NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Example 15: Bottles and caps

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Consider the following example, there are 3 different colored bottles each with a distinct cap, in how many ways can these caps be put on the bottles such that none of the caps are on the correct bottle,
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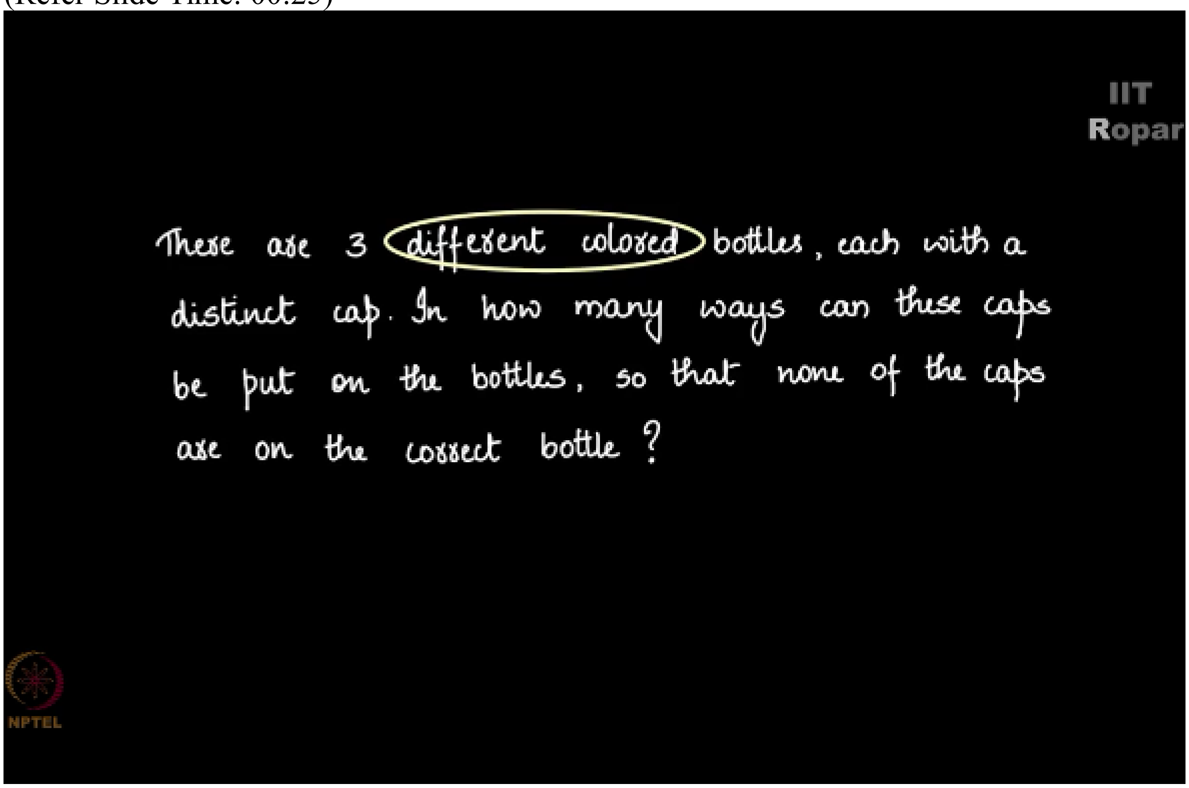

so there are bottles given to you they're different colors and now you must place them in such a way that none of the caps must be on the correct bottle, in how many ways can you do this?

Now let me name the bottles as B1, B2, and B3, you see there are 3 bottles here of red, blue and green color,
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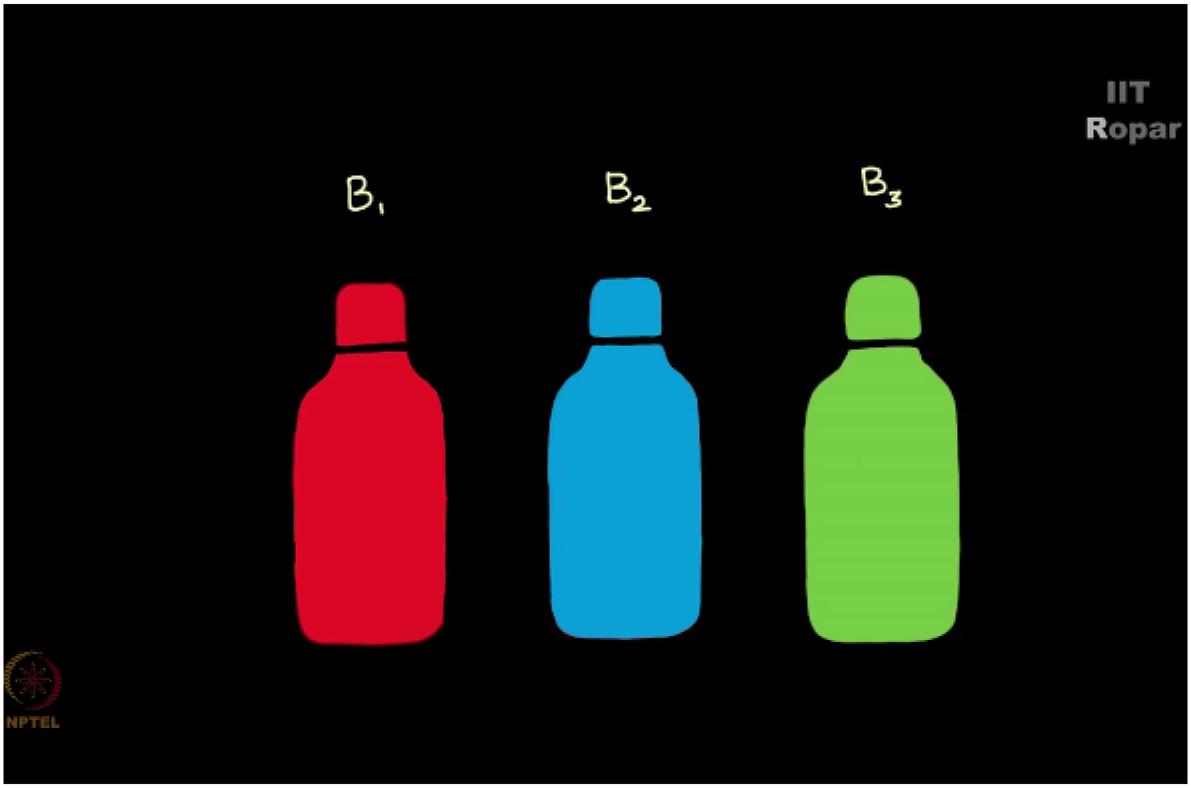

now you must place the caps such that none of the caps must be on the correct bottle, so red cap should not be on the red bottle and so on, in how many ways can you do it? Like this, this, and this, you see here,

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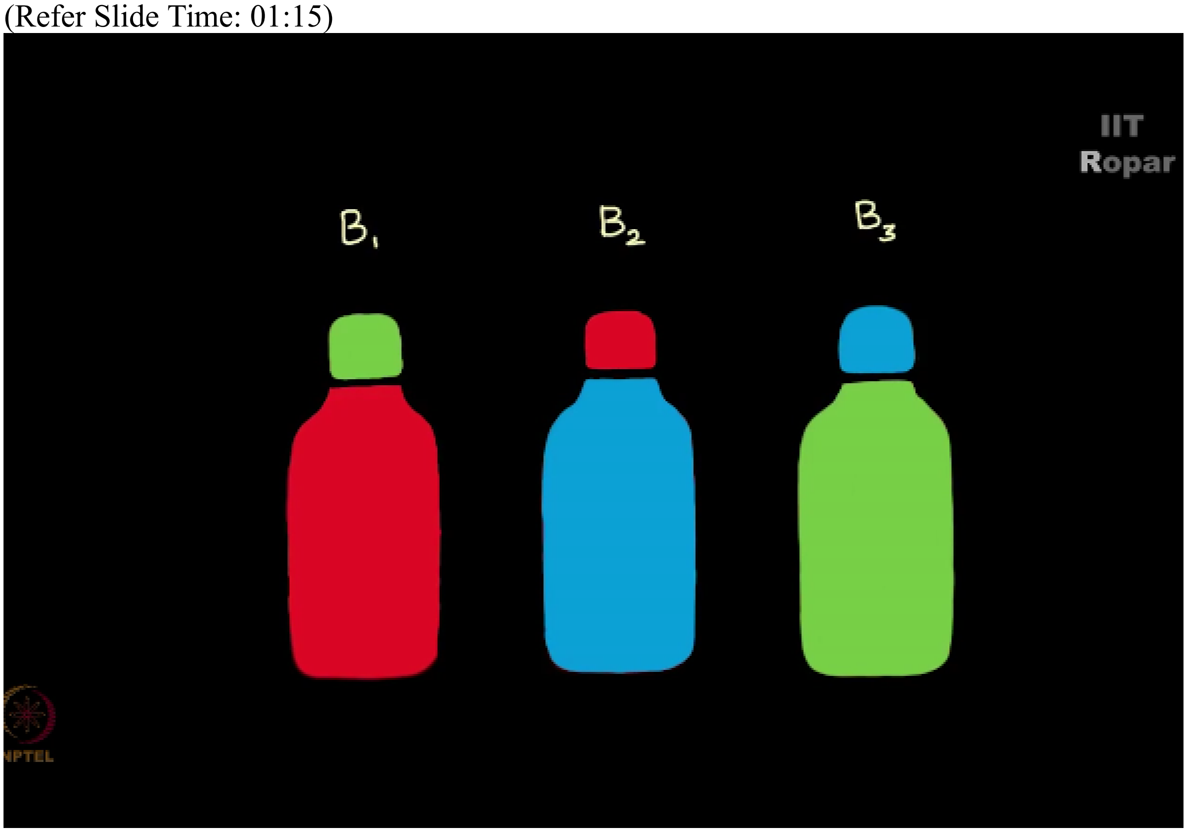

green cap is on the red bottle, red cap is on the blue bottle, and blue cap is on the green bottle, well this is another such permutation,

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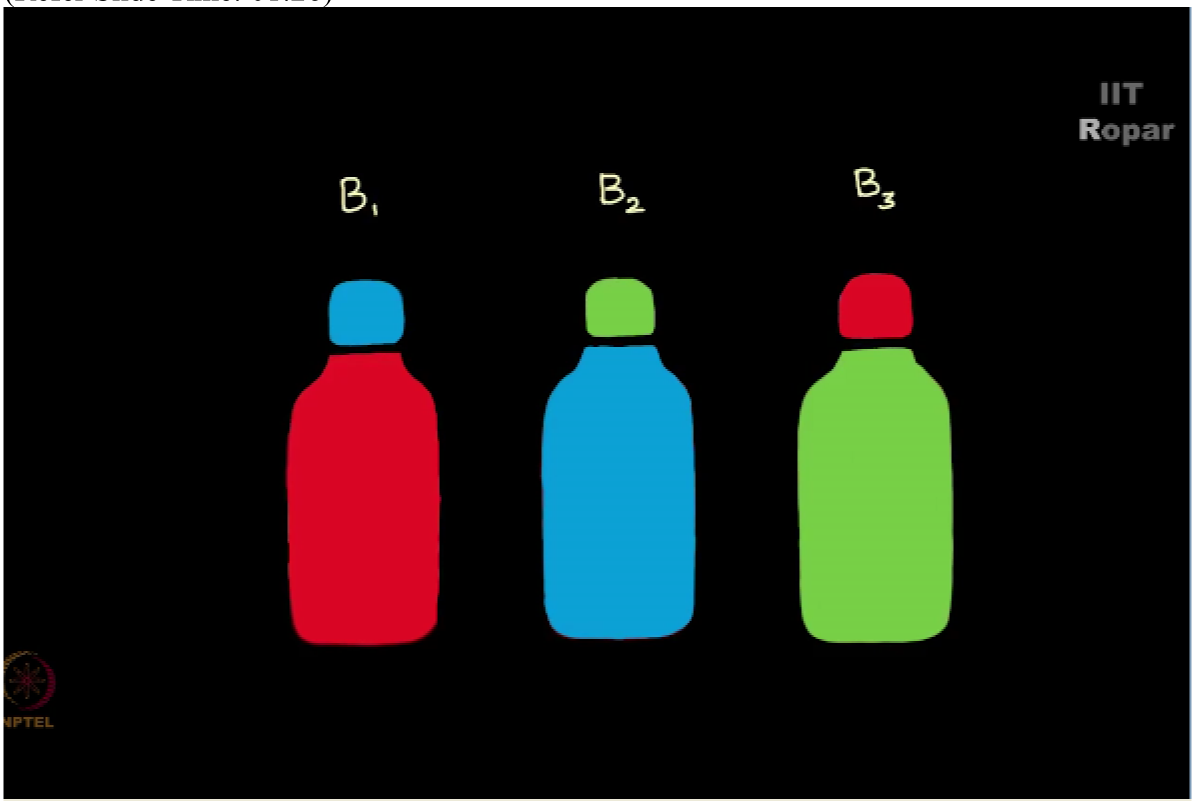

you see here blue on red, green on blue, and red on green, so in how many ways can you do it? Let us see, are these only two possibilities? Let us check, so the conditions here goes like this C1 will be the condition where B1 has itself its cap on itself, C2 is the condition where bottle 2 has its cap on itself and C3 is the condition where B3 has its cap on itself, so these are the 3 conditions.

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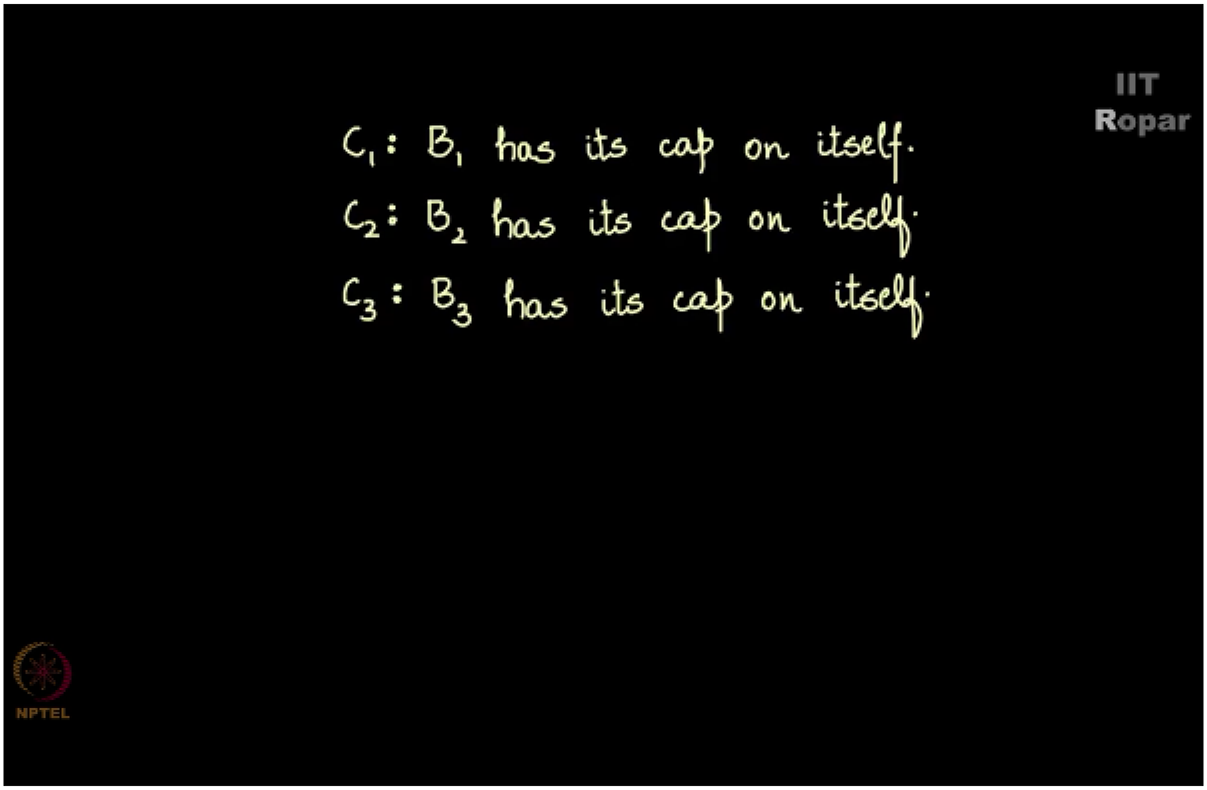

Now the question is what is N(C1 bar, C2 bar, and C3 bar), why? Because you have to find out the number of permutations where none of the caps are on themselves,
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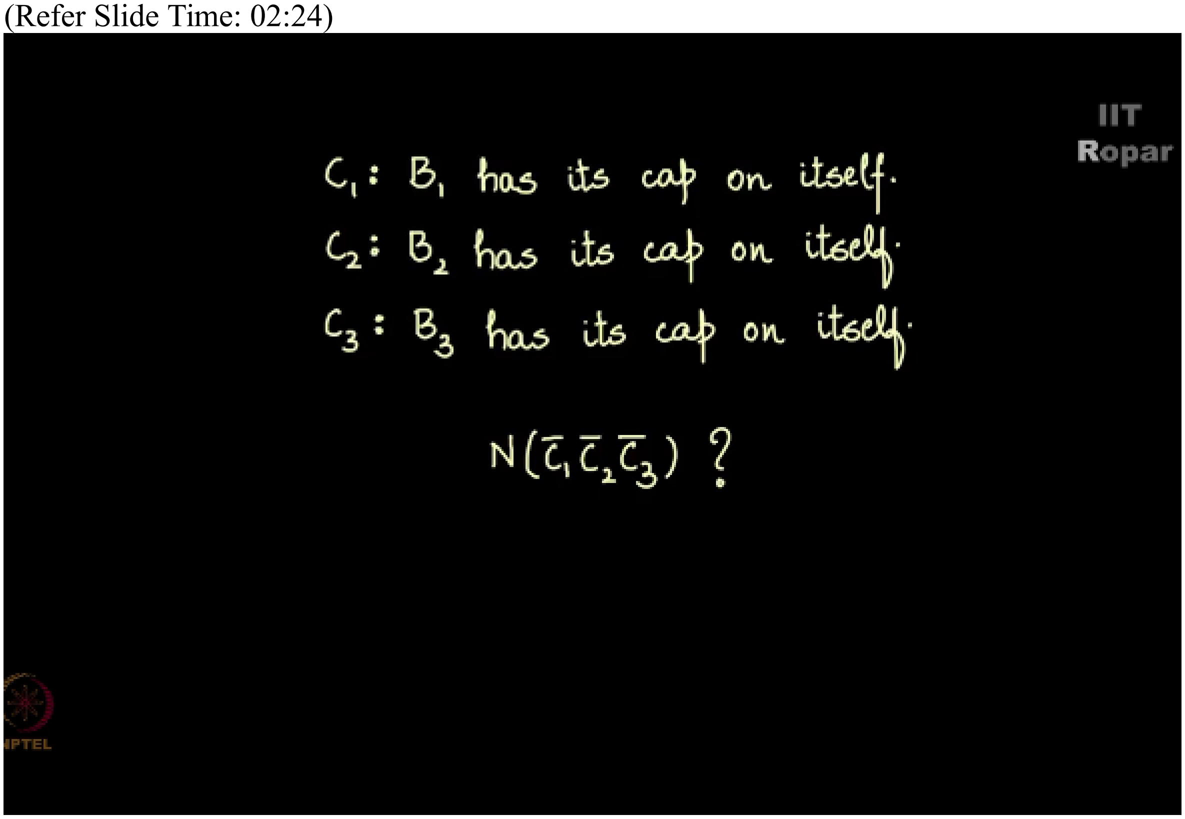

right, the same colored caps that is, so we know that all possible permutations of these 3 bottles where the caps are not on the correct bottles are 3 factorial, right, you can have all possible permutations in 3 factorial ways, right.

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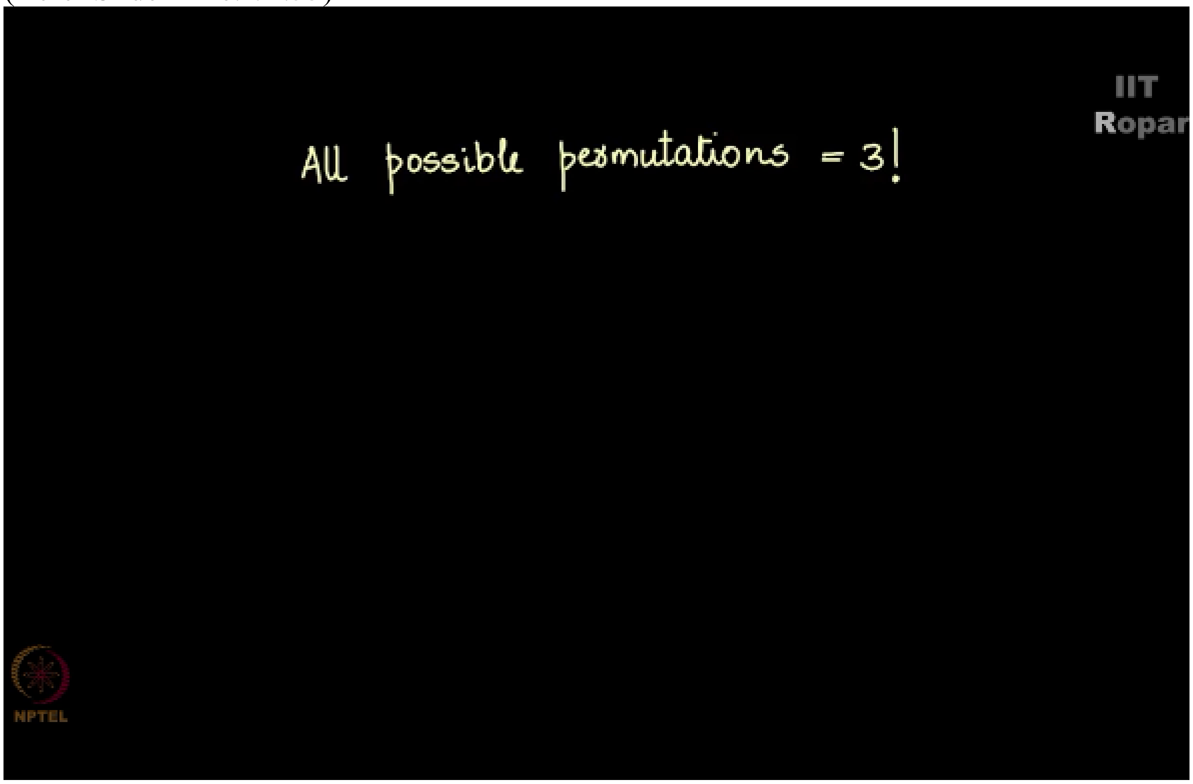

Now you know what is N(C1) here? N(C1) means the number of ways in which B1 has its cap on itself, in how many ways can you do this? It is basically 2 factorial ways, why? Assume B1 has its cap on itself, right, now you have options for the caps of B2 and B3, B2's cap on itself, B3's cap on itself, and B2's cap on B3, B3's cap on B2, these are the 2 possibilities which is precisely 2 factorial, right.

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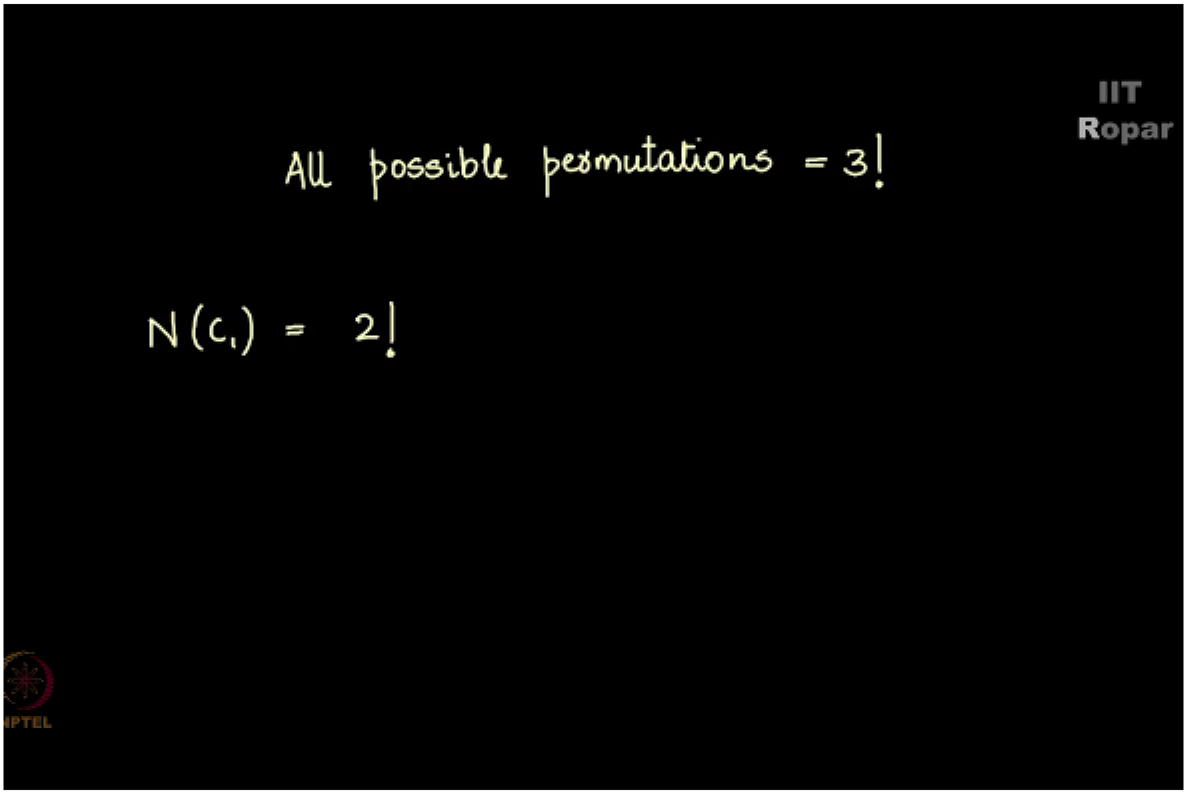

The same holds true for N(C2) as well, which is you're fixing upon the cap of B2 and you're changing the other two, the same holds true for N(C3) as well, you're fixing upon the bottle B3, the cap of it and you are changing the caps of B1 and B2, right, now we have found out N(C1) and N(C2) and N(C3).

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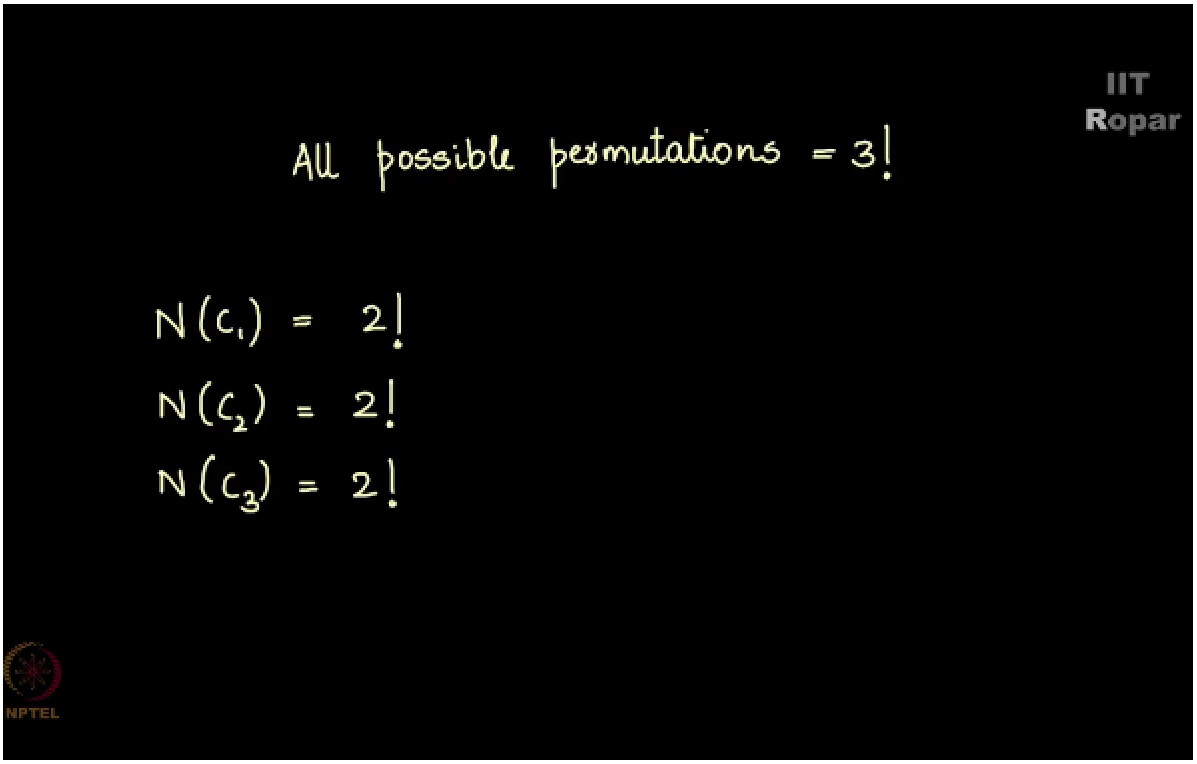

Let us move ahead and see what is N(C1,C2), what do we mean by N(C1, C2)? It is the number of ways of changing the caps where you're fixing C1, C2, that is the cap of bottle 1 and bottle 2 are fixed that is red has red cap on itself, blue bottle has blue cap on itself, right, the only option left is the green bottle, can you do any changes with it? No, it will remain on itself, so N(C1,C2) happens to be 1, don't you think it is the same for N(C2,C3) as well, here you're just keeping the red cap, rather the blue cap and the green cap on itself, the only possibility for red cap is to be on the red bottle, think about it, the same holds true for N(C1,C3) as well, there is only one possibility.

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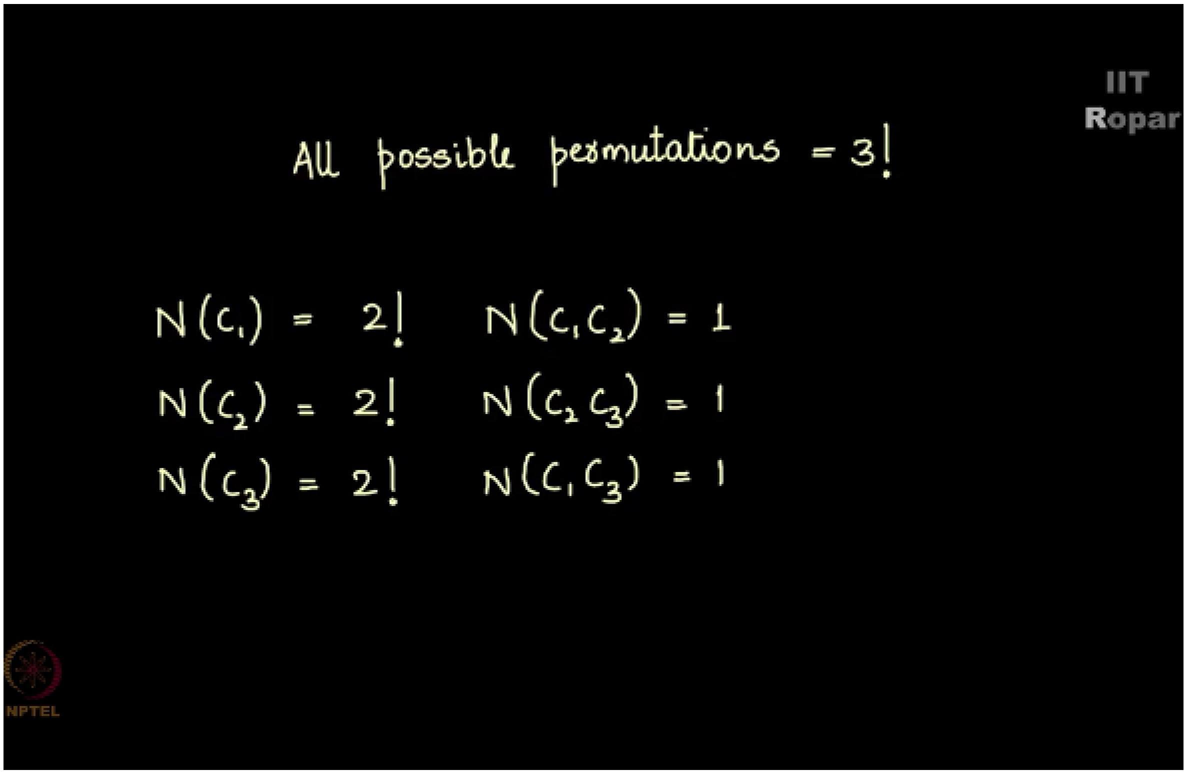

Now what happens to N(C1,C2,C3), you must be very sure that it is 1, you keep all the caps on their respective bottles, in how many ways can you do this? It's just one, right, now we have found out N(C1), N(C2), N(C1,C2) (C2,C3) (C1,C3) and N(C1,C2,C3), these are the available informations.

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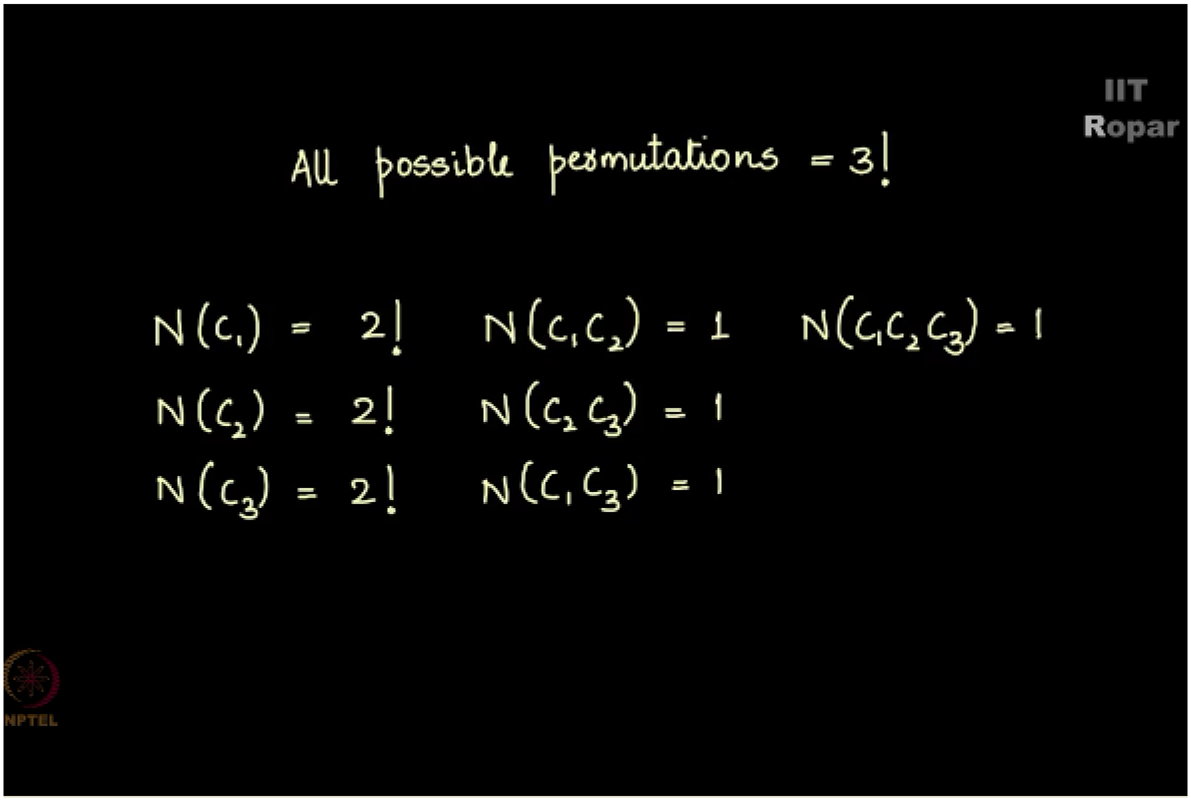

Now the immediate step is to find out N(C1 bar, C2 bar, and C3 bar), what does the formula go like? It goes like this N-N(C1) + N(C2) + N(C3) + N(C1,C2) + N(C2,C3) + N(C1,C3) - N(C1,C2,C3), be very careful with the toggling of + and – signs and the insight information, right, be very careful about it.

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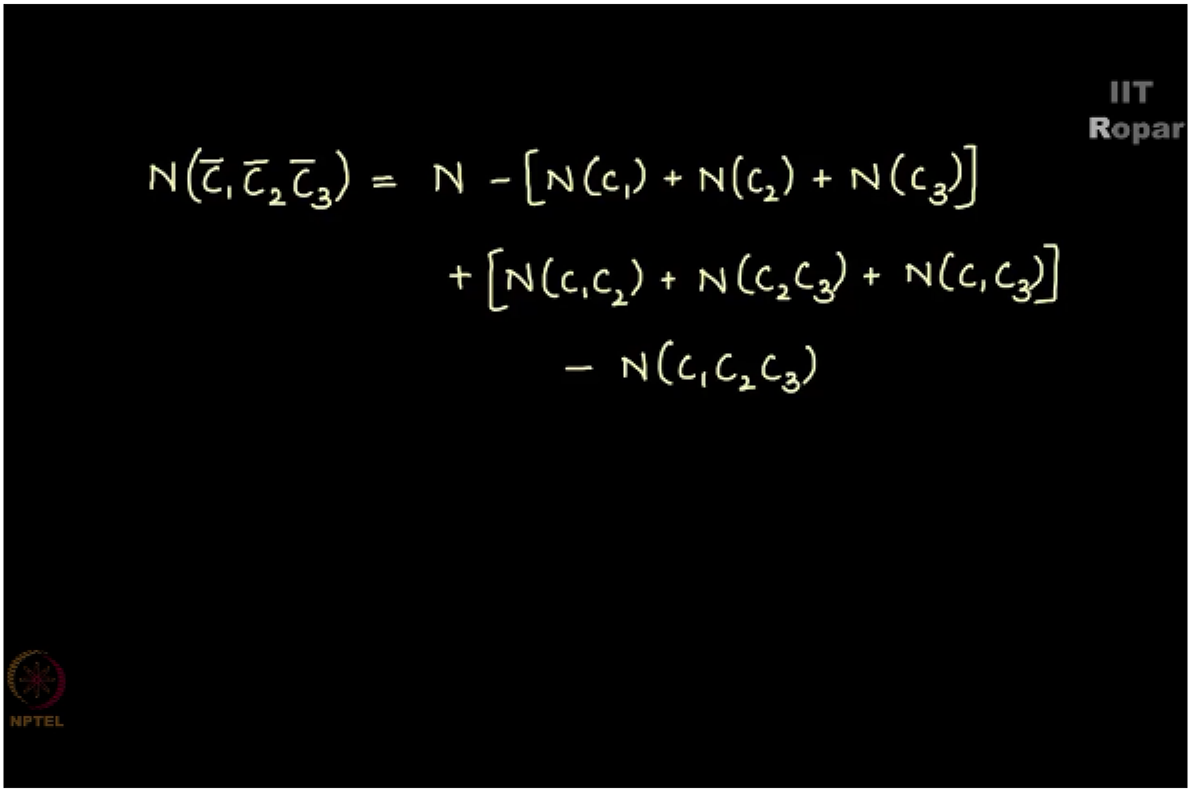

Now substituting the values we get it as 3 factorial – 3 choose 1 x 2 factorial + 3 choose 2 x 1 factorial – 3 choose 3 x 0 factorial, now taking out 3 factorial common we get it as 1 – 1/1 factorial + 1/2 factorial – 1/3 factorial, how will you get this? Expand 3 choose, 1 3 choose, 2 3 choose 3 and the previous steps, it will be very clear.

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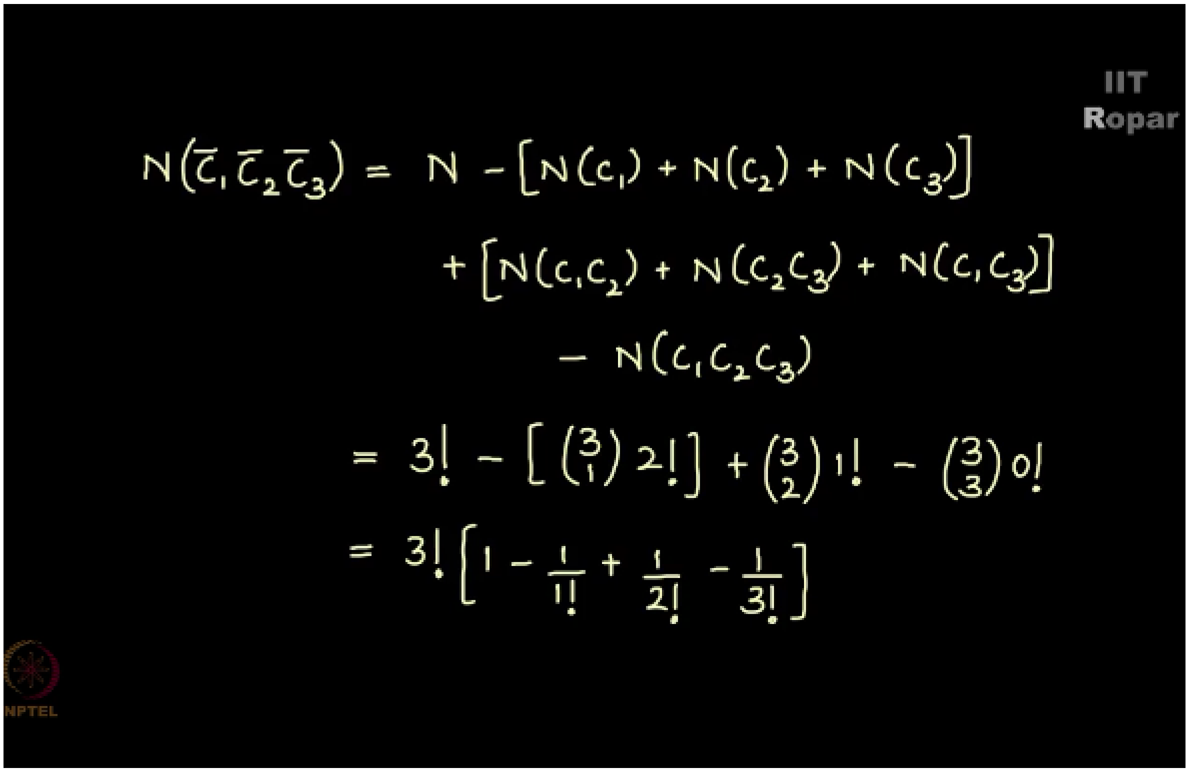

Now solving this we see that N(C1 bar, C2 bar, C3 bar) happens to be 2, well what does this mean? This means that the number of ways in which these caps can be put on the bottles so that none of the caps are on the correct bottle, the number of ways is 2, so in these many ways you can arrange the caps in such a way that none of the caps are on their respective bottles, so what is the moral of the story? The moral of the story is that the number of derangements of 3 items is 2, don't you think so, instead of 1, 2, 3 I just had 3 bottles here, you can check it out with the numbers 1, 2, 3, you will see that the number of derangements is 2.

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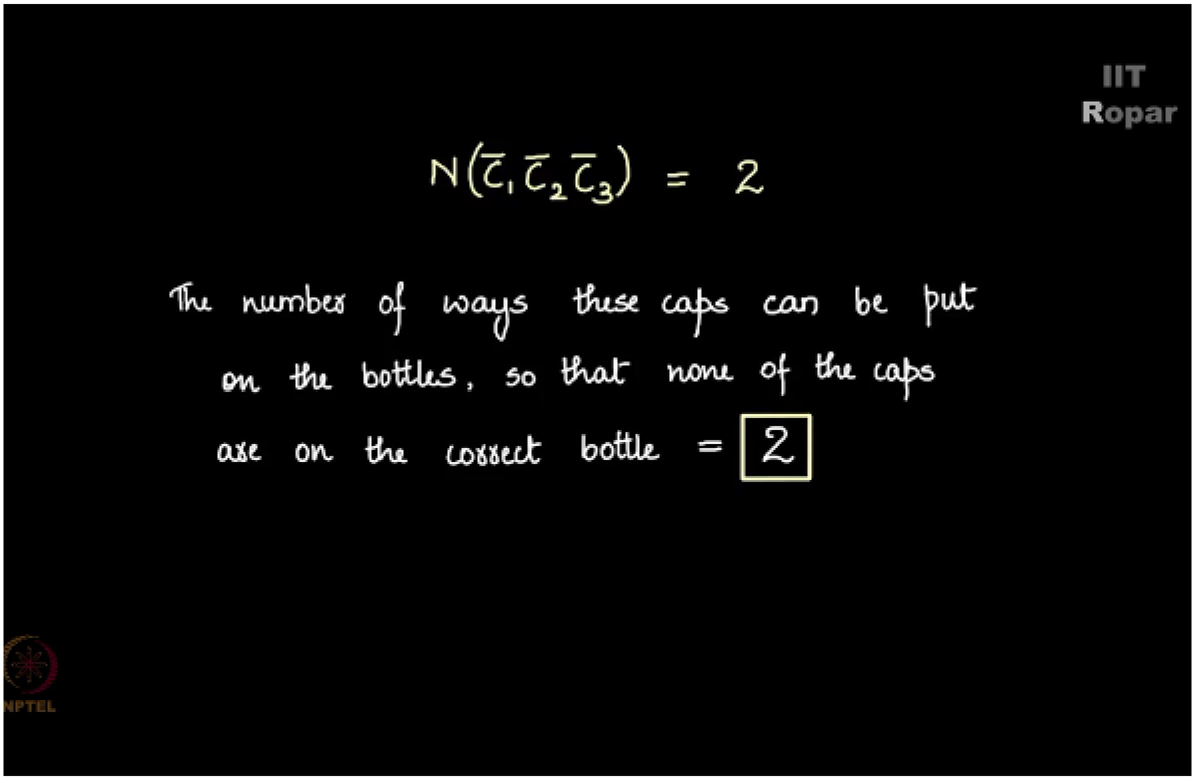

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