**NPTEL**

**NPTEL ONLINE CERTIFICATION COURSE**

**Discrete Mathematics**
**Principle of Inclusion and Exclusion**

**Example 12 : Integer solutions of an equation**

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How many integers solutions are there for X+Y+Z = 20, where X < 7, Y < 8, and Z < 9,
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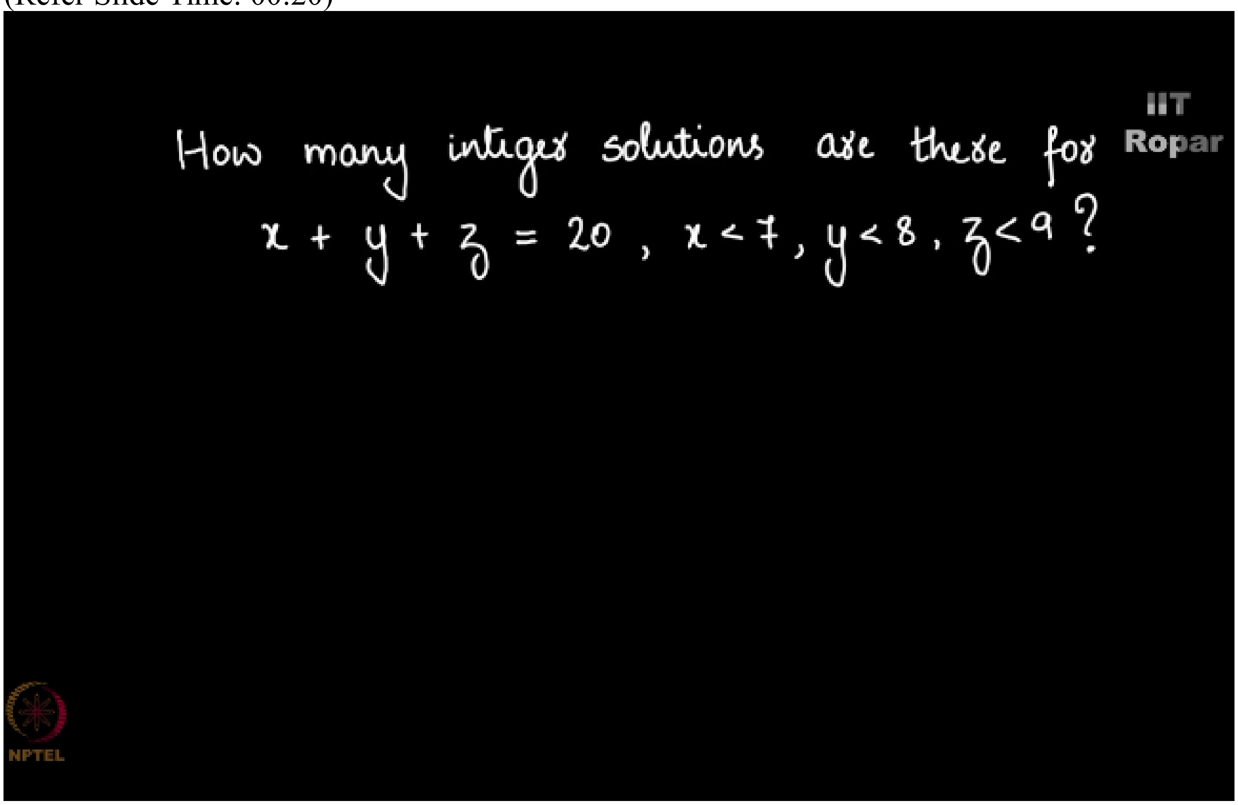


all three of them are strictly less than, all possible solutions for this is given by 3 + 20 – 1 choose 20, following the formula N+R-1 choose R, which is 22 choose 20 which is 22 x 21/2 cancelling 2 and 22 we get it as 231,
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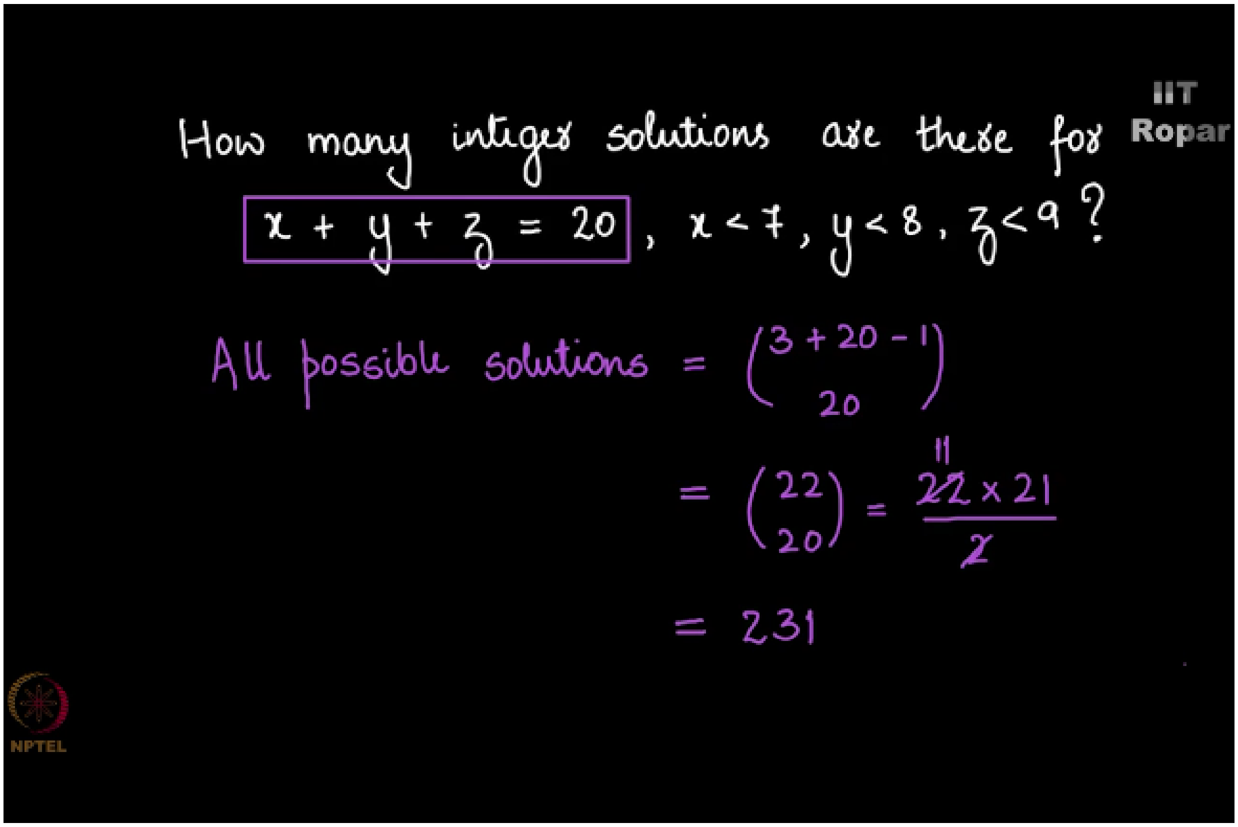


now this is all possible solutions of X+Y+Z = 20, but we are over counting here and we must remove some.

Let C1 be the condition which says that the solutions where X is greater than or equal to 7, and C2 be the condition for solutions where Y is greater than or equal to 8, and C3 be the condition for the solutions where Z is greater than or equal to 9.

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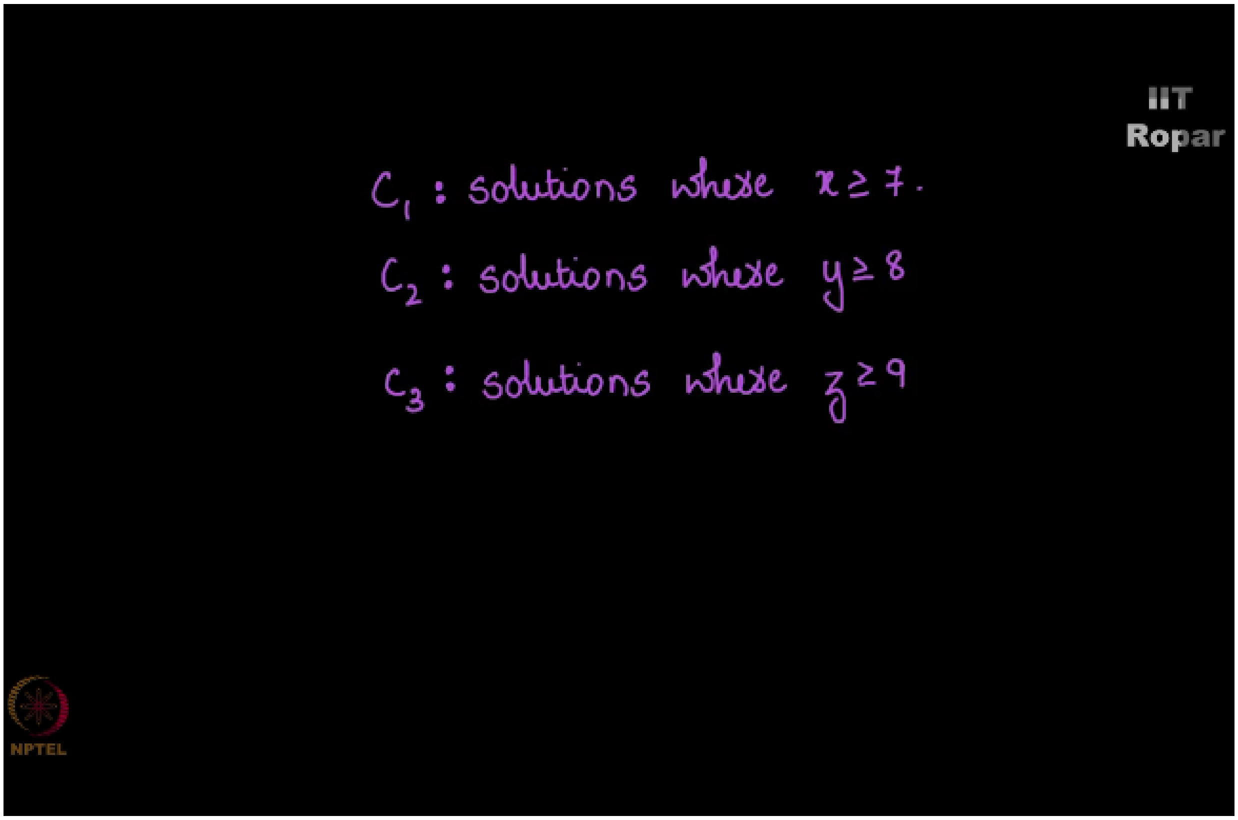


Now what is the question, we have to find out N(C1 bar, C2 bar, C3 bar), now C1 is those solutions where X is greater than or equal to 7,

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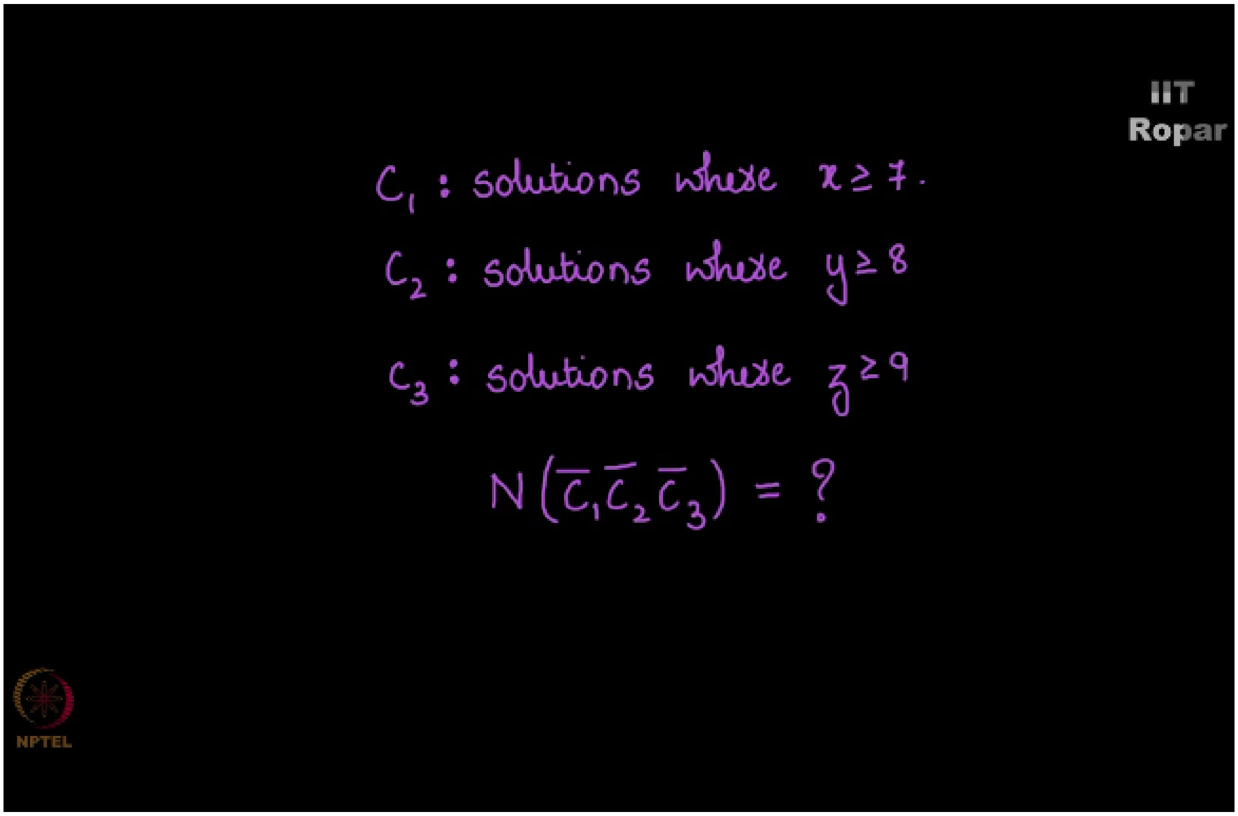


how many of them are there? Let us see, N(C1) happens to be the following X+Y+Z = 20, now boils down to X+Y+Z = 13, right, there I had to have X greater than or equal to 7, now I'm removing 7 from both sides and I have got the equation X+Y+Z = 13.

Now the number of solutions for this equation is 3+13 – 1 choose 13 which is 15 choose 13 which is 15 x 14/2 which is 105.
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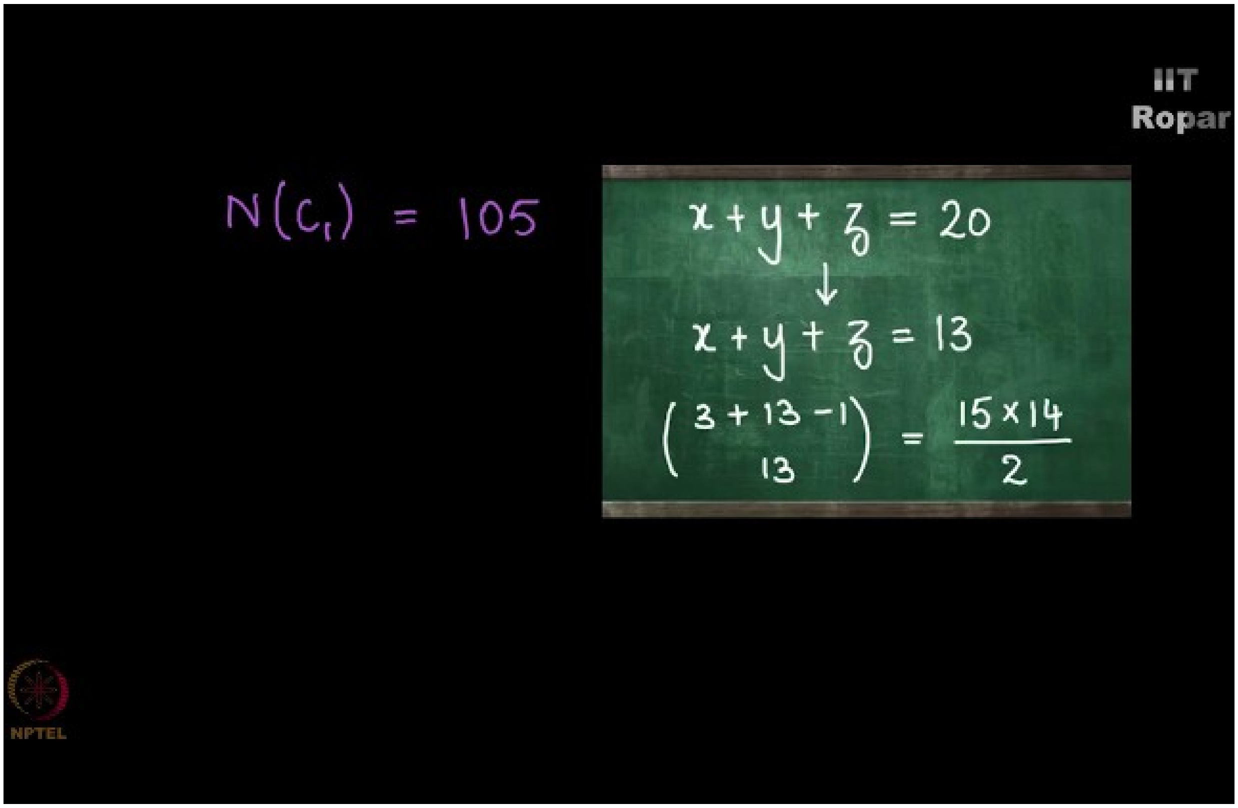


Now N(C2) happens to be the number of solutions for X+Y+Z = 12, how did I get this? I removed 8 from both the sides, now the number of solutions for this is 3+12-1/12 which is 14 choose 12, and it is 14 x 13/2, cancelling 2 and 14 we get 91.

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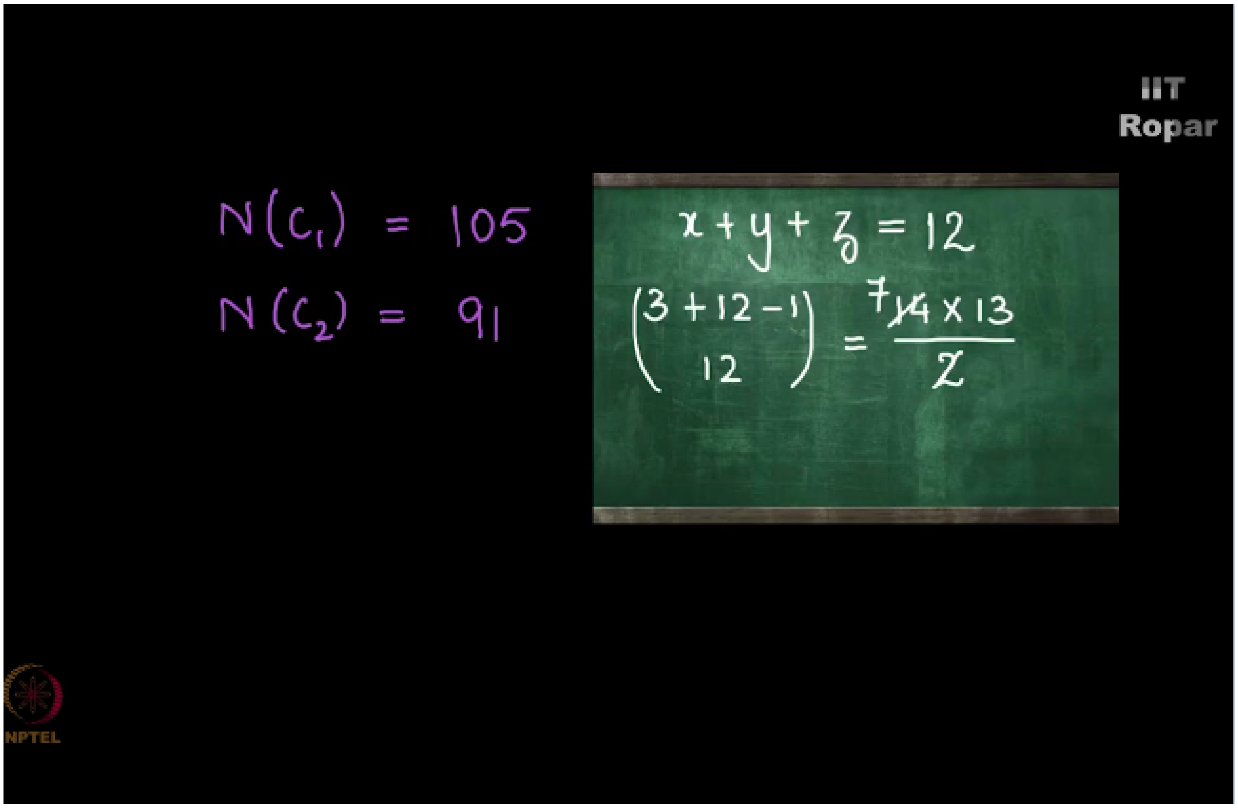


N(C3) will be those solutions where Z is greater than or equal to 9, and how do I obtain that? Removing 9 on both sides I get the equation X+Y+Z =11, and 3+11-1 choose 11, which is 13 choose 11, 13 choose 11 is 13x12/2 which is 78.

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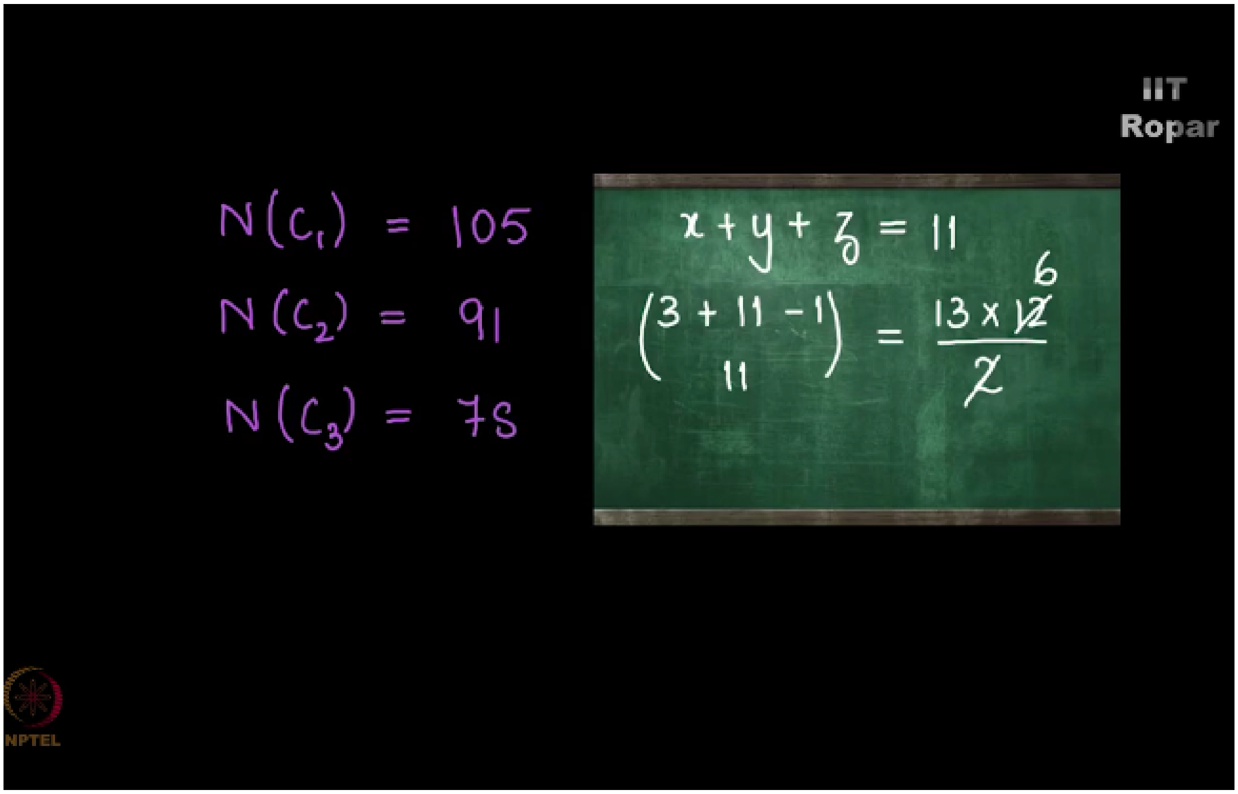


Now moving ahead to calculating N(C1,C2), C1,C2 means C1 should happen as well as C2 which means we must find the number of solutions for X+Y+Z = 20 where X is greater than or equal to 7 and Y is greater than or equal to 8, right, now removing both this conditions what do I get? It is equivalent to the equation X+Y+Z = 5, because 7+8 is 15, right, now for this we have 3+5-1 choose 5 which is 7 choose 5, which is 7 x 6/2 which is 21 ways, so N(C1,C2) is 21,

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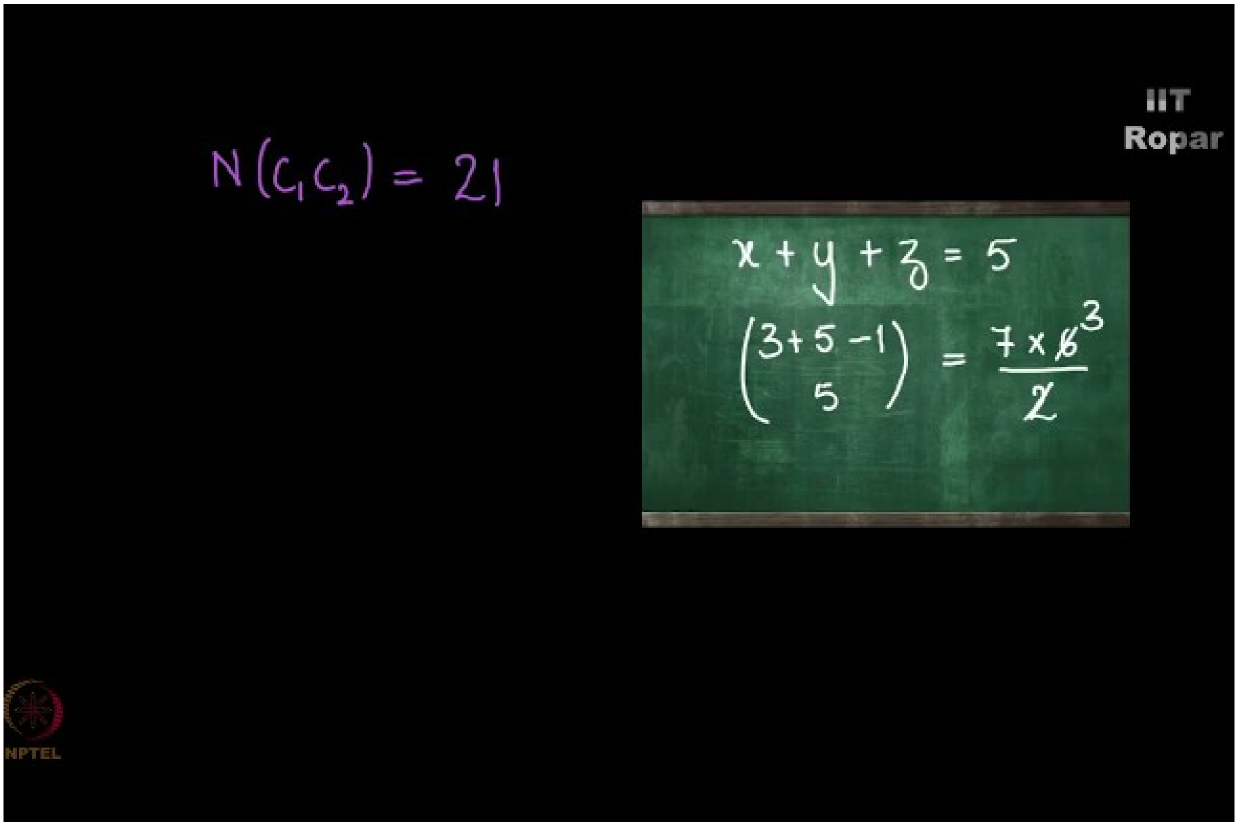


N(C2,C3) happens to be X+Y+Z=3, how did I get this? On removing 8+9 which is 17 from both sides of the original equation, now the number of solutions for this is 3+3-1 choose 3 which is 5 choose 3 and solving this we get it as 10, so N(C2,C3) is 10.

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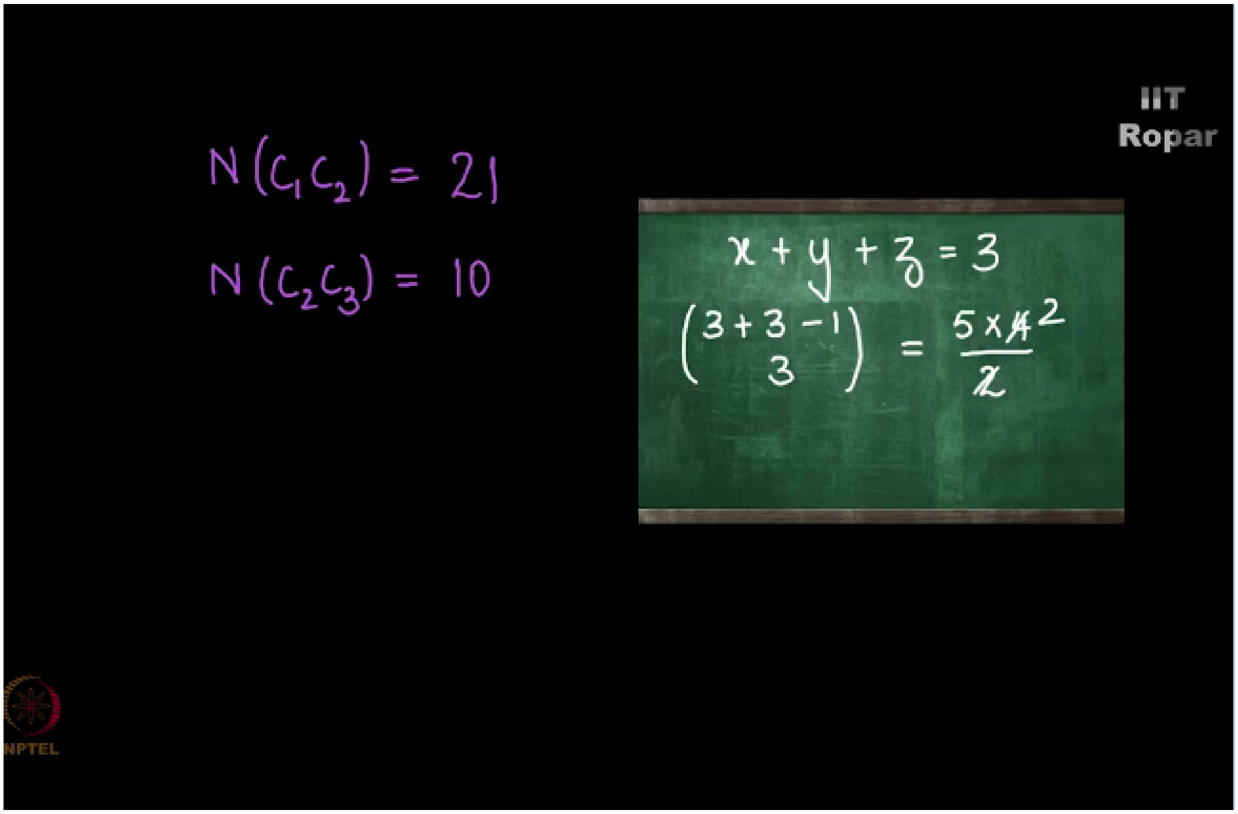
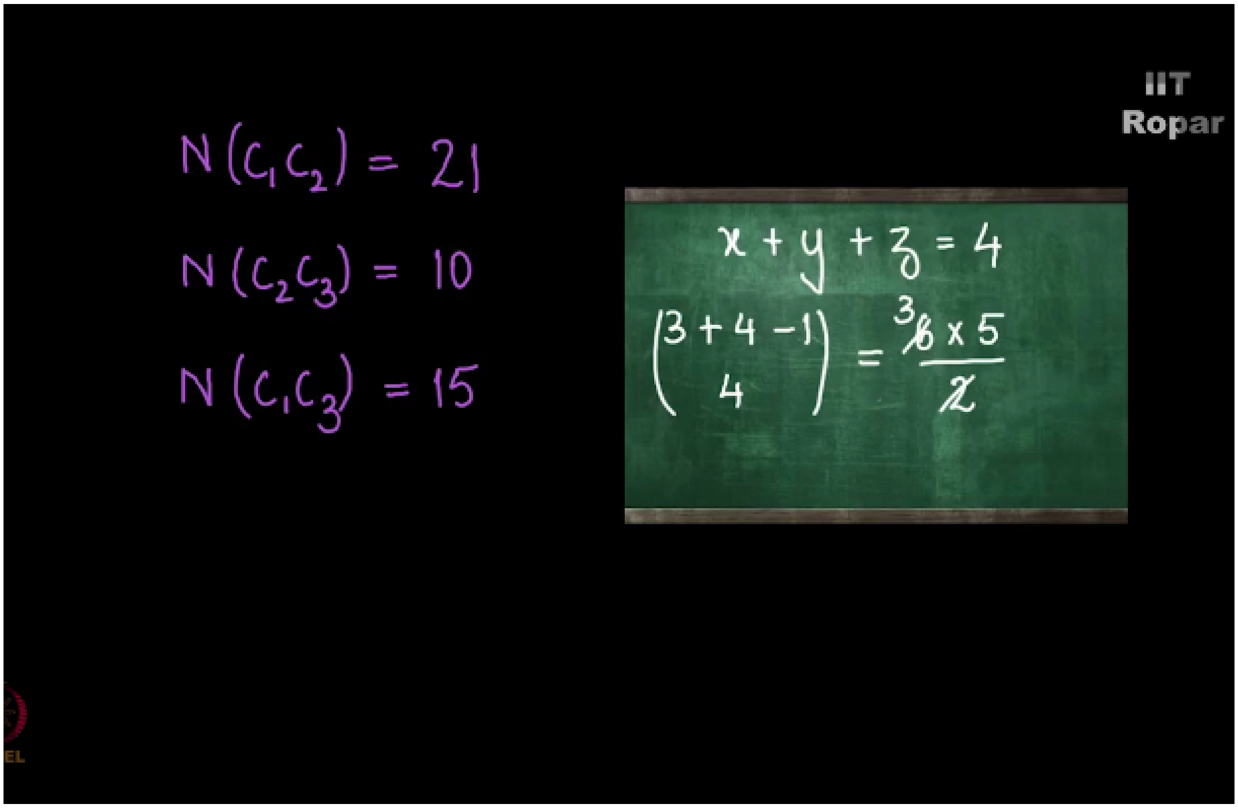



N(C1,C3) is, we get it by solving X+Y+Z =4, right, how did I get 4? Adding 7 and 9 which is 16, and removing 16 on both sides from the originally equation which is now the solution for X+Y+Z=4 is 3+4-1 choose 4 which is 6 choose 4, and 6 choose 4 is equal to 6x5/2 = 15, (Refer Slide Time: 05:11)




now N(C1,C3) is 15, what will N(C1,C2,C3) be? N(C1,C2,C3) if I see if all of them should happens simultaneously will there be such a solutions, even if I substitute 7, 8 and 9 in place of X, Y, Z I will obtain 7+8 is 15, 15+9 is 24, 24 as the answer which will be an invalid solution, and hence N(C1,C2,C3) is 0,

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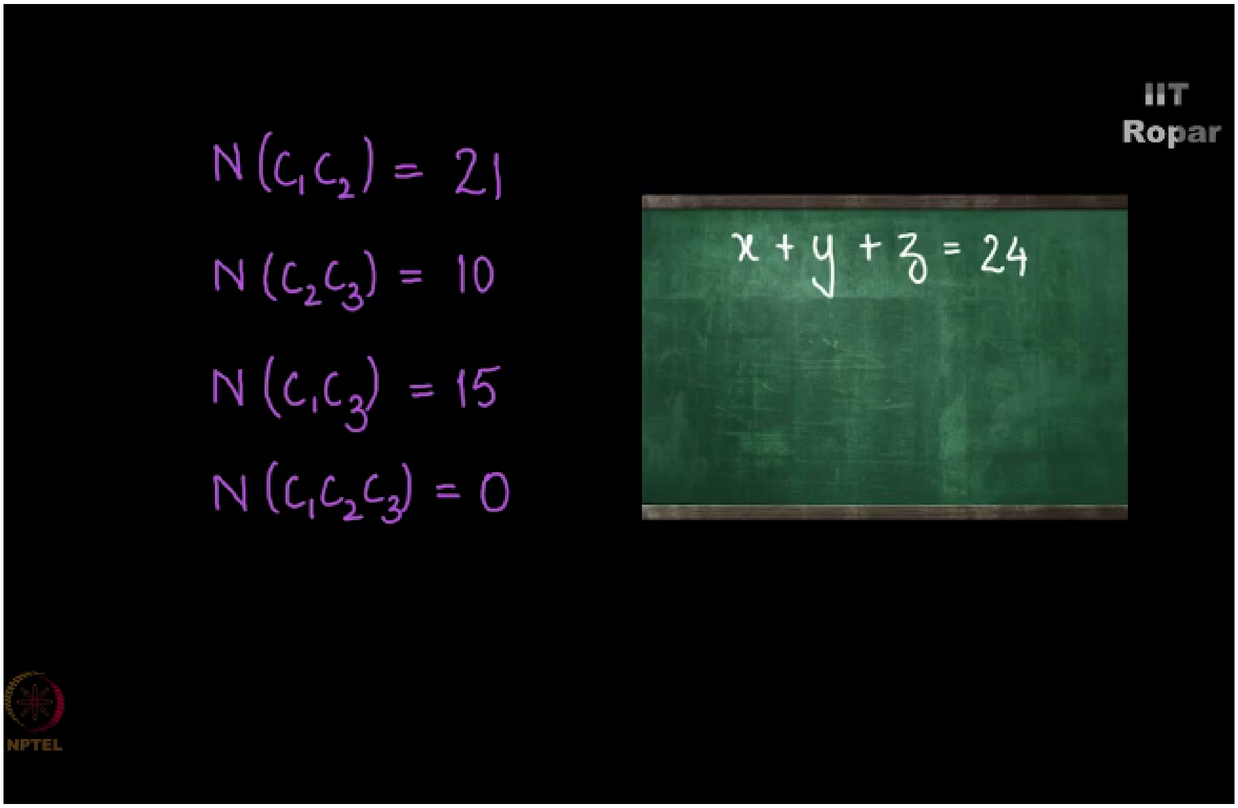


now what is the final answer? N(C1 bar, C2 bar, C3 bar) will be N-N(C1) + N(C2) + N(C3) + N(C1,C2) + N(C2,C3) + N(C1,C3) - N(C1,C2,C3) so we get it as 231 – 274 + 46 and the final answer is 3,

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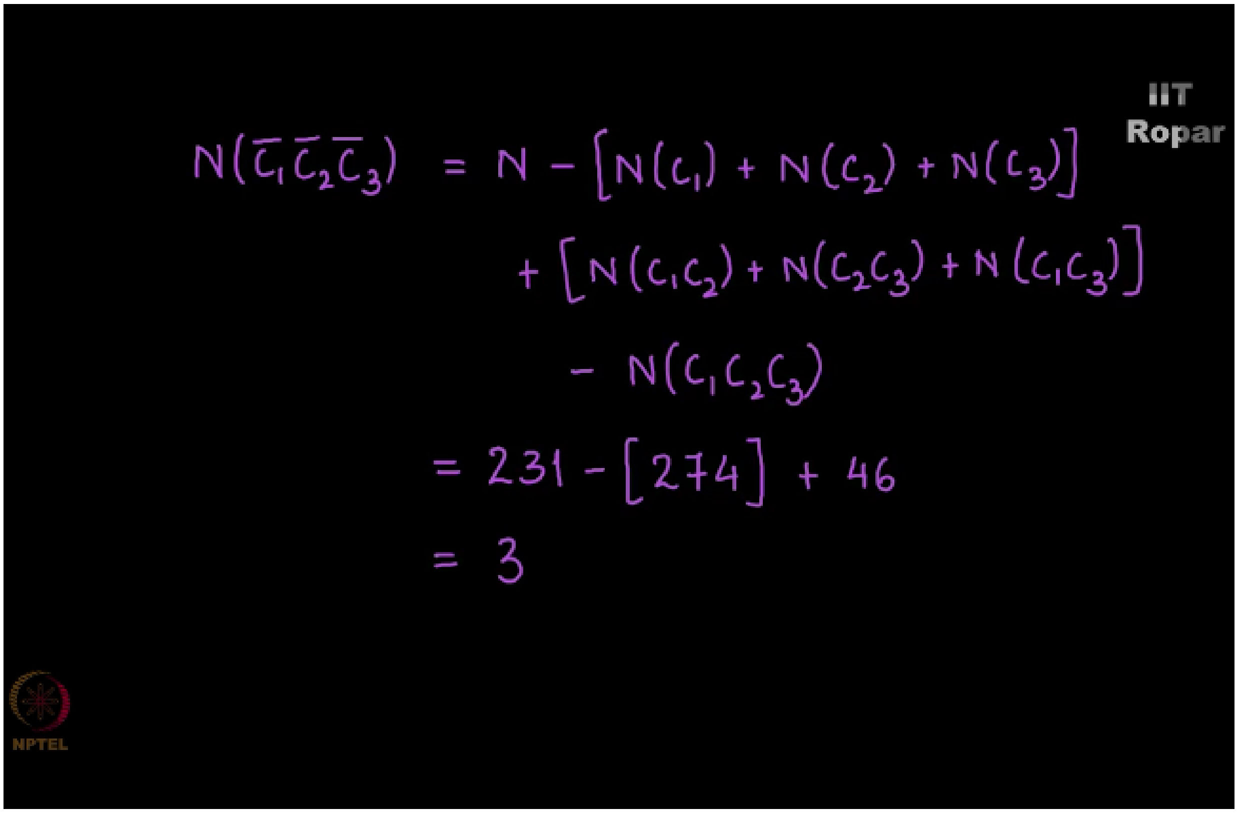


so there are 3 solutions for the equation X+Y+Z = 20 where X is less than 7, Y is less than 8 and Z is less than 9.

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