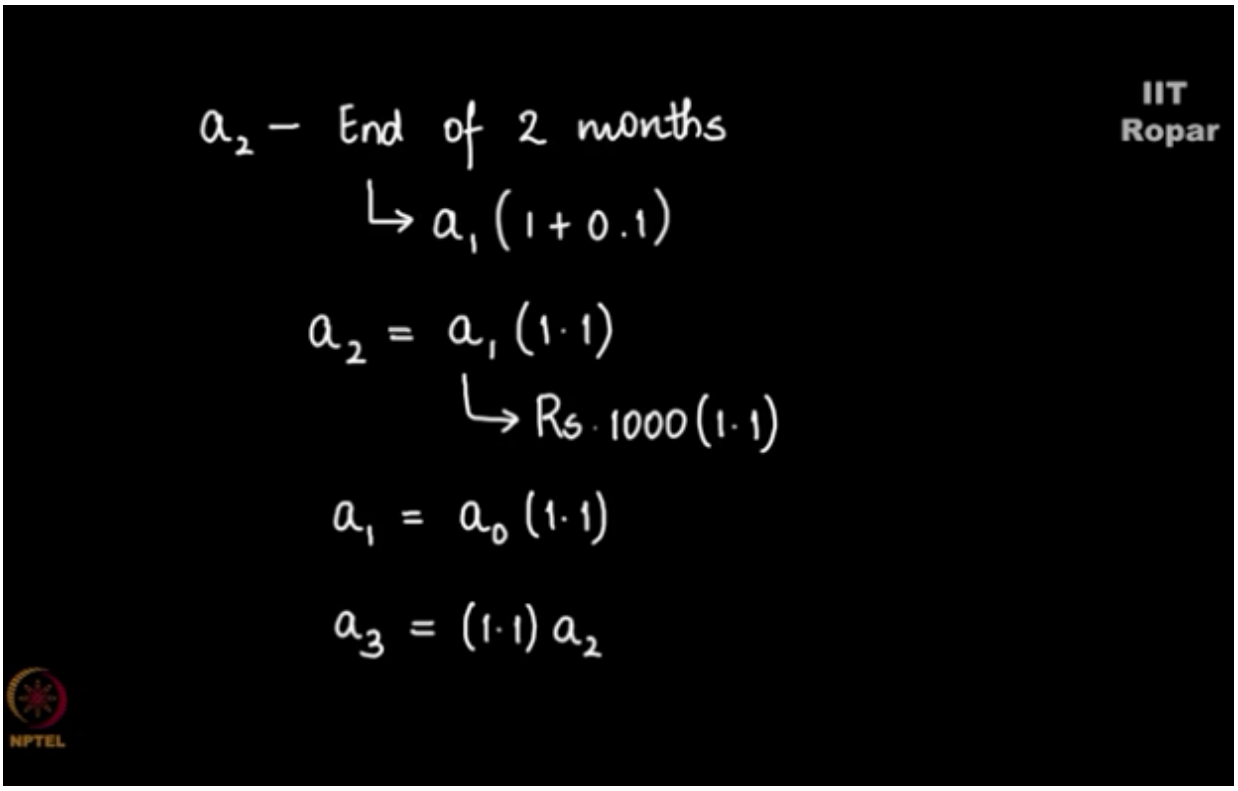

but in place of A2 you can remove and then write 1.1 times A1, in place of A1 you can write again 1.1 times A naught so you get 1.1 cube times A naught, so A3 is 1.1 cube times A naught which is 1000, so A3 is 1.1 cube times 1000, which is 1331 at the end of 3 months,

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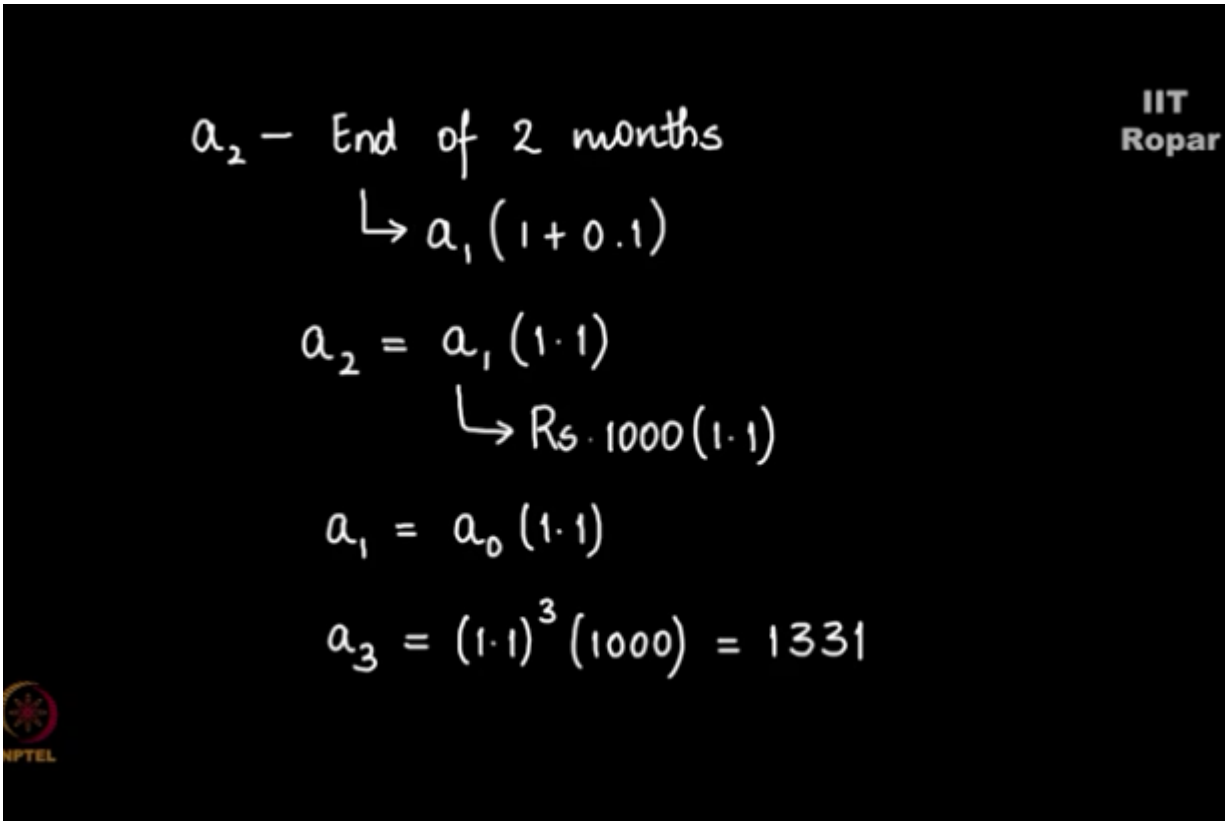

so if you're asked to calculate at the end of 12 months what will be the answer? The recurrence relation here is in general AN = 1.1 times AN-1, with this being the recurrence relation, please note look at the definition of recurrence relation, it says you express that Iths times by the previous terms that is precisely what's happening here, AN is expressed as AN-1 times something, basically the previous terms, correct, so AN = 1.1 times AN-1 is the recurrence relation and the question is at the end of 12 months which is A12, what will be your returns?

Given that A naught is 1000, please note that is very important that's called the initial condition, correct, A naught is 1000, what is A12? If AN is 1.1 times A to the N-1, we know that AN is simply 1.1 to the power of 12 times 1000, we saw that just now which is actually equal to 3138.42 paise, let's say rounded off to 3138,

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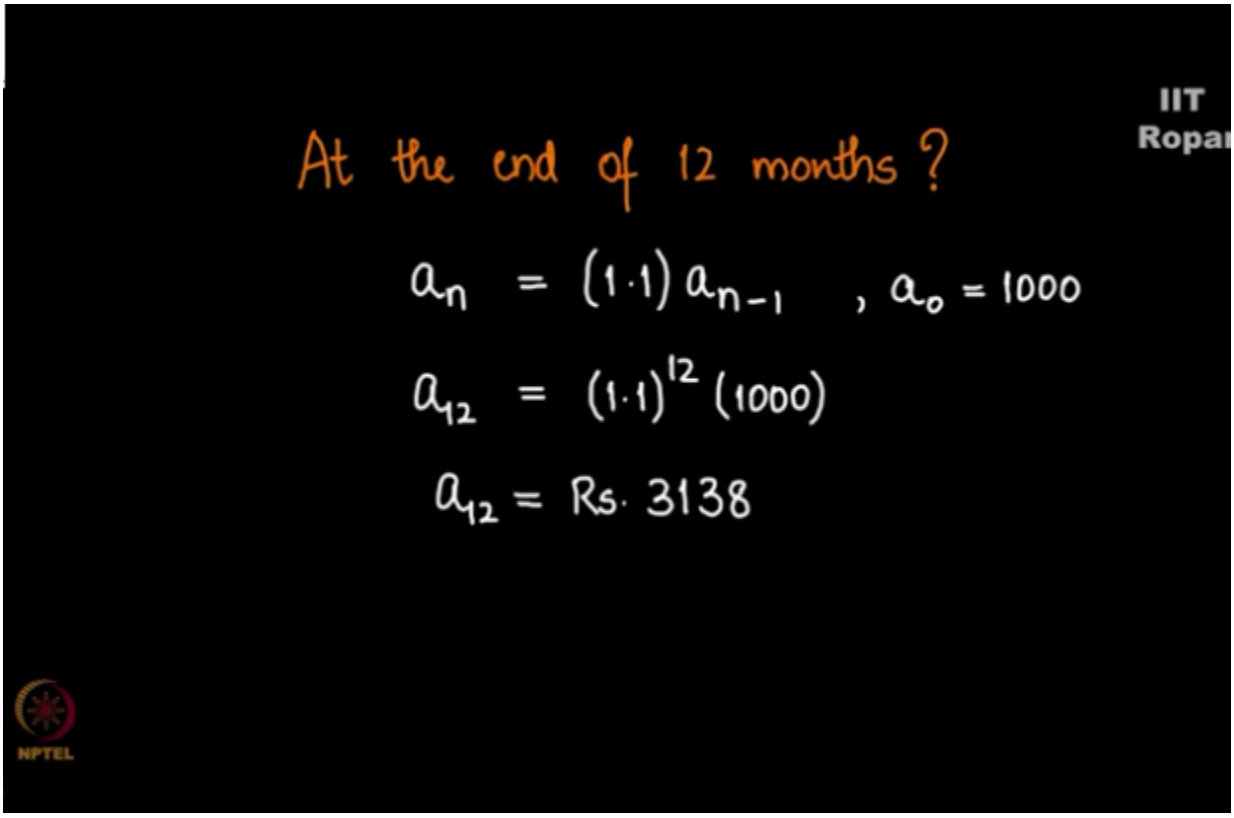

three thousand one hundred and thirty eight rupees is what you will be getting at the end of 1 year, if you keep compounding it every month by 10%, it's a lot more than thrice the money that you invested in, correct, so don't worry much about compound interest and your school days where I used to solve a lot of problems, the idea here is to get motivated to think on the lines of recurrence relation.

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