NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Example 17: Even integers and their places
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In how many ways can integers 1, 2, 3 so on up to 10 be arranged such that no even integer is in its natural place?
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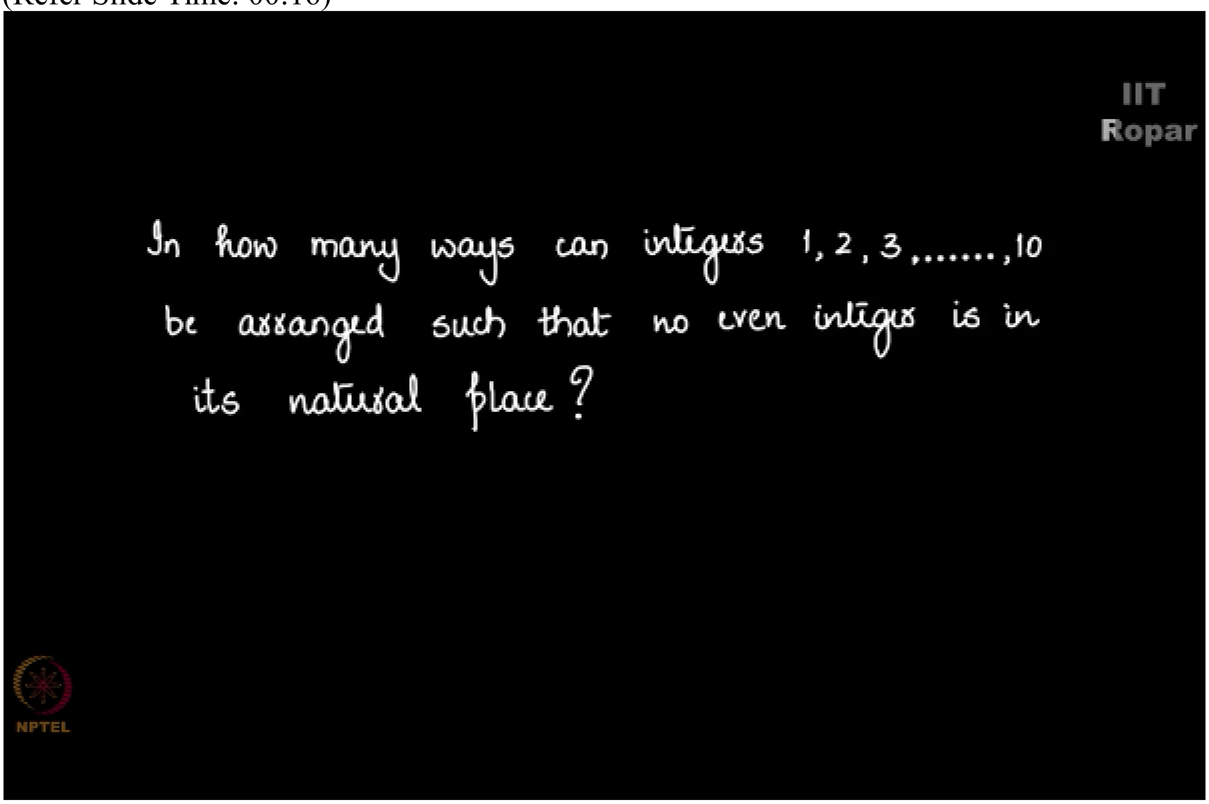

What do we mean by this? We have been given integers 1, 2, 3 up to 10, and we have to arrange them in such a way that no even integer is in its original position. Let us see how we can do this, consider the 10 integers, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10,
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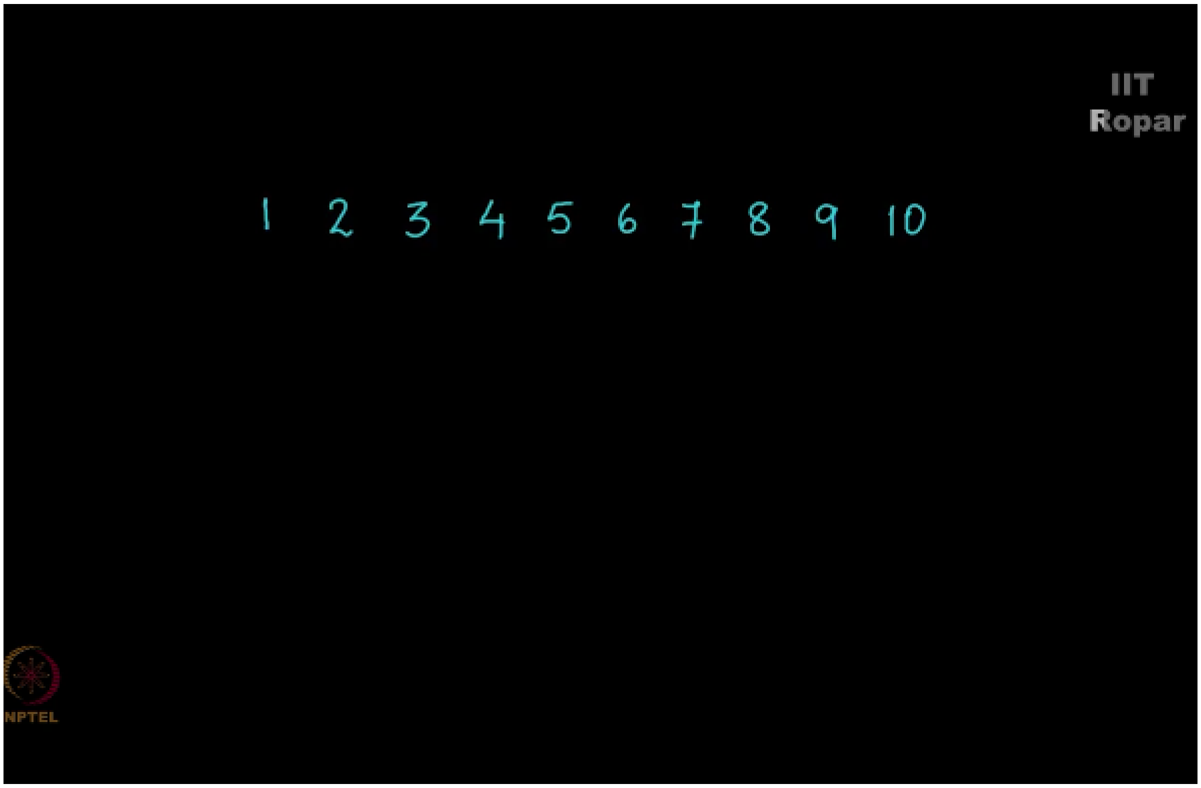

what are the even integers here? 2 is here, 4, 6, 8 and 10, you see these are the even integers and they must not be in their original position, in these positions.

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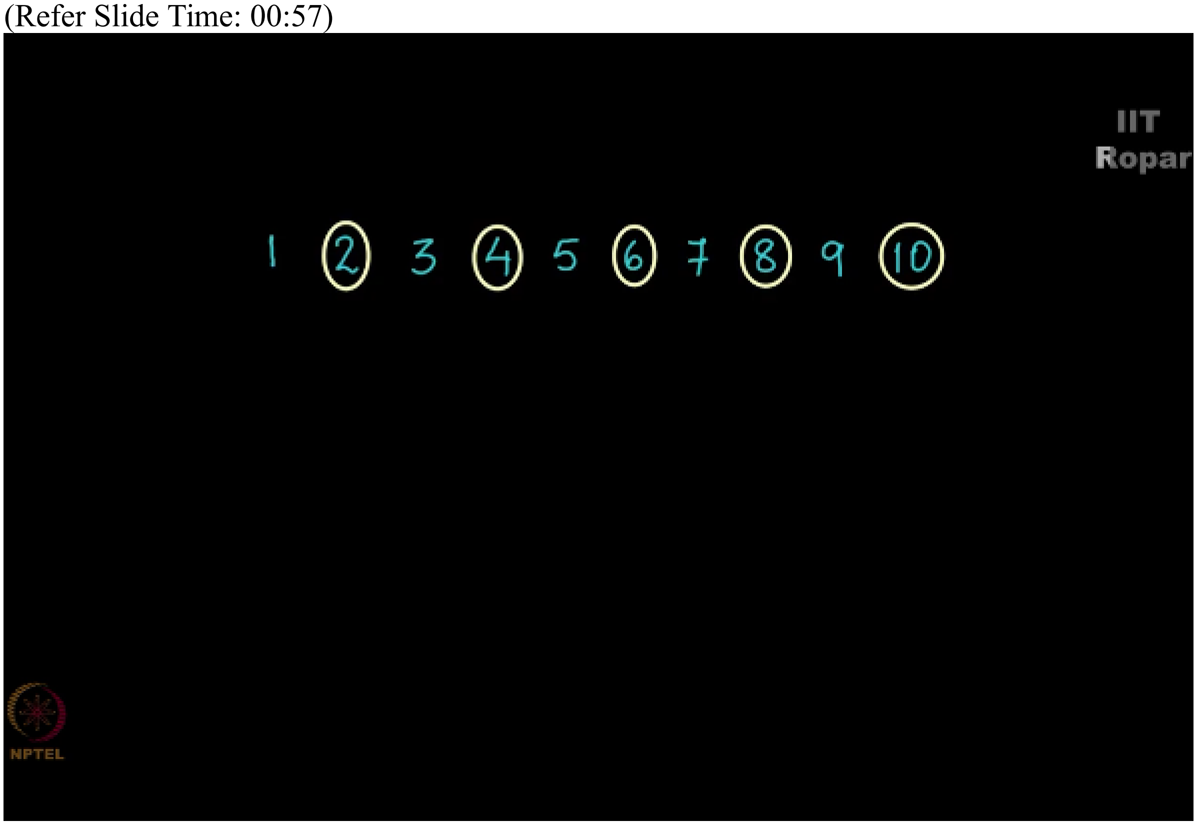

Now how can I place them? I can arrange these numbers something like this, 6 can come here, 8 can come here, 2, 10 and 4, or 8, 2, 10, 4 and 6 this can be another possible arrangement or 10, 6, 4, 8, 2 this is another possible arrangement,

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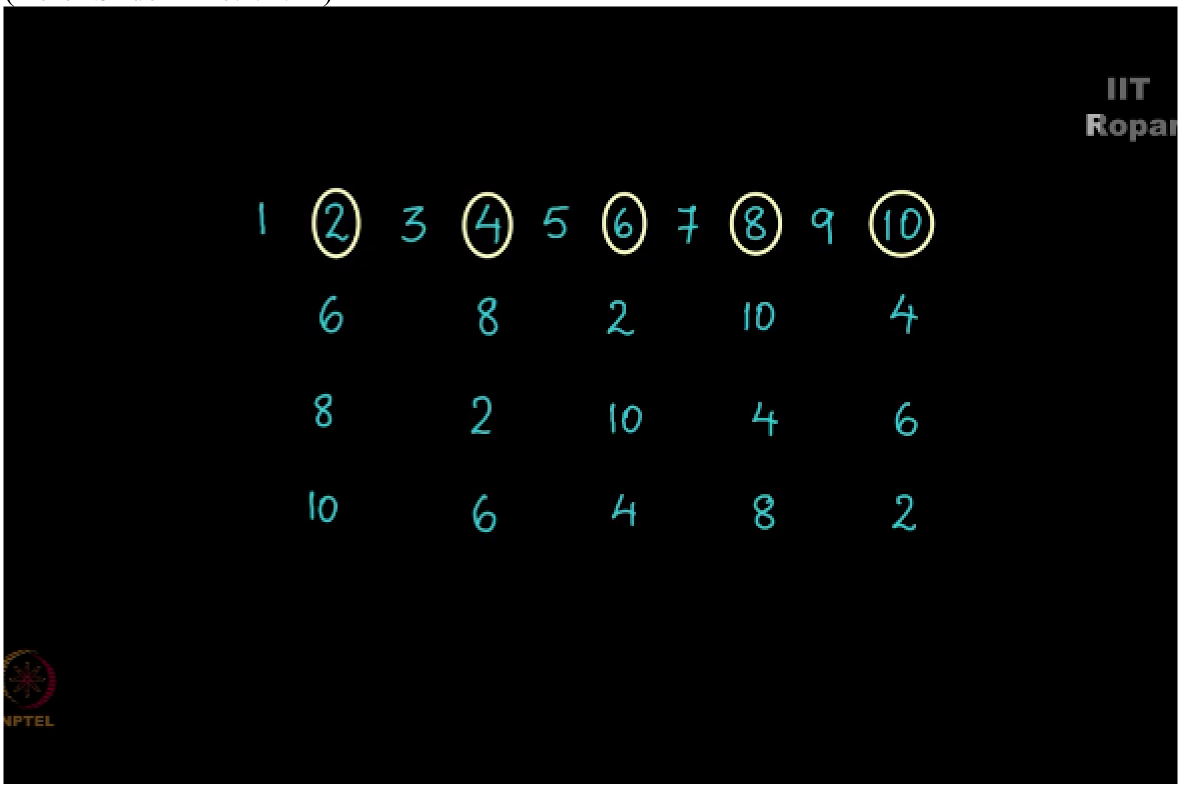

so you see in how many ways can the integers 1 to 10 be arranged such that no even integer is in its original position, I have shown you a few positions of the even integers.

Now let us start writing down the conditions, condition 1 is 2 is in its original position or in its own position, condition 2 is 4 is in its own position, and condition 3 is 6 is in its own position, condition 4 is 8 is in its own position, and condition 5 is 10 is in its own position,

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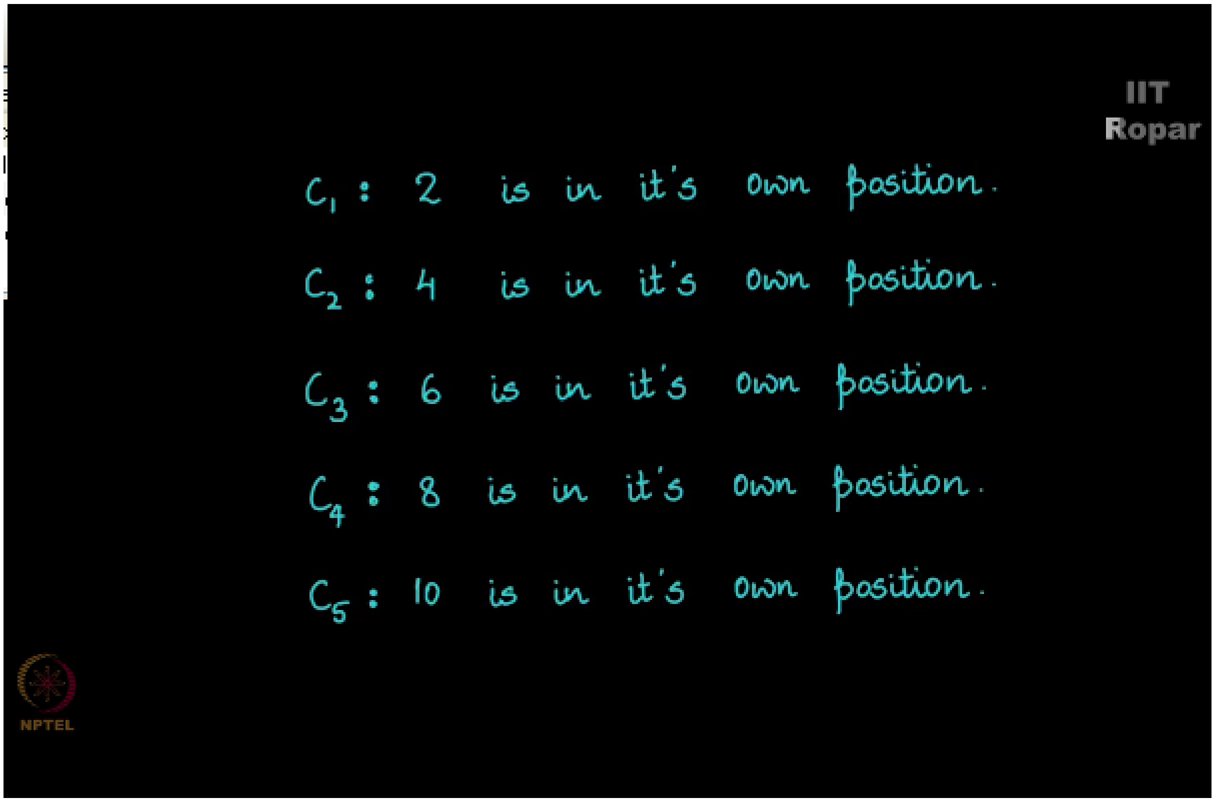

you see these are the 5 conditions, you might want to pause the video here and think about the problem for 5 to 10 minutes and probably you can try it out yourself, I'll be also giving you the solution now.

So you see we have the conditions here, what should we actually calculate? We must find out N(C1 bar, C2 bar, C3 bar, C4 bar, C5 bar), we must find out this, why?
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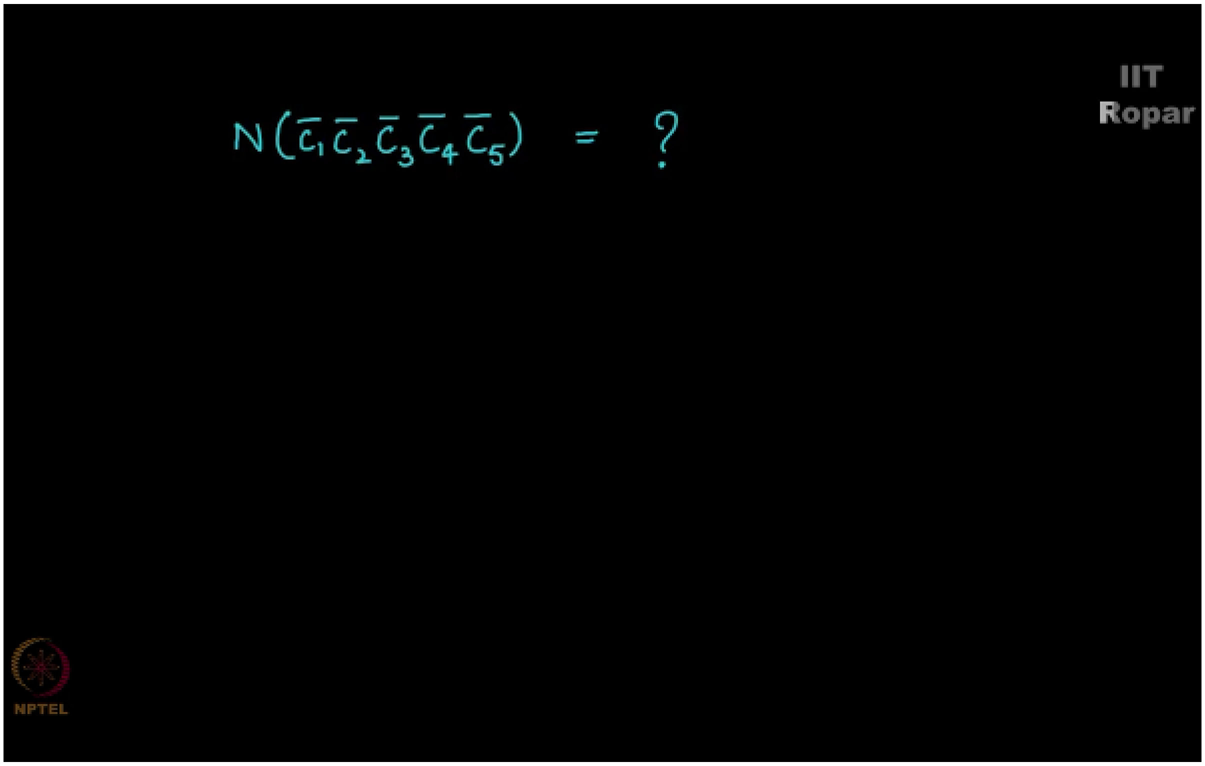

Because we are concerned only about the position of the even integers, we are not bothered about how to place the odd integers, the even integers must not take their position, right.

Now what is N(C1), before that let me tell you all possible permutations, all possible permutations happens to be 10 factorial, you must be knowing, why? Because there are 10 integers and you can arrange them in 10 factorial ways,
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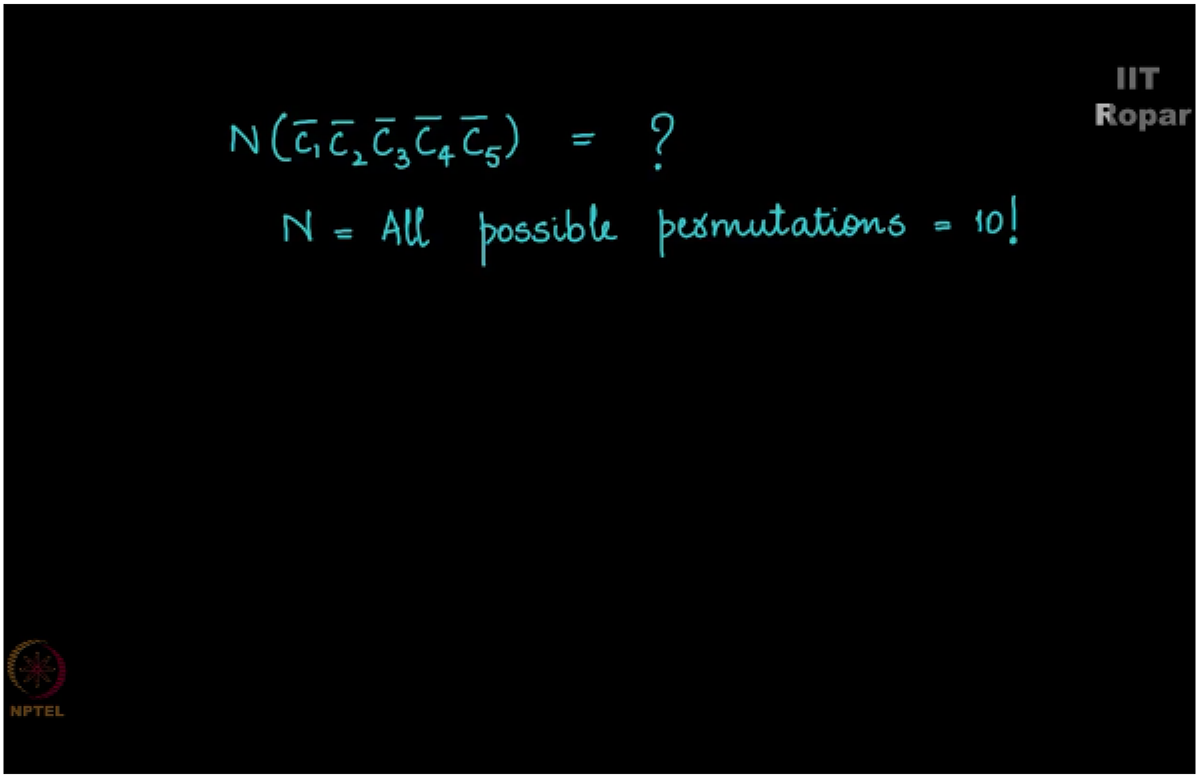

well, you must be knowing that we are over counting here, this is not the final answer because 10 factorial includes all those permutations where 2 is in its own position, 4 is in its own position and so on, so we must calculate only those conditions where even integers are not in their own position, right.

Now let us start by seeing what is N(C1)? N(C1) happens to be something like this, consider these 10 slots here,

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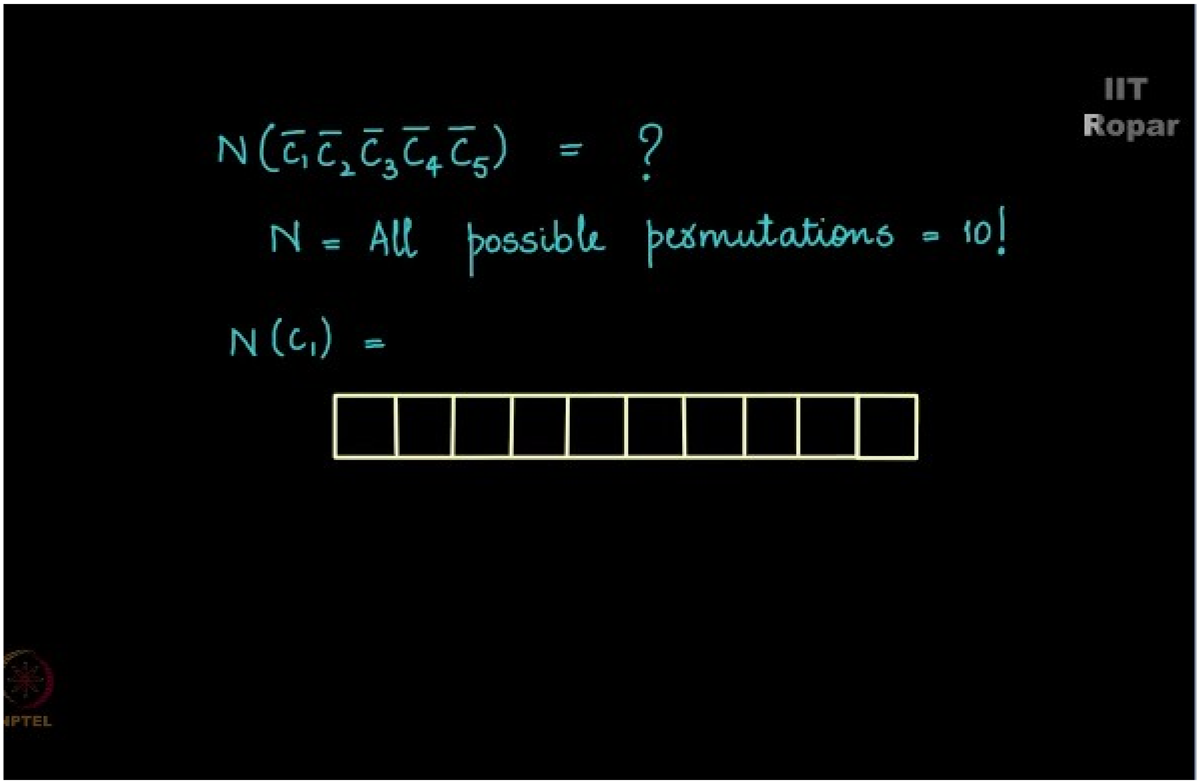

now 2 must be in its own position according to condition 1, so I placed 2 here, now I can arrange the rest of the 9 integers however I want, so something like this is a possible permutation,

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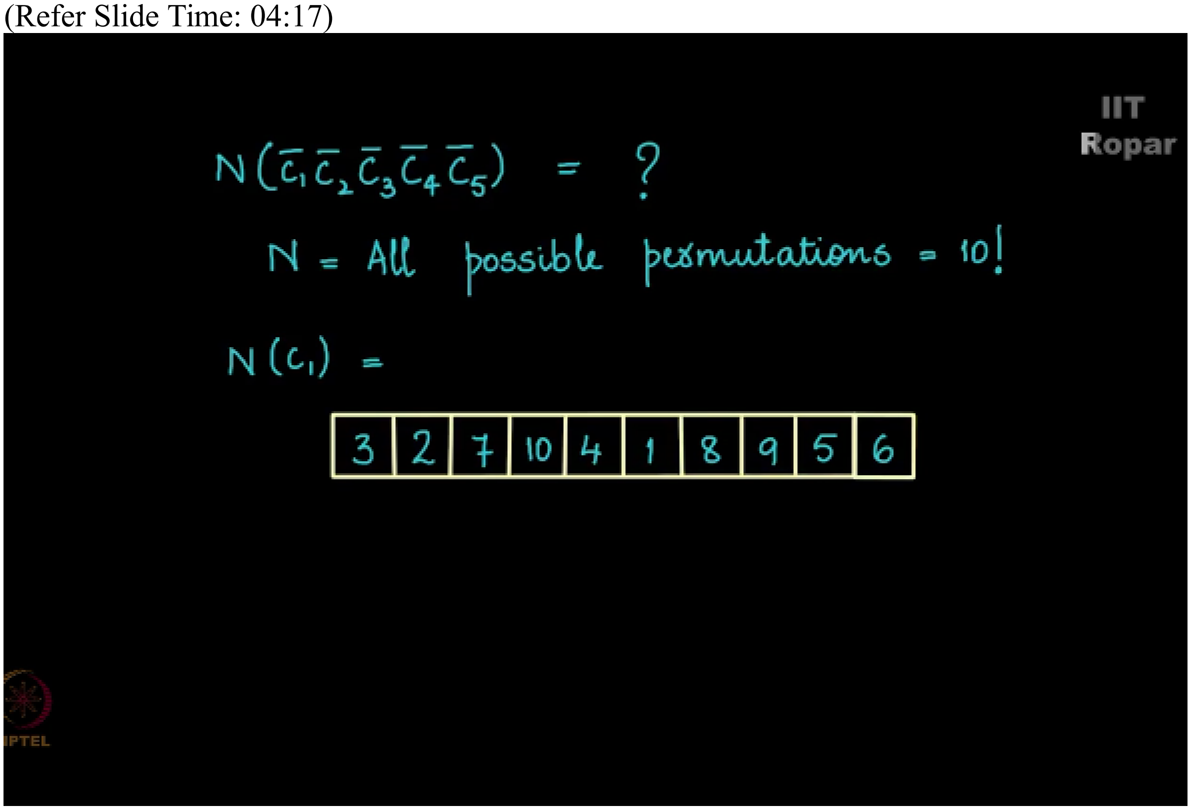

well, something like this is also another permutation,
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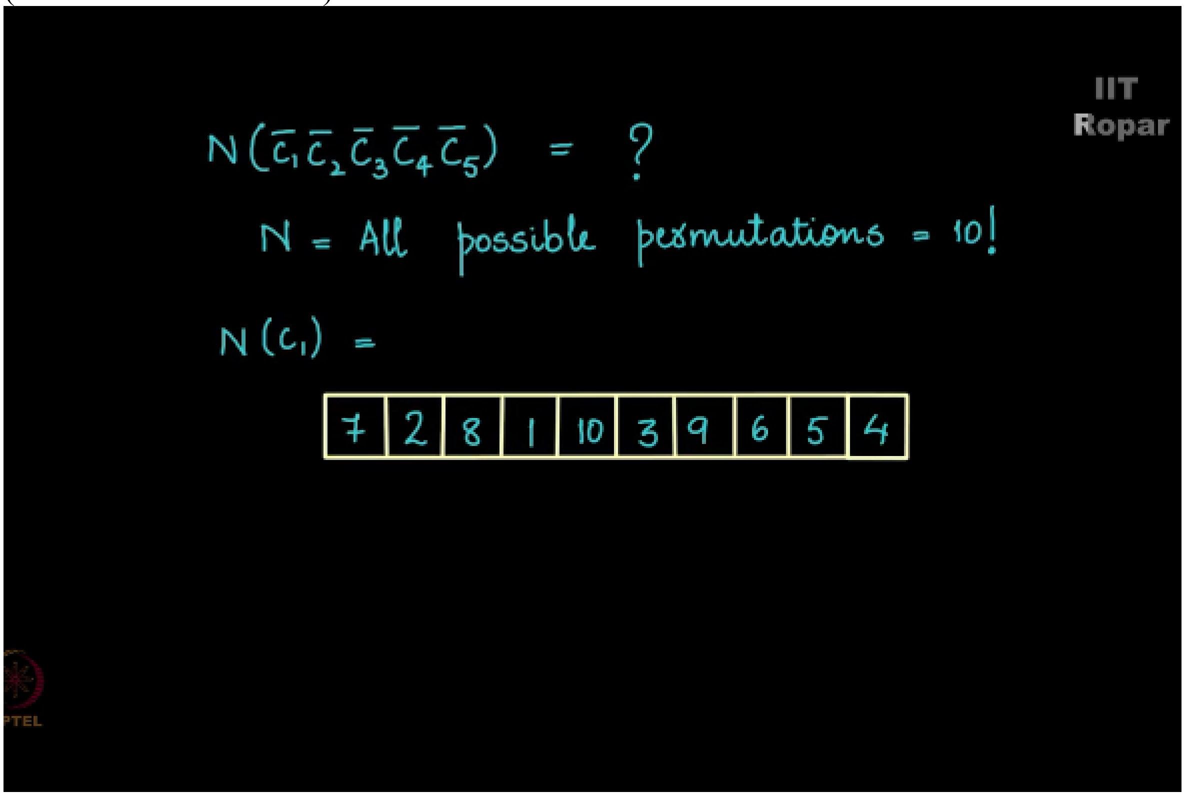

so apart from 2 we have 9 free integers and they can be permutated in all possible ways.

So in how many ways can 9 integers be permuted? 9 factorial, so N(C1) happens to be 9 factorial, because 2 is getting fixed here, we cannot change 2's position. Now N(C2), you must be very sure that N(C2) is also 9 factorial, why? You see 4 is getting fixed here,
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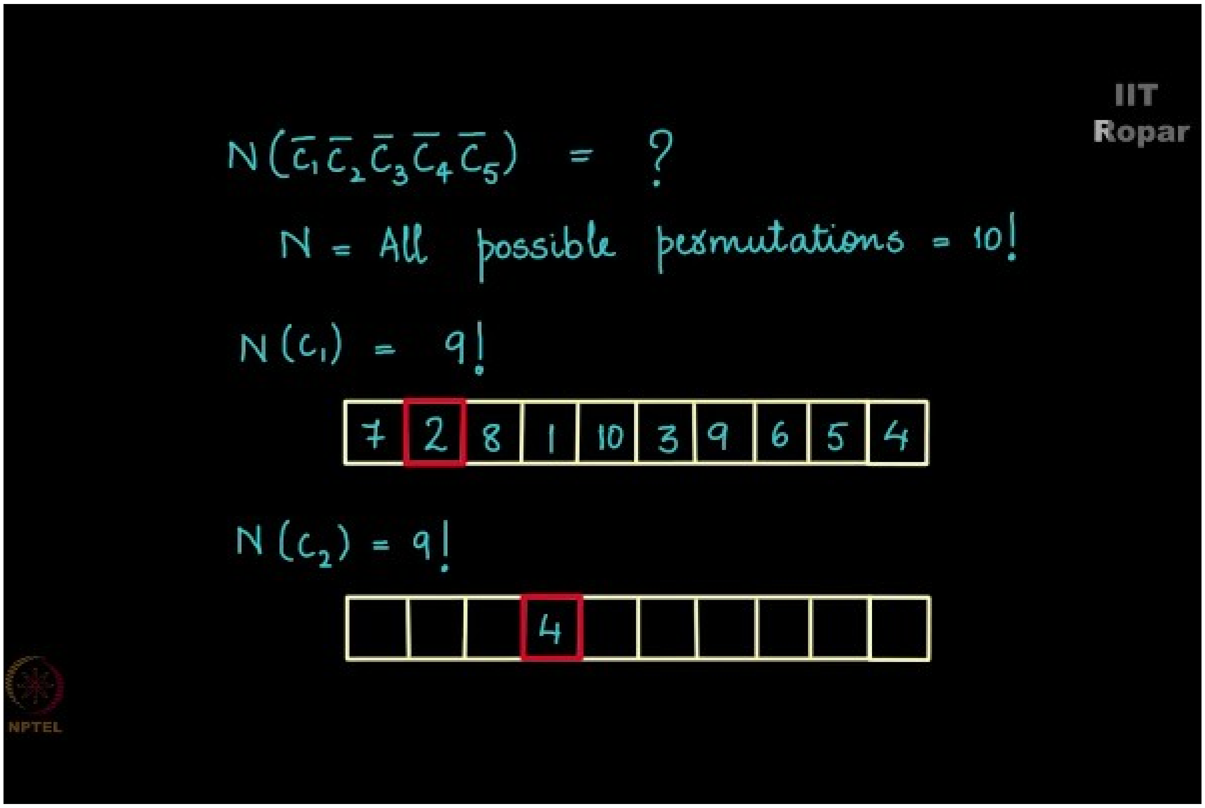

4 is in its own position and I can vary the position of other integers, and we have 9 integers left and they can be permuted in 9 factorial ways.

N(C3) is those permutations where 6 is getting fixed, and it is 9 factorial in number. Now N(C4) is where 8 is in its own position and those permutations where others can be arranged in earth, anyways so the number of those permutations are 9 factorial, 6 is fixed, 8 is fixed, so we have calculated this, the last one remaining is N(C5) and you see this is also N factorial, why? Those permutations where 10 is fixed and apart from that all others can be permuted in different ways.

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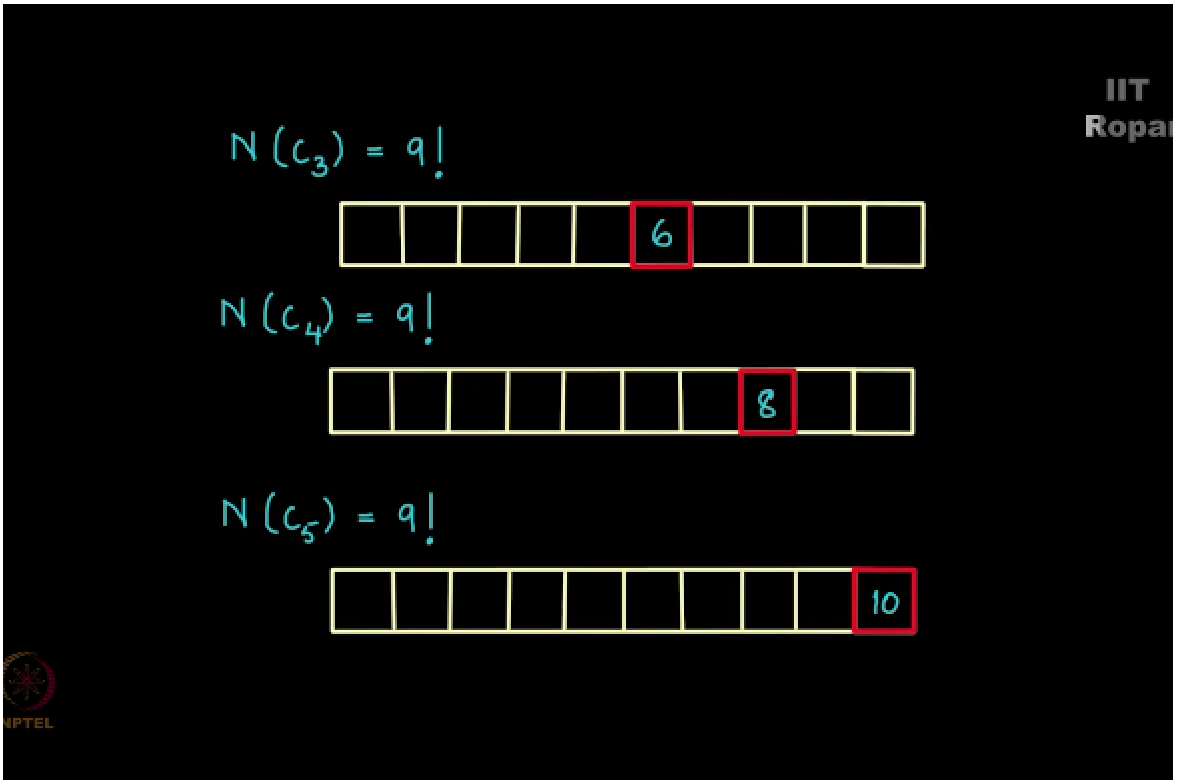

Now we have calculated N(CI) where I is in the range from 1 to 5, now let us move on to see what is N(C1,C2)? It happens to be 8 factorial, well, you must be sure of the reason, I'm not going to explain that, N(C1,C2) is what? It is those permutations where 2 is in its own position as well as 4 is in its own position, so these 2 are fixed, how many integers are free then? 8, and hence the number of ways in which you can arrange 8 integers is 8 factorial.

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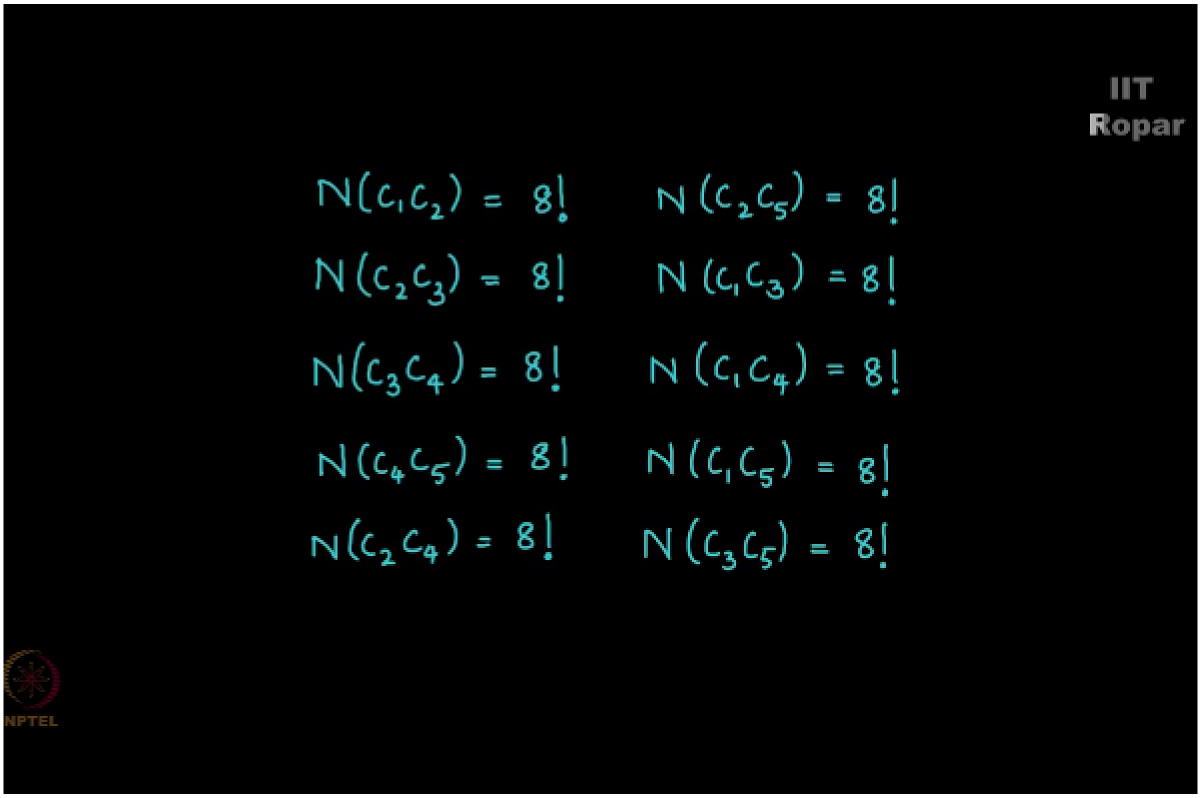

Well, it holds true for these conditions as well, N(C2,C3) (C3,C4) and so on, all of them will be 8 factorial, right, well you can check them individually if you are interested.

Now moving ahead let us see what is N(C1,C2,C3)? Now C1 is fixed, C2 is fixed, and C3 is fixed, 2, 4 and 6, the remaining integers are 7 in number and they can be arranged in 7 factorial ways, and now in general N(CI,CJ,CK) happens to be 7 factorial where I, J, K lies between 1 to 5, this was the condition where 3 are involved, 3 integers.

What if 4 integers are fixed? N(C1,C2,C3,C4) happens to be 6 factorial for the same analogous reason, now N(CI,CJ,CK,CL) is 6 factorial where I, J, K,L lie in the range 1 to 5, you must be knowing the reason now, and hence the last one to be found out is N(CI,CJ,CK,CL,CM) it is 5 factorial, 5 integers are fixed and we have 5 remaining, and hence they can be arranged in 5 factorial ways, right.

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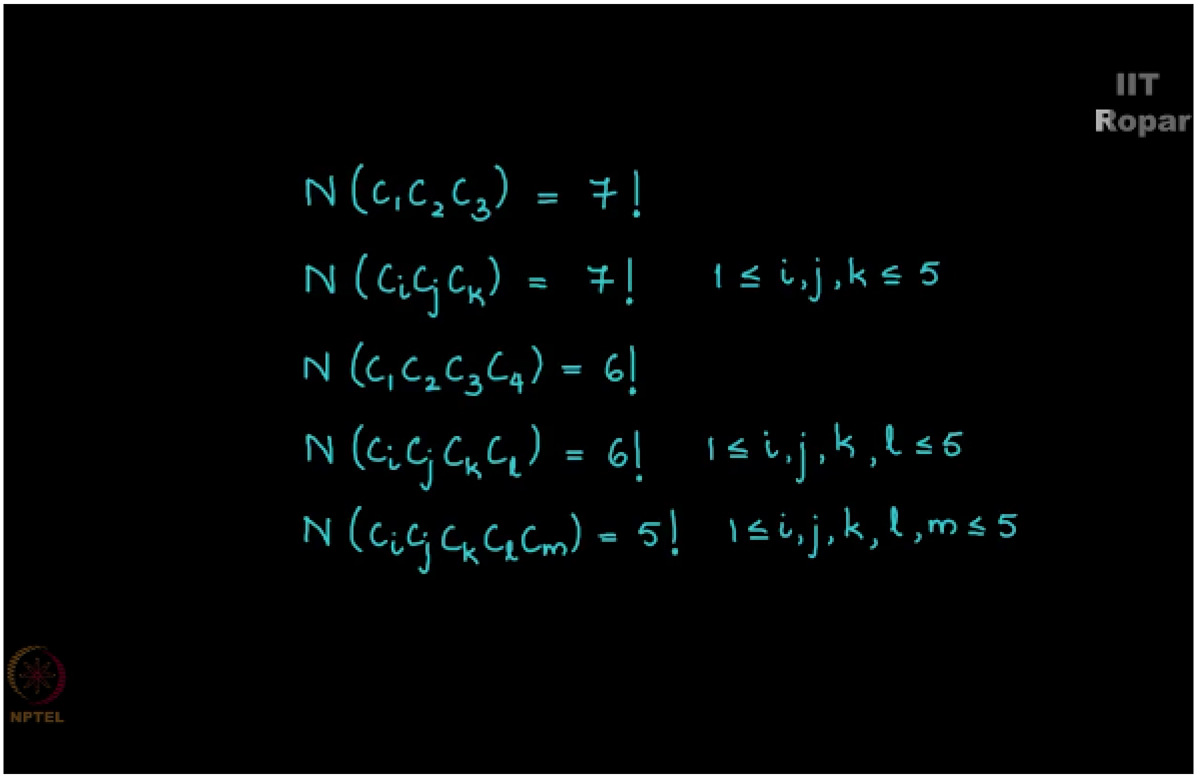

So here I,J,K,L,M come in the range 1 to 5, now what is the final answer? N(C1,C2,C3,C4) we have calculated all of them, now let us see N(C1 bar, C2 bar, C3 bar, C4 bar and C5 bar), this is 10 factorial, the all possible permutations – 5 choose 1 into 9 factorial + 5 choose 2 into 8 factorial, you must be knowing why I am choosing, why I am writing as 5 choose 1 and 5 choose 2, right, -5 choose 3 into 7 factorial + 5 choose 1 into, rather 5 choose 4, it is same as 5 choose 1 into 6 factorial – 5 choose 5 into 5 factorial,
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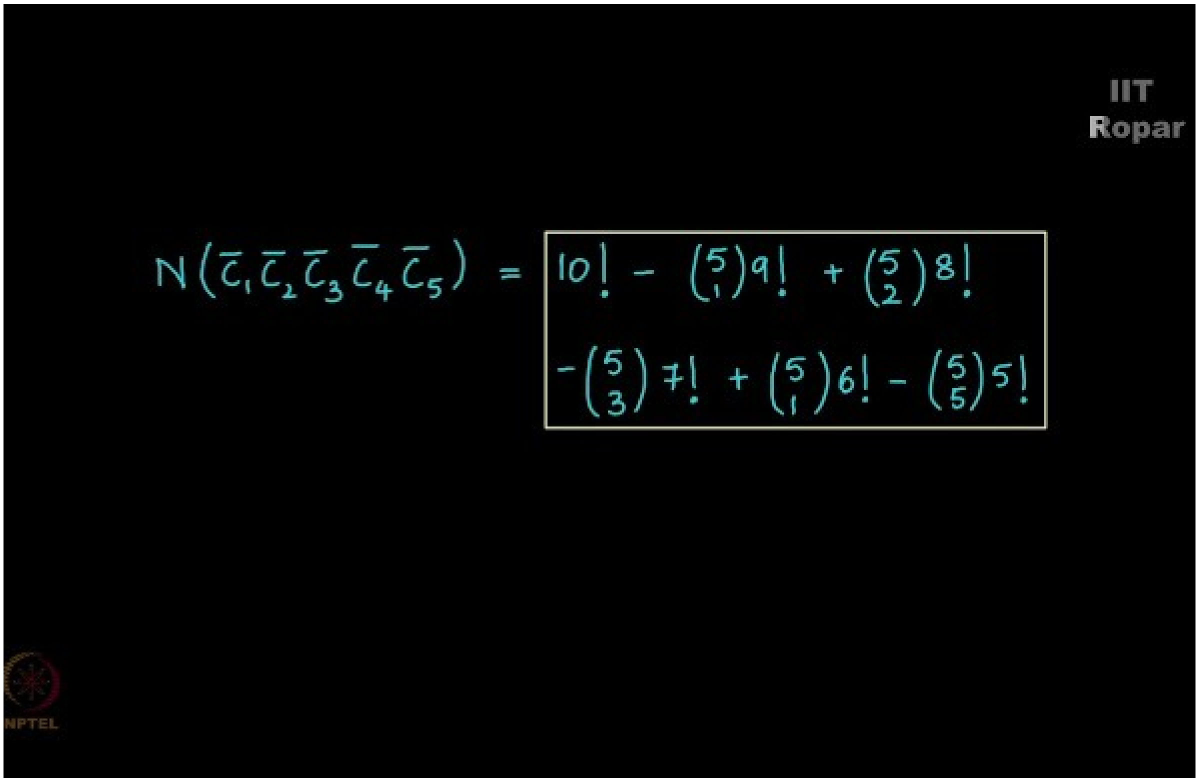

so in these many ways the integers 1 to 10 can be arranged such that no even integer is in its original position.

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