**Discrete Mathematics**
Principle of Inclusion and Exclusion

Example 16: Self grading

By
Prof. S.R.S Iyengar
Department of Computer Science
IIT Ropar

Consider the following question, a teacher gives a test to 4 students, he wants them to evaluate each other's test papers, without grading their own papers, in how many ways can he give the test papers to evaluate to the students, right.
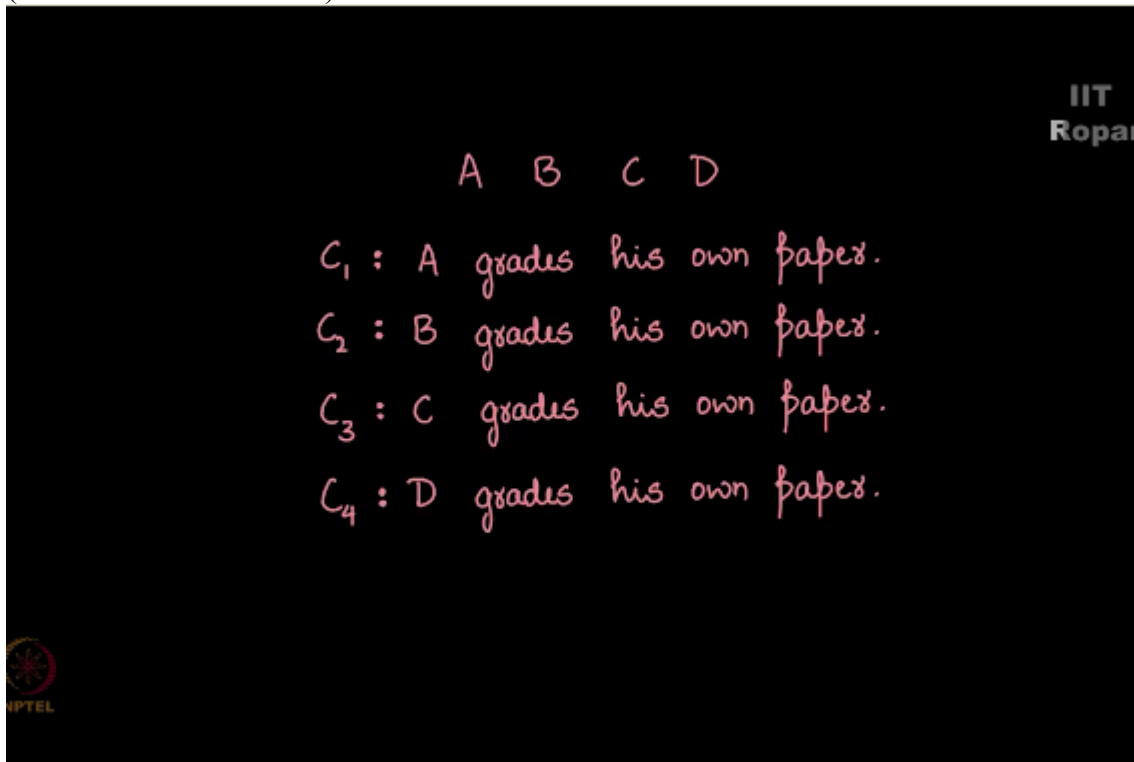(Refer Slide Time: 00:25)
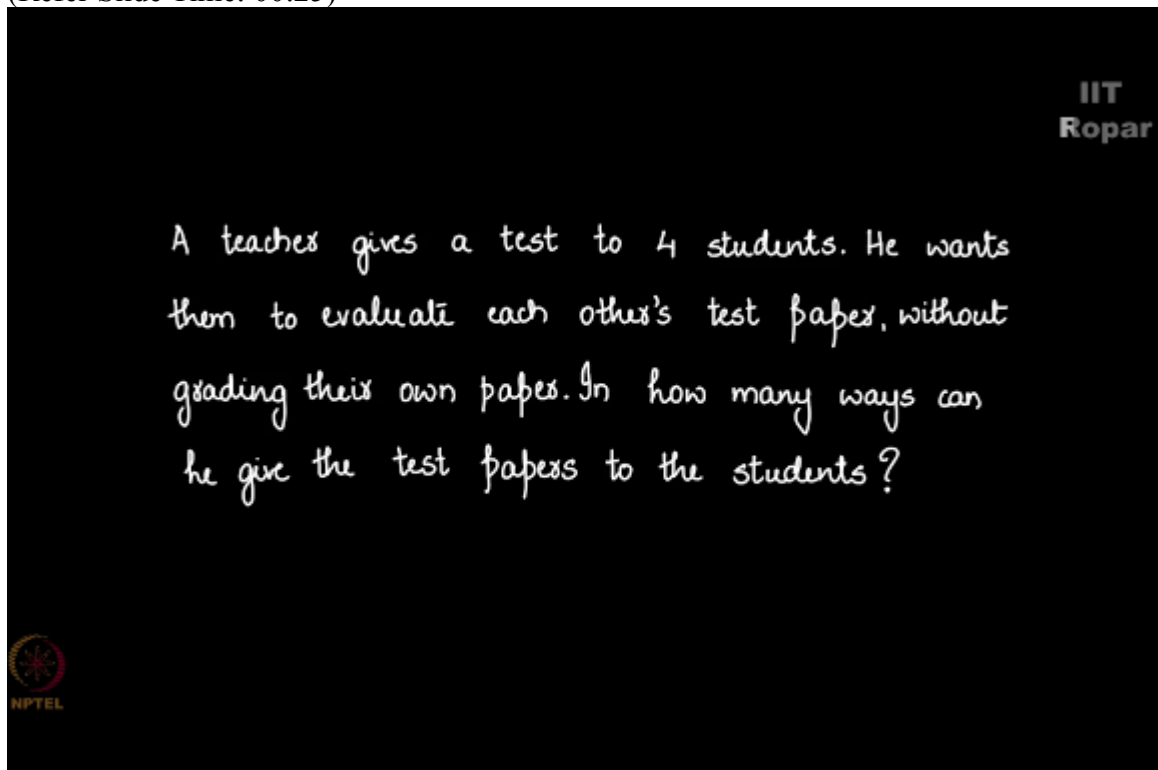
Now he's not evaluating their test papers here, he wants the students to do that, and there are 4 students here and they have to evaluate each other's papers without correcting their own papers, so we have these 3 points to keep in mind here, right.

Now let me name the four students as A, B, C and D, now you must be knowing the derangements problem by now and I suggest you do not use the formula directly, pause the

video here write down the conditions possible for this question formulate the conditions and proceed further.

Now if you are finding the difficult let me do it, you can also watch the video now, conditions C1 is A will grade his own paper, condition 2 is B will grade his own paper, and C3 will be C grade his own paper, and condition 4 will be D grades his own paper, right, these are the four conditions we have.
(Refer Slide Time: 01:41)

Now we have to find out the number of ways in which the teacher can distribute the test papers to the students, how? Such that nobody will evaluate their own papers that is precisely asking N(C1 bar, C2 bar, C3 bar and C4 bar),
(Refer Slide Time: 02:07)

A   B   C   D

C_1 : A grades his own paper.

C_2 : B grades his own paper.

C_3 : C grades his own paper.

C_4 : D grades his own paper.

$$N\left(\overline{C_1}\,\overline{C_2}\,\overline{C_3}\,\overline{C_4}\right)$$

now it is equivalent to the derangements problem of 4 numbers, you have already seen that.

Let me enumerate a few possibilities here A can grade B's paper, B can grade D's paper, C can grade A's paper, and D can grade C's paper, this is one such possibility, other way would be A can grade C, B can grade A, C can grade D, and D can grade B,
(Refer Slide Time: 02:43)

Derangements of 4 numbers.

A   B   C   D
↓   ↓   ↓   ↓
C   A   D   B

yes, so this is another such possibility, they can go on enumerating, but how many such possibilities are there? Well, we know that D4 has to be found out here and it is equal to 4 factorial – well the formula will come to help us here, right, you need not really write N(C1), N(C2) at this point, you can use the formula at this stage, once you know the conditions it is same that things are clear in your mind.

Now 4 factorial – 4 choose 1 x 3 factorial + 4 choose 2 x 2 factorial – 4 choose 3 x 1 factorial + 4 choose 4 x 0 factorial,
(Refer Slide Time: 03:42)

Derangements of 4 numbers.

$$A \quad B \quad C \quad D$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$C \quad A \quad D \quad B$$

$$D(4) = 4! - \binom{4}{1}3! + \binom{4}{2}2! - \binom{4}{3}1! + \binom{4}{4}0!$$

now simplifying this will give us the answer, this is equal to 4 factorial – 4 factorial/1 factorial x 3 factorial x 3 factorial + 4 factorial/2 factorial x 2 factorial x 2 factorial – 4 factorial/3 factorial x 1 factorial x 1 factorial + 4 factorial/0 factorial x 4 factorial x 0 factorial, well I have just expanded all the 4 choose 1, 4 choose 2, 4 choose 3 and 4 choose 4.
(Refer Slide Time: 04:23)

$$D(4) = 4! - \binom{4}{1}3! + \binom{4}{2}2! - \binom{4}{3}1! + \binom{4}{4}0!$$

$$= 4! - \frac{4!}{1!\,3!}3! + \frac{4!}{2!\,2!}2! - \frac{4!}{3!\,1!}1! + \frac{4!}{4!\,0!}0!$$

Now the next step would be to take out 4 factorial common, so it is 4 factorial x 1 – 1 factorial + 1 – 2 factorial + 1 -3 factorial + 1/4 factorial, now this can also be written as 4 factorial x 1-1 + 1/2 – 1/6 + 1/24, right, I'm just writing the values of the factorials, and simple calculation will tell me that this is equal to 9,
(Refer Slide Time: 05:04)

$$D(4) = 4! - \binom{4}{1}3! + \binom{4}{2}2! - \binom{4}{3}1! + \binom{4}{4}0!$$

$$= 4! - \frac{4!}{1!\,3!}3! + \frac{4!}{2!\,2!}2! - \frac{4!}{3!\,1!}1! + \frac{4!}{4!\,0!}0!$$

$$= 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right]$$

$$= 4!\left[1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right]$$

$$= 9$$

so in 9 ways the teacher can give the test papers to 4 students to evaluate where every student will evaluate others papers and not his own.

IIT MADRAS PRODUCTION

Founded by
Department of Higher Education
Ministry of Human Resources Development
Government of India

www.nptel.iitm.ac.in

Copyrights Reserved