NPTEL

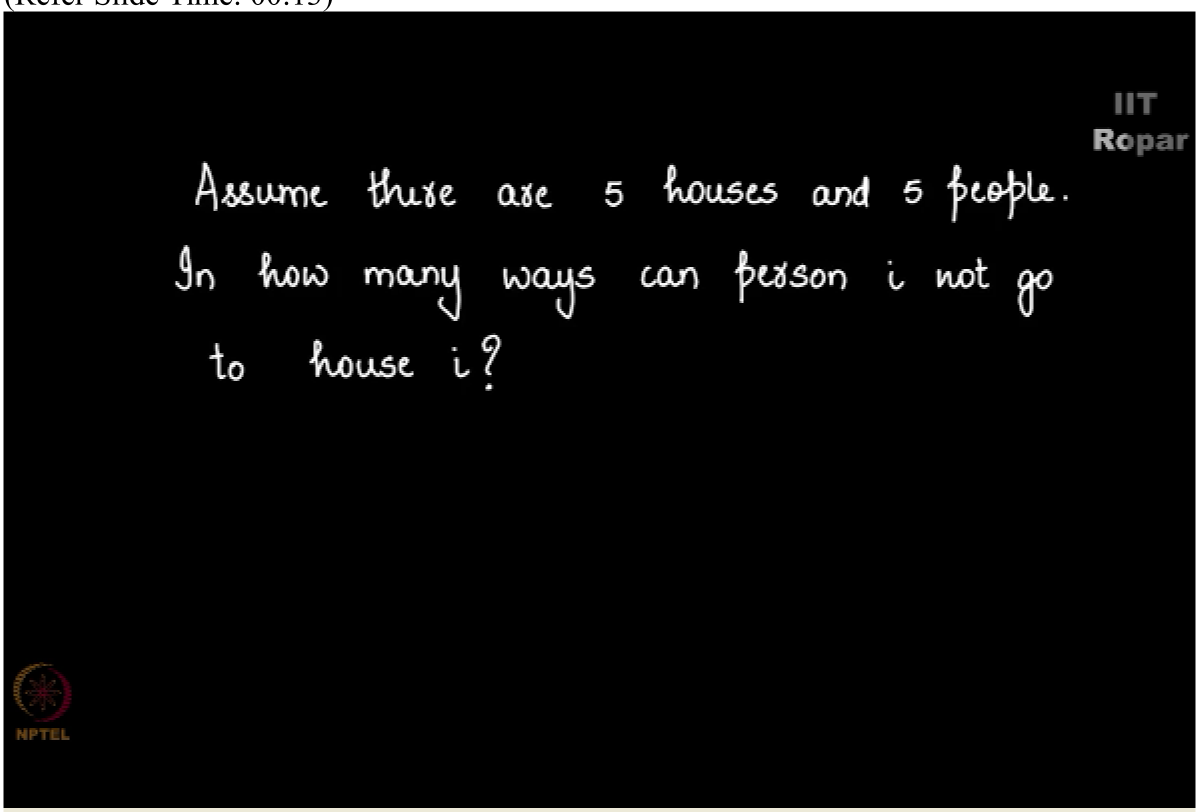
NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

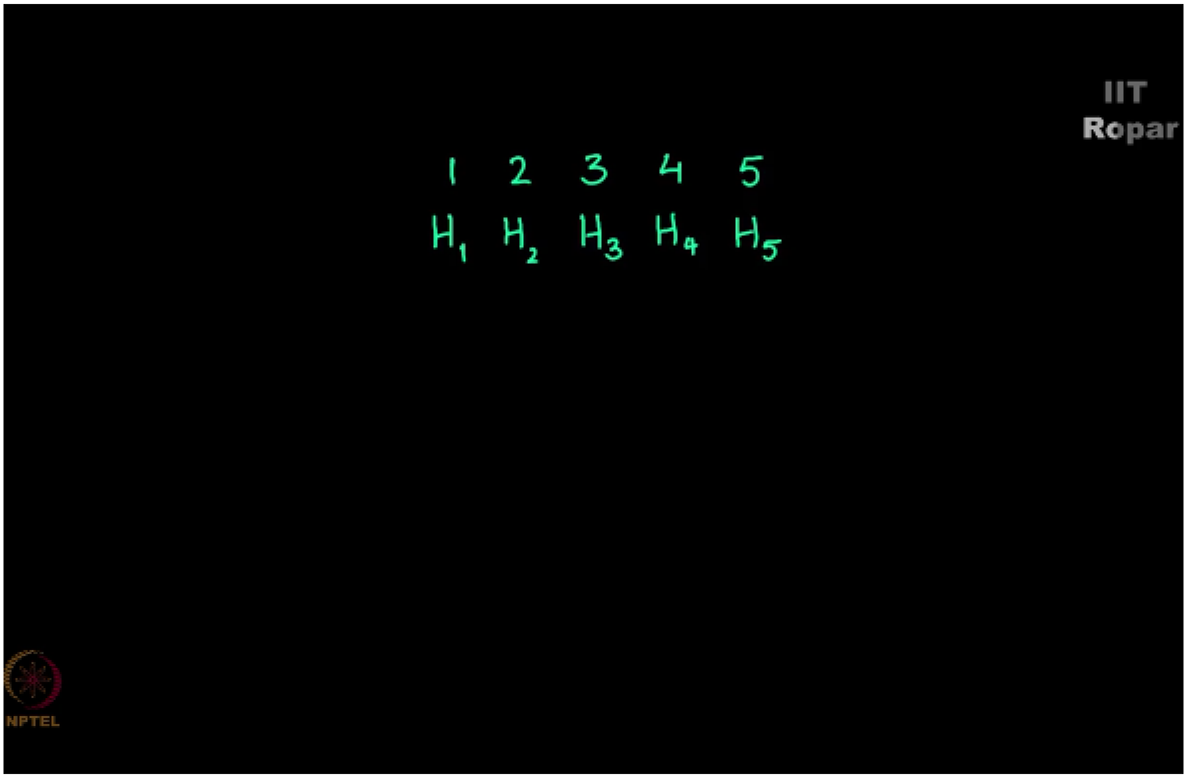
Example 14: No one in their own house

By
Prof. S.R.S Iyengar
Department of Computer Science
IIT Ropar

Assume there are 5 houses and 5 people, in how many ways can person I not go to house I,
(Refer Slide Time: 00:13)

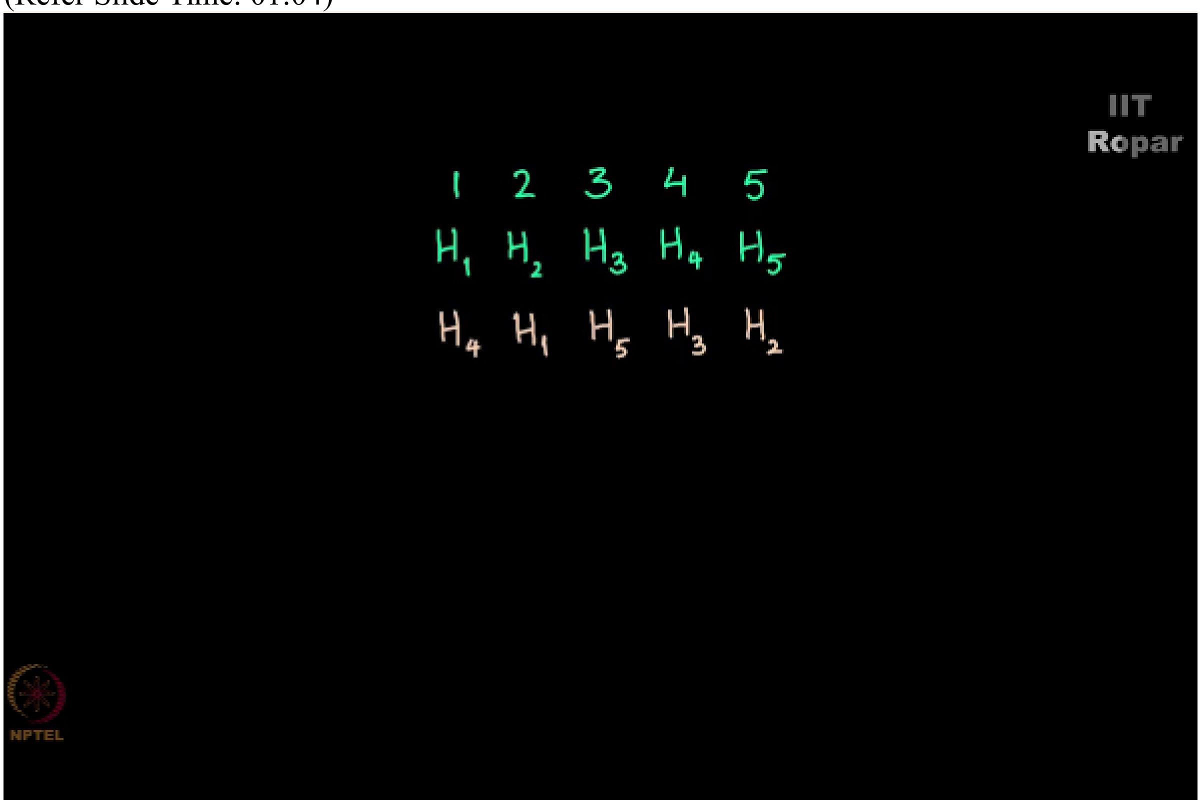


you see we have 5 houses and 5 people, person I cannot go to house I, what does it mean let me tell you, we have 5 people 1, 2, 3, 4, 5 and 5 houses like this H1, H2, H3, H4 and H5,
(Refer Slide Time: 00:36)



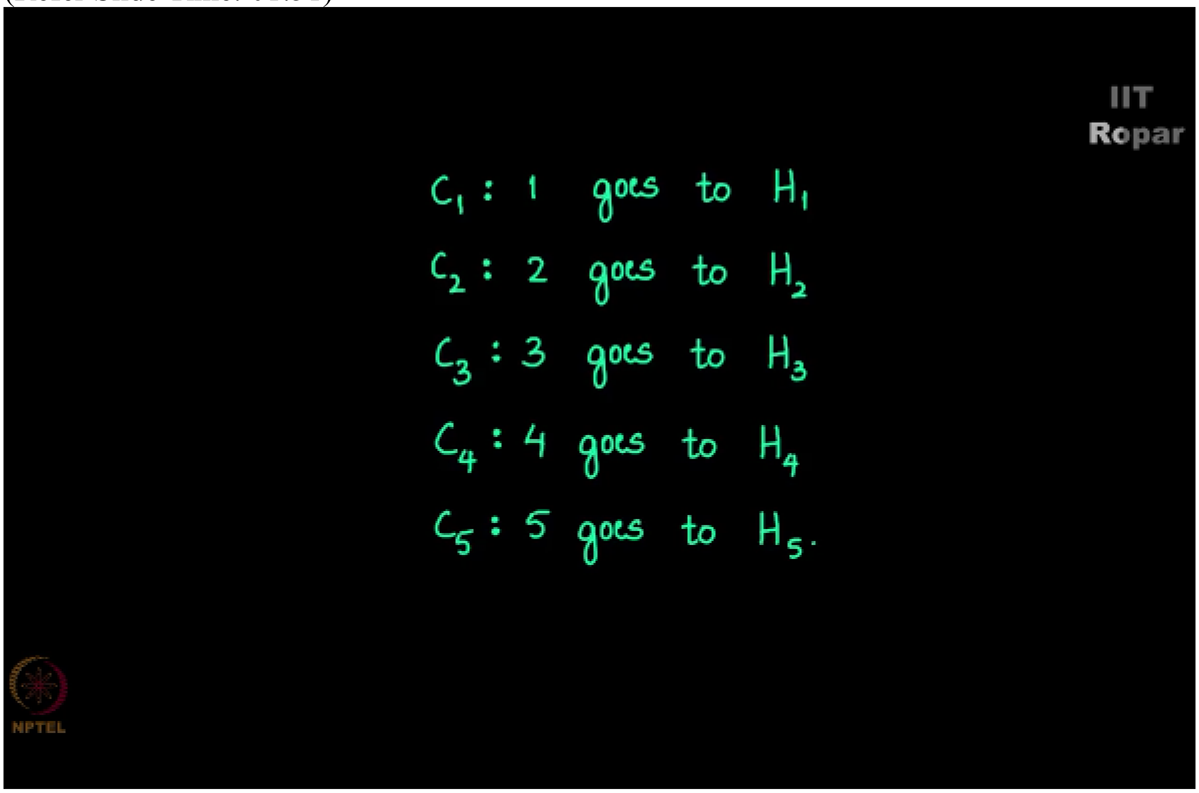
but the condition given is that 1 cannot go to H1, 2 cannot go to H2, 3 cannot go to H3 and so on, so person I cannot go to HI, a possible enumeration would be something like this, 1 going to H4, 2 going to H1, 3 going to H5, 4 going to H3, and 5 going to H2, this is possible.

(Refer Slide Time: 01:04)



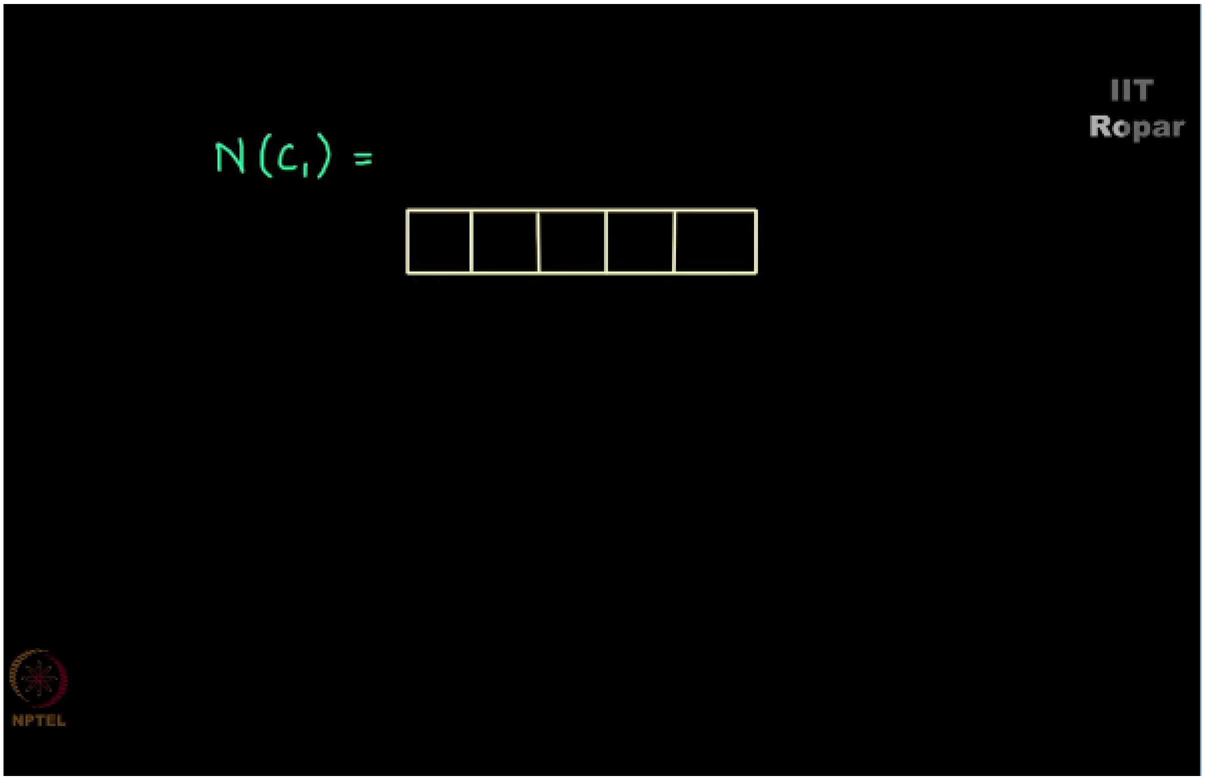
Now let us see in how many ways these people cannot go to their respective houses, so what do we have to find out here? We have to find out basically first let us find out what are the conditions, right CI's that is, let me write condition one as 1 going to his house H1, or condition two being 2 going to H2, and three would be 3 going to H3, condition four would be 4 going to H4, condition five would be 5 going to H5,

(Refer Slide Time: 01:51)



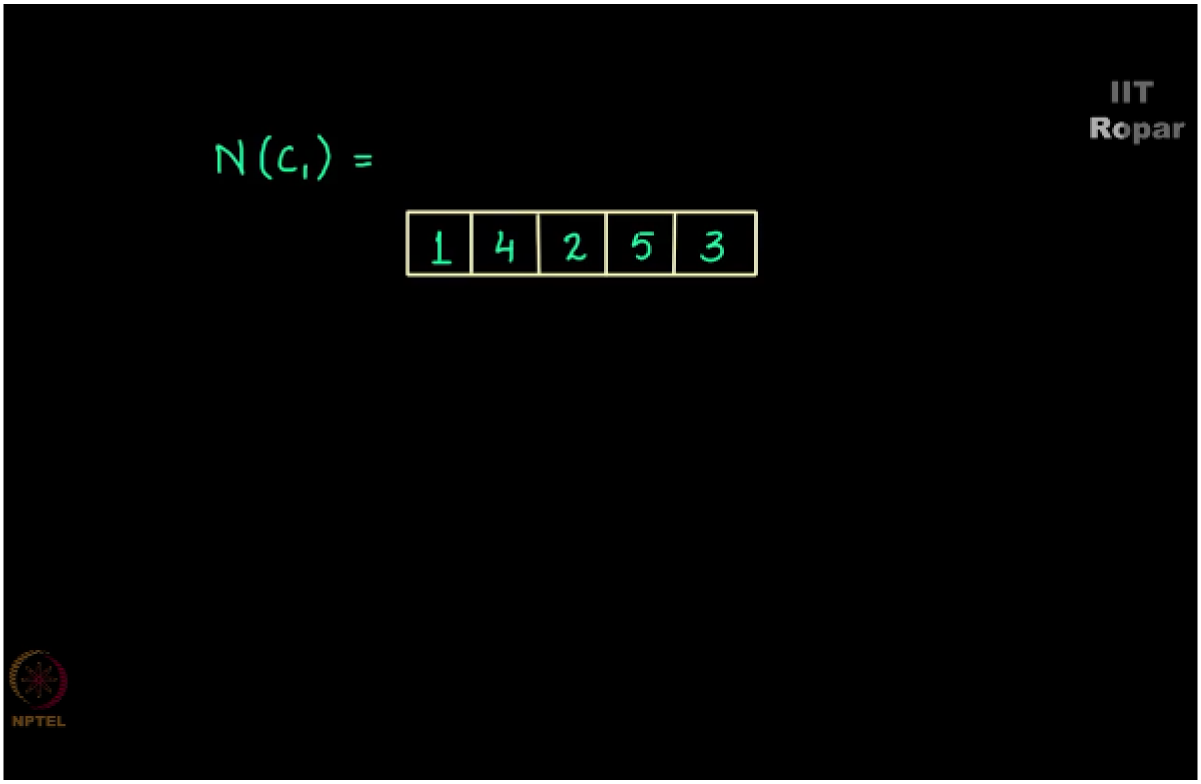
so these are the conditions given to us, right, we have formulated them, now let us proceed further and find out what is N(C1), you see these slots given here,

(Refer Slide Time: 02:10)



these are the houses N(C1) would mean those enumerations where 1 goes to house number 1 itself, right like this 1 is in 1.

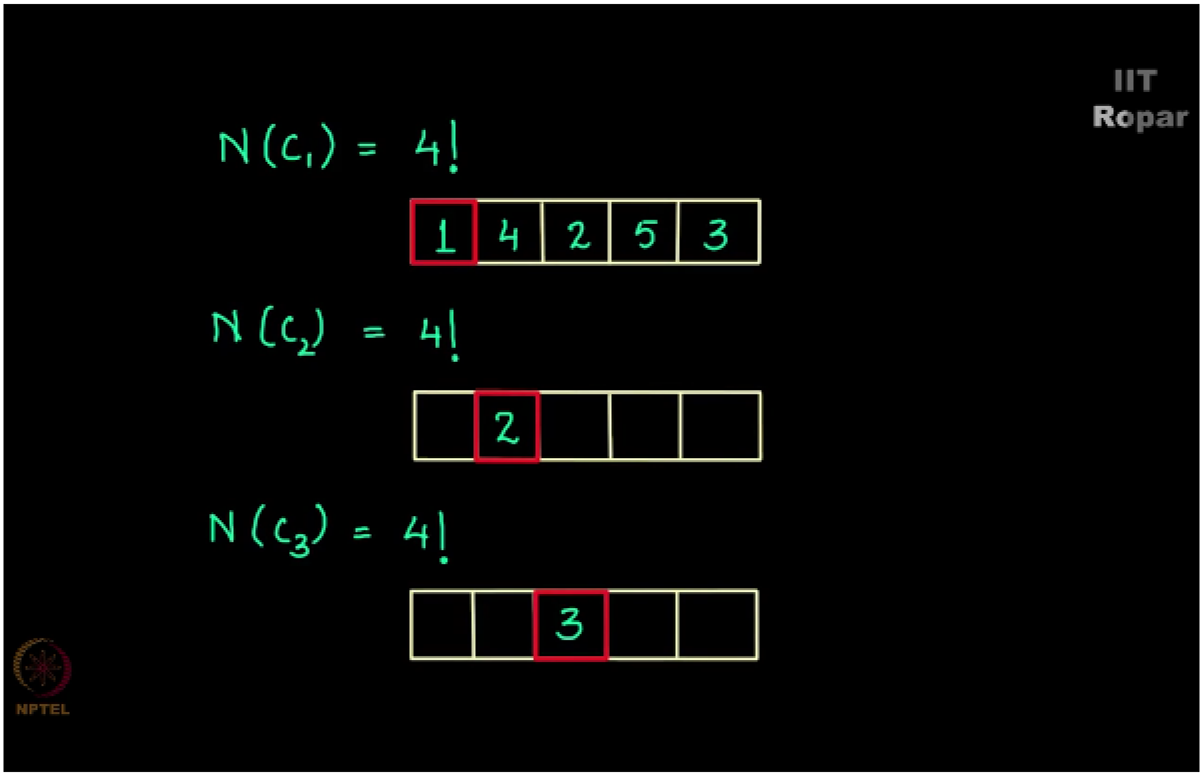
Now we are fixing this which means 1 goes to 1 and rest of them can permute in all possible ways, so in how many ways can 4 people do it? 1 is fixed so we cannot consider 1, so out of 5, 4 can be enumerated, one such way is this, right, 4, 2, 5, 3,
(Refer Slide Time: 02:46)



so this can be done in 4 factorial ways, do you see that? 1 is fixed and hence we consider 4 people and hence the 4 factorial.

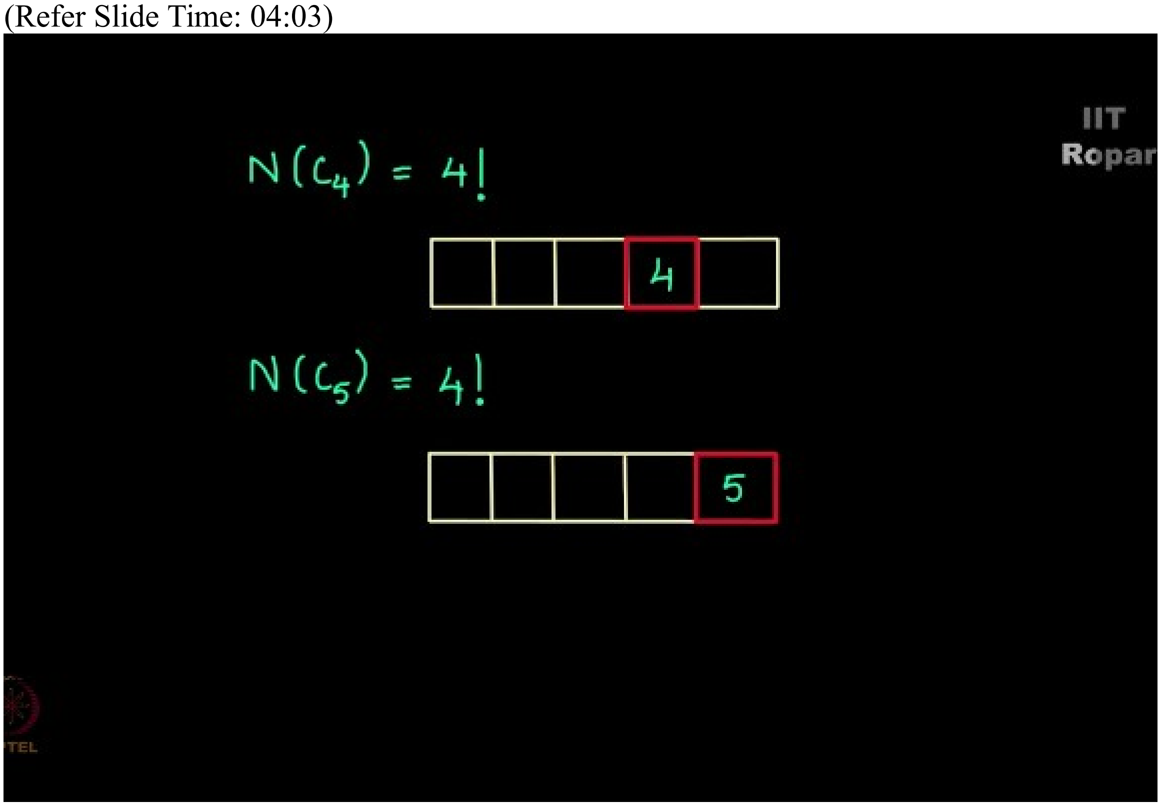
Now for N(C2) again it is 4 factorial, why? Because now 2 is fixed other than 2 all the others 1 3, 4 and 5 can be enumerated in all possible permutations, right, now for N(C3) again it is 4 factorial, why? 3 will go to house 3 it is fixed and the other 4 of them can be permuted in for all possible ways.

(Refer Slide Time: 03:29)

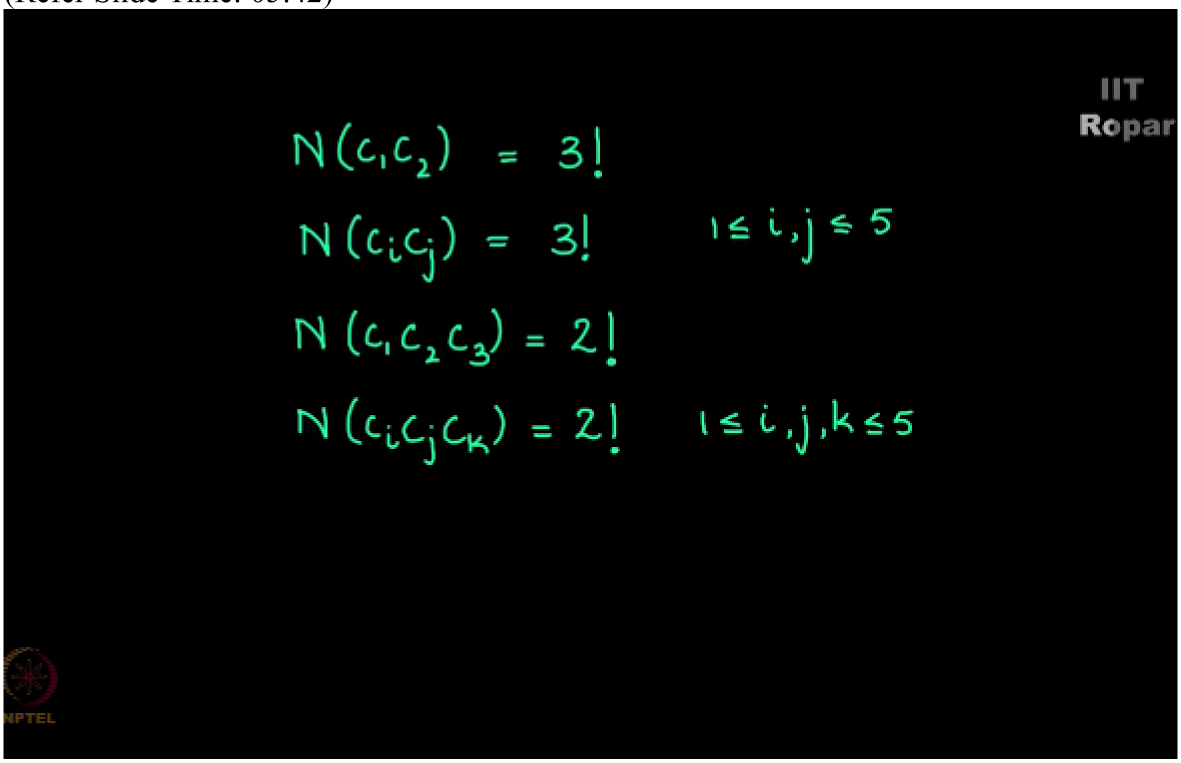


Now N(C4) again has the same analogous reason as 4 is fixed, and the remaining 4 can be permuted in 4 factorial ways same for C5 as well, so condition number of elements in condition 5 following condition 5 is 4 factorial, why? Because 5 goes to house 5 itself and the rest of them can go in 4 factorial ways to the other houses, 5 is fixed here.

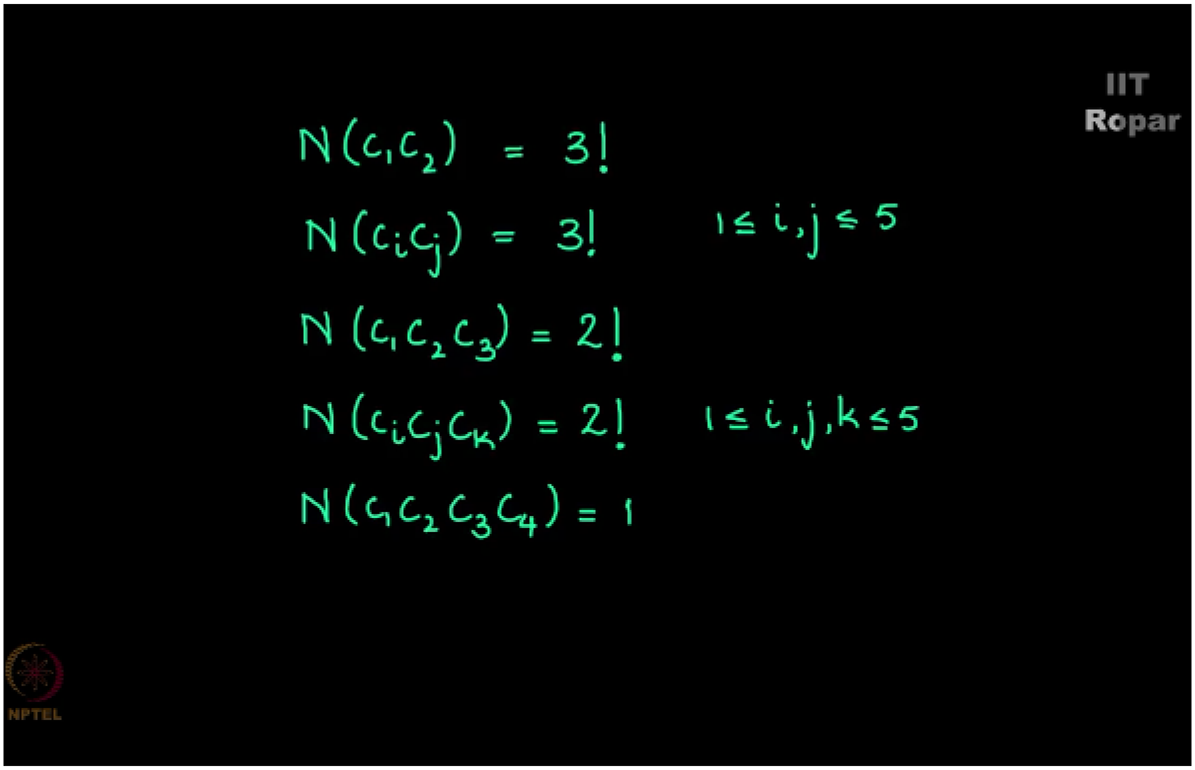
(Refer Slide Time: 04:03)



So now we have found out what is N(C1), N(C2), N(C3) and N(C4) (C5) right, now the next step is fixing 2 conditions, N(C1, C2) 2 what will this be? You can see that it is 3 factorial, how? Well you must not be really stuck here because earlier you saw that one person goes to his own house, now 2 people are going so how many remain? 3 people and they can be permuted in 3 factorial ways, this holds true for C1, C2 where 1 goes to house 1, and 2 goes to house 2, but don't you think it is the same for others as well, what do I mean by that? N(CI,CJ) is 3 factorial for all IJ between 1 to 5, in general this is 2, true, now what if 3 people go to their respective houses? What do I mean by that? N(C1,C2,C3) go to house 1, 2, 3 respectively, for the remaining 2 people 4 and 5 how many permutations are possible? 2 permutations which means N(C1,C2,C3) is 2 factorial, can this be true in general? Yes, which means N(CI,CJ,CK) is 2 factorial for I, J, K lying between 1 to 5.
(Refer Slide Time: 05:42)

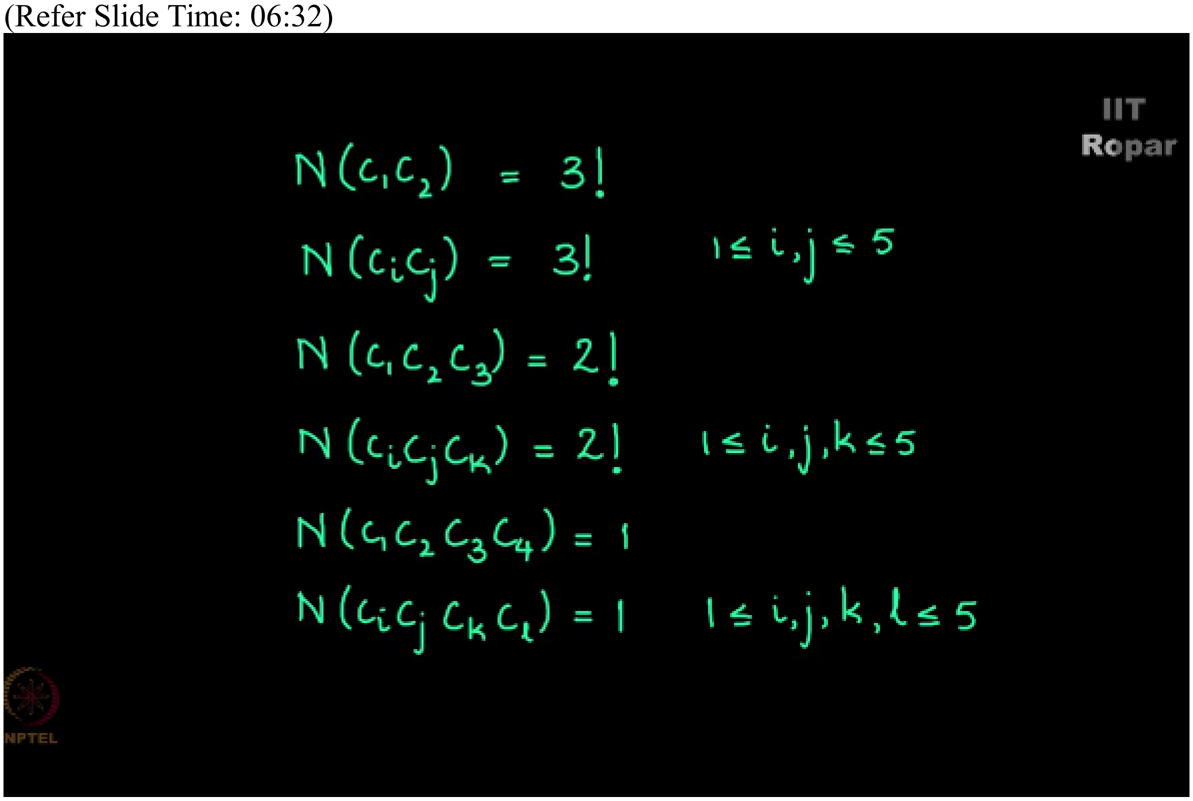


Now we have 5 people in all, and we have covered for 3 conditions, N(C1,C2,C3,C4) you see here what is happening, 1 goes to house 1, 2 goes to house 2, 3 goes to house 3, 4 goes to house 4, the remaining person 5 has only possibility to go to house 5,
(Refer Slide Time: 06:13)



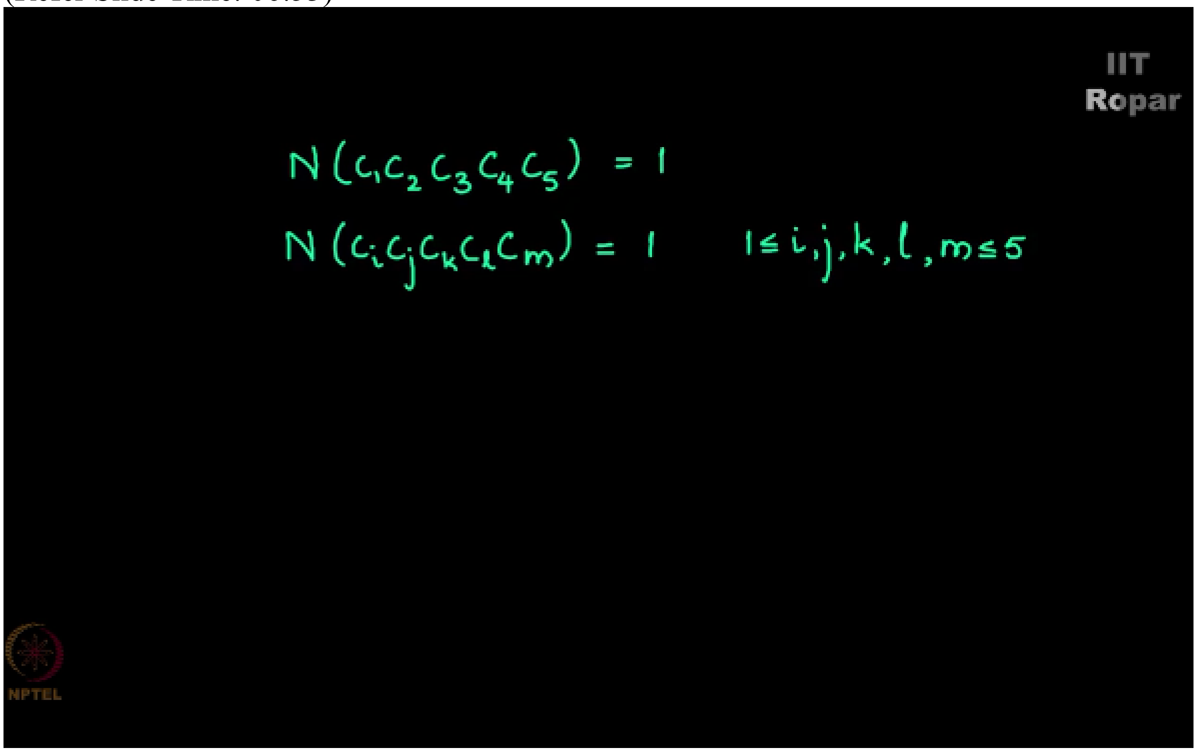
so this is 1 which means it can be done only in one way, well this is true in general,

(Refer Slide Time: 06:32)

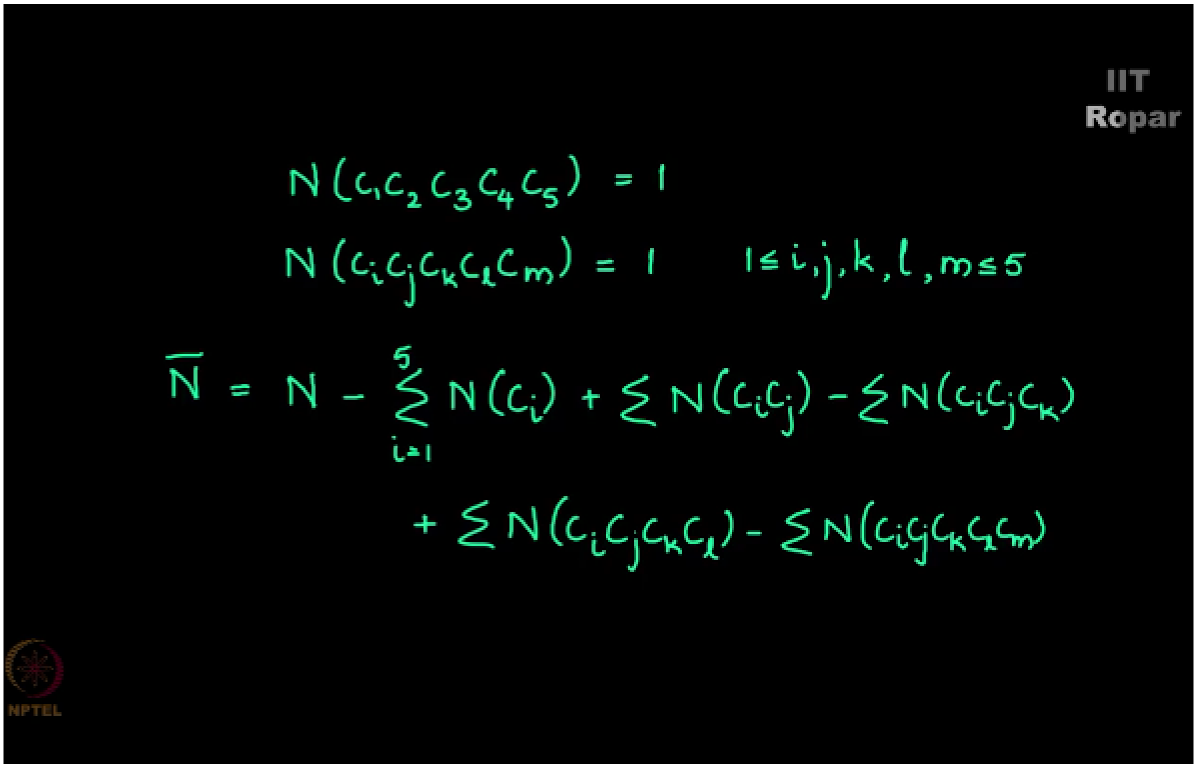


so for all other cases it is true as well, it is 1 that is one permutation for CI, CJ, CK, CL, what would N(C1,C2,C3,C4,C5) mean? The number of permutations where every person goes to

their own house, this is obviously 1, right, so N(CI,CJ,CK,CL,CM) is 1 for all of them between 1 to 5.
(Refer Slide Time: 06:53)

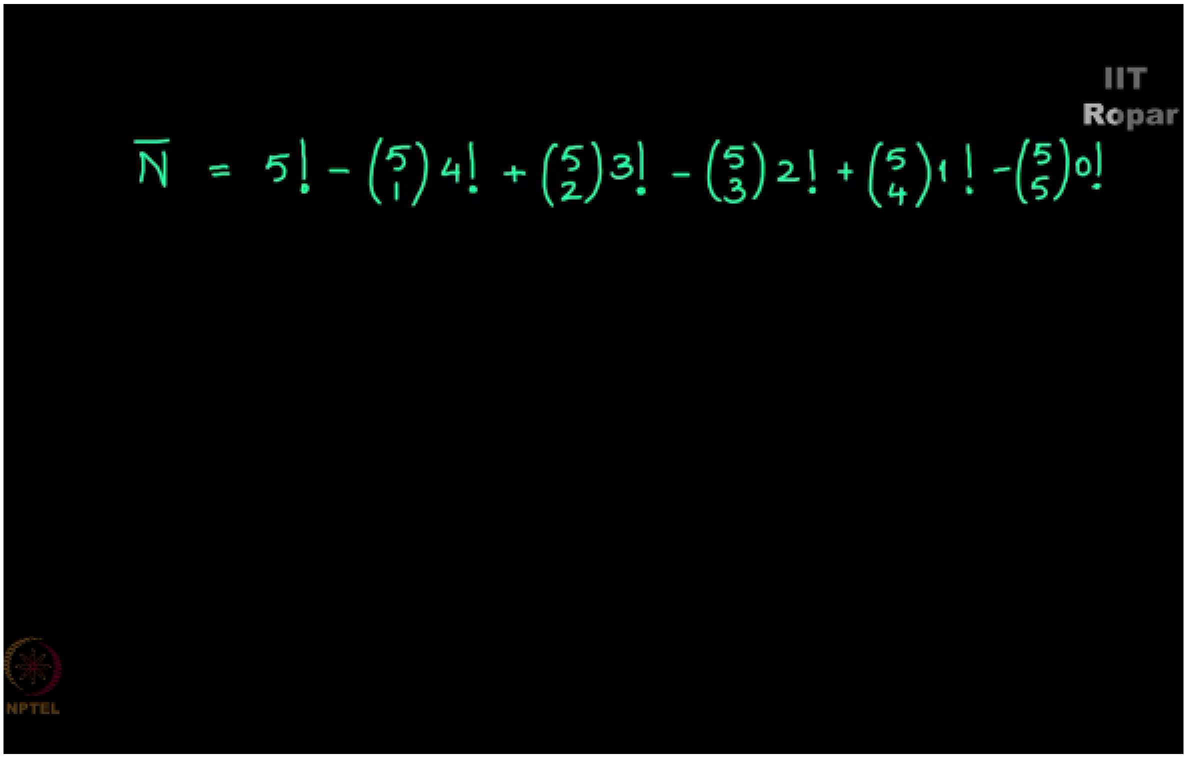


Now the question is what is N bar? Which is N(C1 bar, C2 bar, C3 bar, C4 bar, and C5 bar)? It is N all possible permutations - summation N(CI) where I is from 1 to 5 + summation N(CI,CJ) again I, J between 1 to 5 - summation N(CI,CJ,CK) + N(CI,CJ,CK,CL) – N(CI,CJ,CK,CL,CM),
(Refer Slide Time: 07:40)



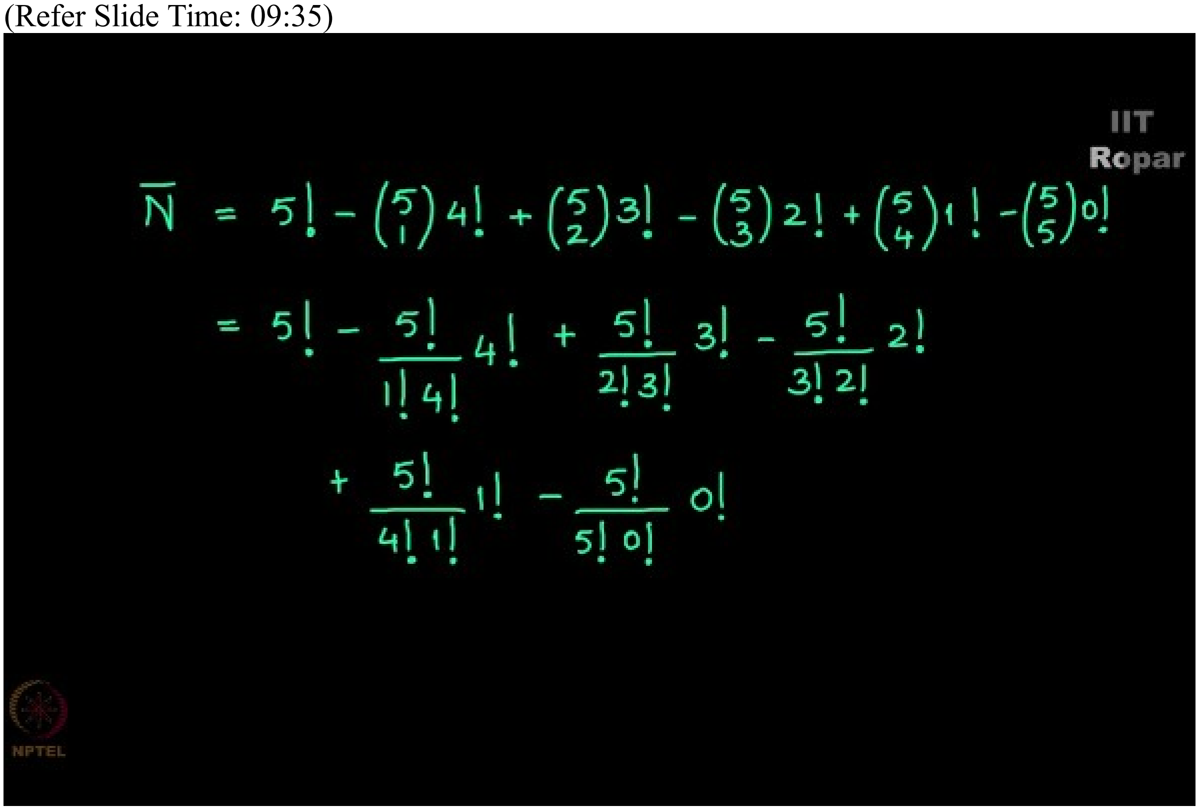
so this is the formula to find out that how many ways can I not go to HI, right, so N here is what? It is all possible permutations, so N bar happens to be all possible permutations is 5 factorial, 5 factorial - 5 choose 1 into 4 factorial, why did I write 5 choose 1, because 4 factorial is true for all the 5 of them, right instead of writing 5 I have written 5 choose 1 which is 5 itself, you can also see it as one person out of the 5 people, right, choosing 1 out of the 5 + 5 choose 2 x 3 factorial, this is for C1,C2 rather CI,CJ - 5 choose 3 x 2 factorial this is for 3 people + 5 choose 4 x 1 factorial - 5 choose 5 into 0 factorial.

(Refer Slide Time: 08:52)



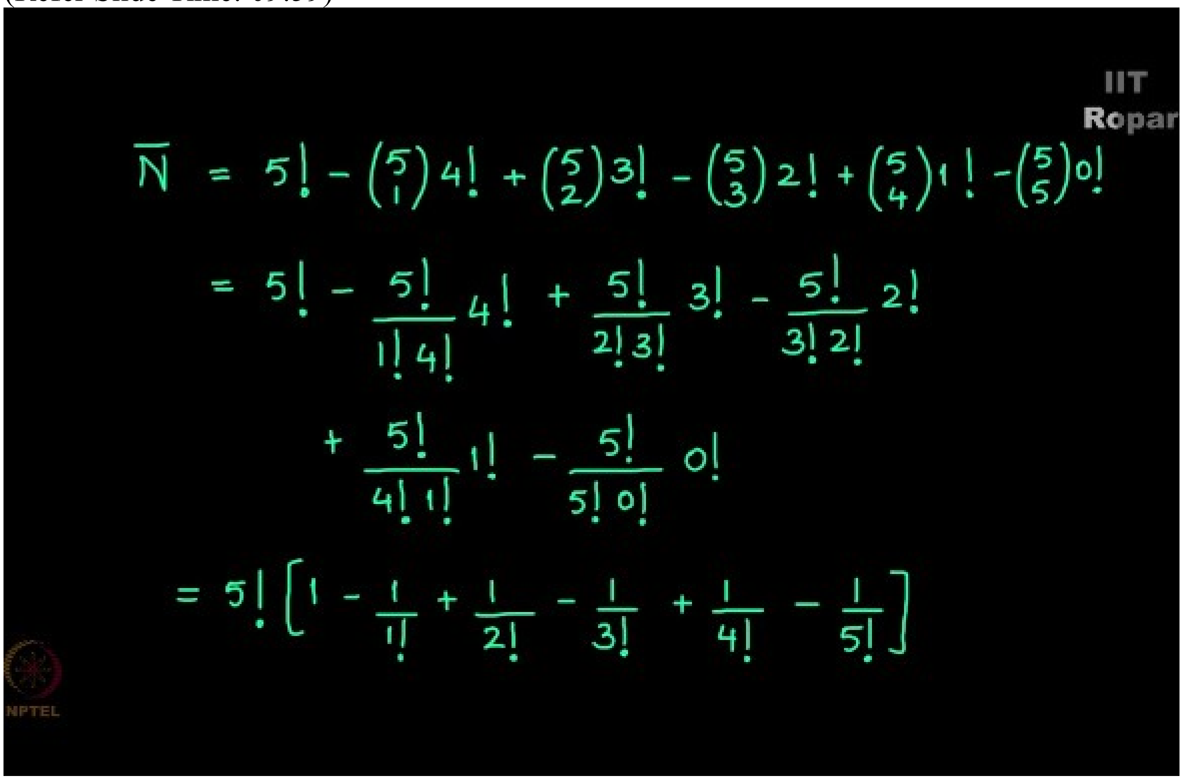
Now this can be written as 5 factorial -, if I expand each one of them I can write it like this 5 factorial/1 factorial x 4 factorial x 4 factorial + 5 factorial/2 factorial x 3 factorial x 3 factorial - 5 factorial/3 factorial x 2 factorial x 2 factorial + 5 factorial/4 factorial x 4 x 1 factorial x 1 factorial and so on, well you are expected to write all of this yourself by now.

(Refer Slide Time: 09:35)



Now I take out 5 factorial common here and cancel out the common terms, what do I get? 5 factorial x 1 -1/1 factorial + 1/2 factorial – 1/3 factorial + 1/4 factorial – 1/5 factorial, do you see this?

(Refer Slide Time: 09:59)



It's something very familiar to us, well this is the answer for the question that person I cannot go to house I, he cannot go to house I in these many ways.