NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Principle of Inclusion and Exclusion

Example 11 - Seating Arrangement - Part 2

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In the previous video we had seen what is N(C1) N(C2) and N(C3), now let us see what is N(C1,C2)? Let us first understand what is C1,C2, consider this table I had told you that earlier C1 was locked,
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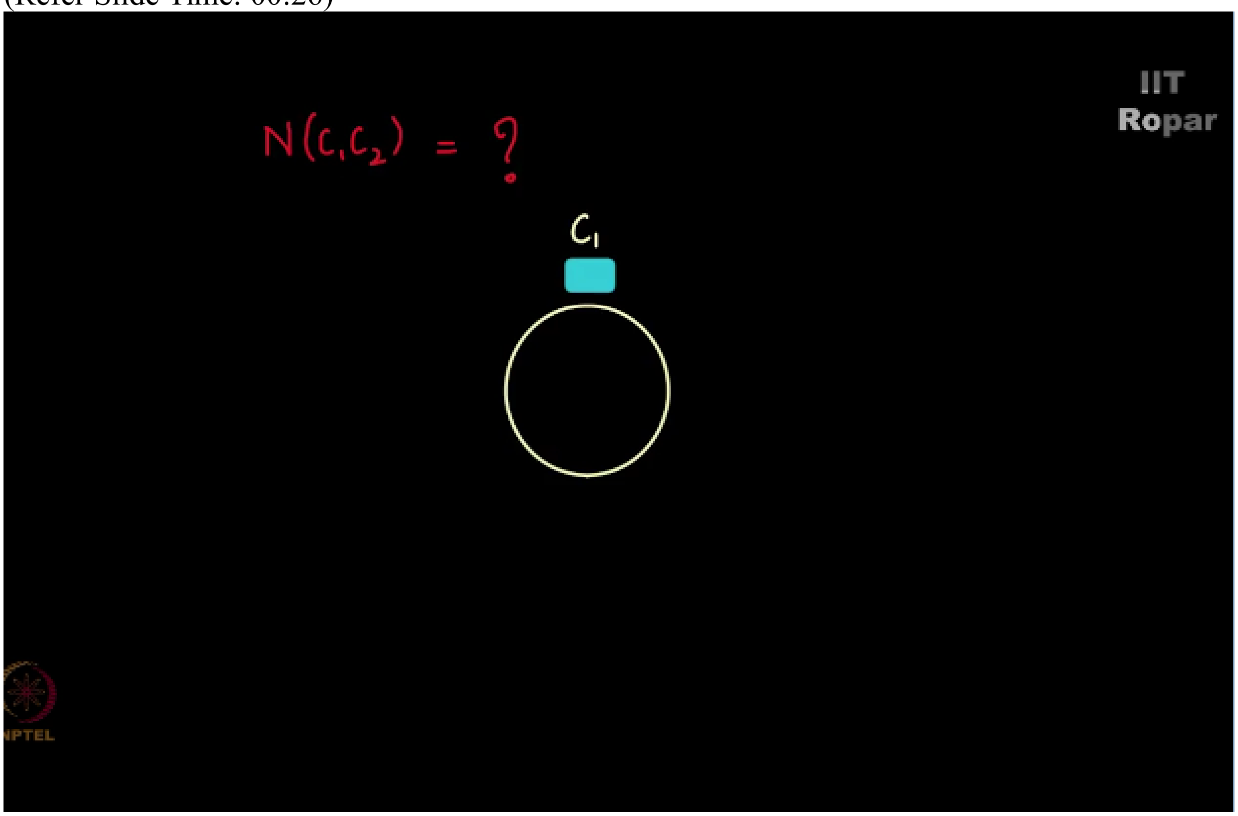

which means we had considered those permutations where C1 or C2 or C3 alone was fixed, but now this is a condition where C1 and C2 are both locked, which means C1 and C2 these both couples are, couple 1 and couple 2 are both sitting beside each other, now out of 6 people 4 are gone, how many remain? 2 people let me name them as P1, P2, so we have people as P1, P2 here, and the 2 couples C1, C2.
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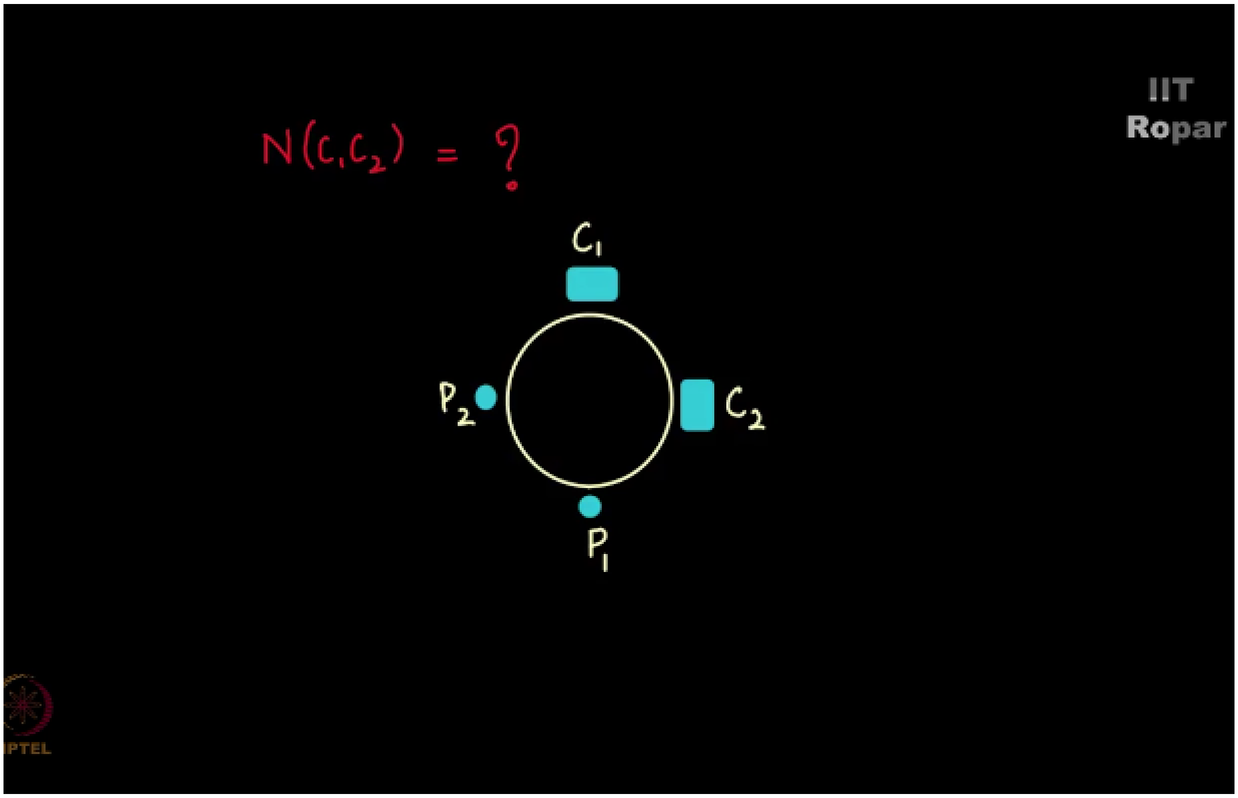

In how many ways can these arrangements happen here, permutations of these 4 people, please remember I am considering C1 as 1 object, C2 as another object, do not get confused with C1, C2 and couple 1 and couple 2, what do I mean by C1 and C2 when I write the diagram is couple 1 and couple 2, so couple 1 is considered as one object, couple 2 is considered as another object and I have P1, P2, how many permutations are possible? C1, C2, P1, P2 is one possible permutation, P1, C1, C2, P2 is one possible permutation, another one is P1, C1 and P2, C2, this is another possible permutation so on and so forth we can go on listing,

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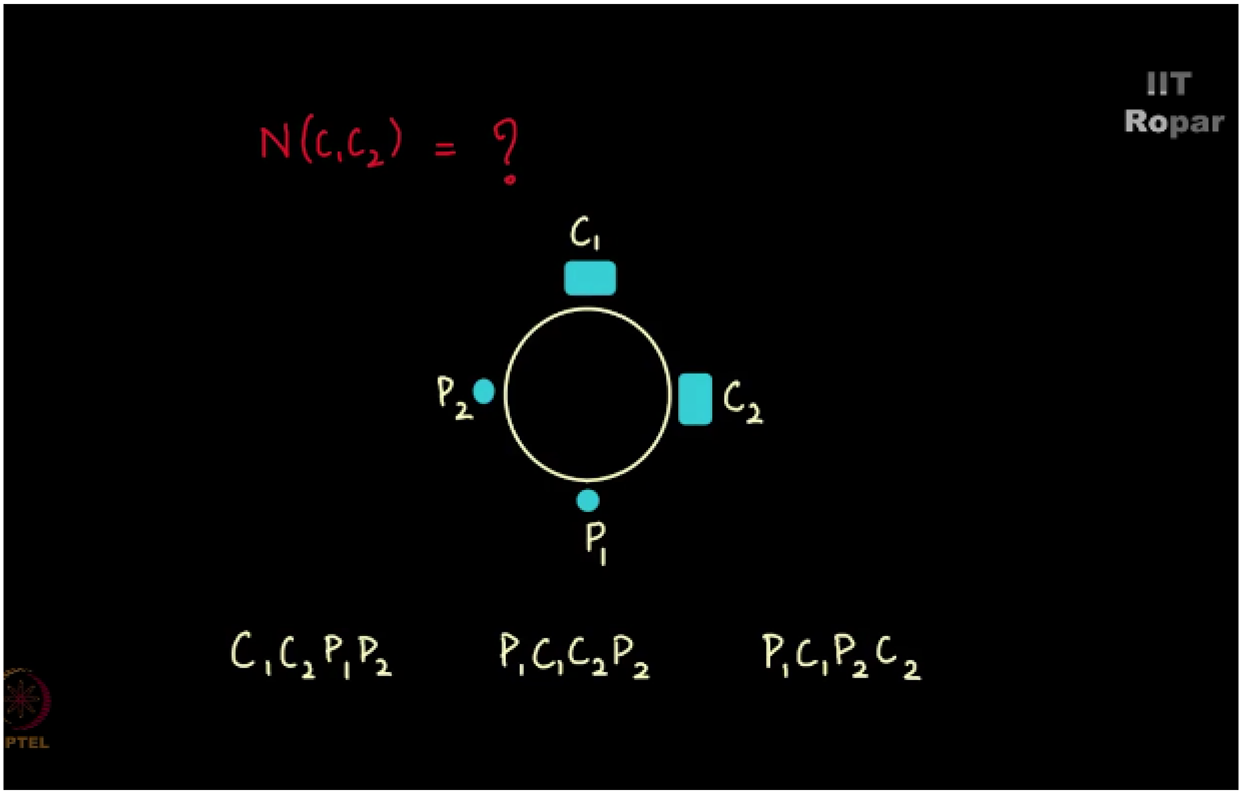

but how many of them are there let us see, for 4 distinct objects as we had seen earlier from the formula they can be arranged in 3 factorial ways, how? 4-1 factorial which is 3 factorial ways, earlier in the previous video I had proved this to you by showing bijection, I'm not going to repeat the same well, but you can do it all by yourself.

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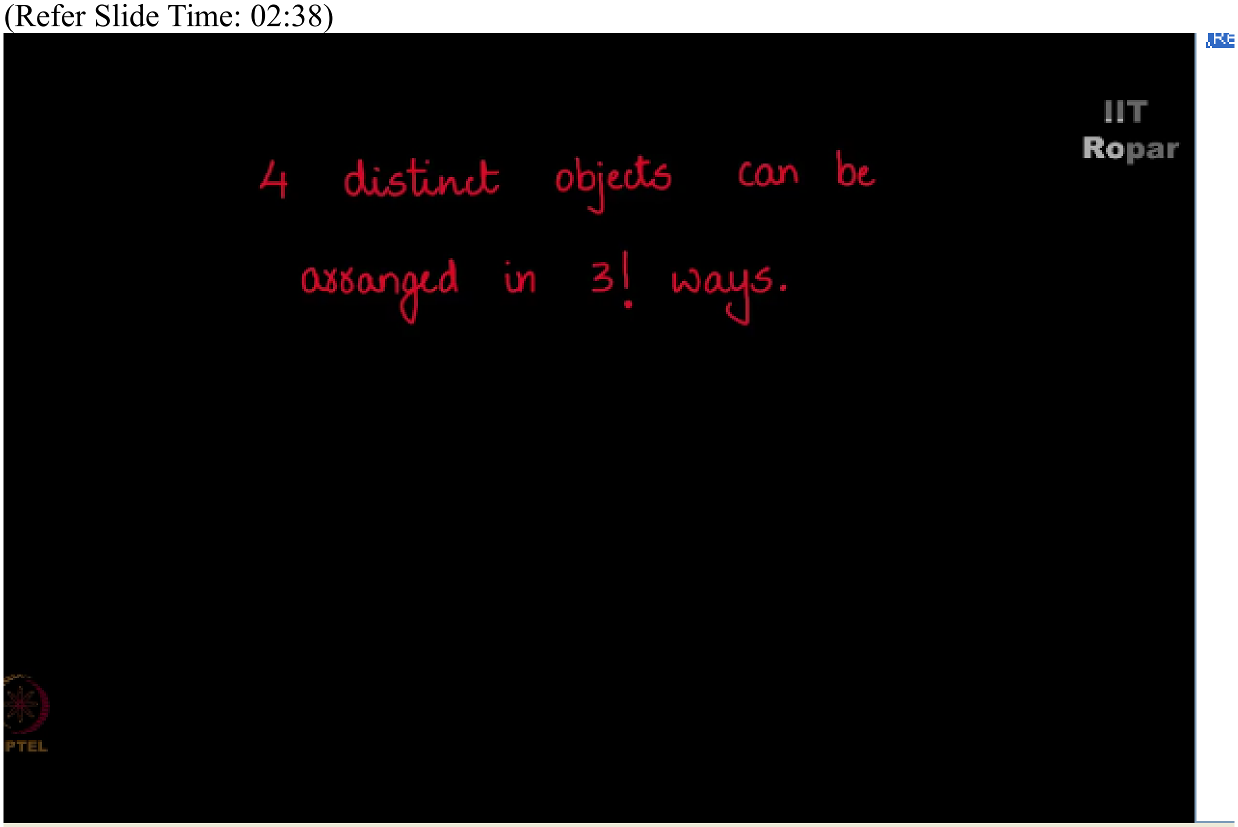

In the earlier video what we had done is we had appended C1 in the bisection proof, but you need not take C1 always for that matter out of 5 objects which we had considered there you can take any one and append any one, right, and you will be obtaining the same answer, so according to that 4 distinct objects can be arranged in 3 factorial ways and hence N(C1,C2) happens to be 2 square x 3 factorial, how is it 2 square? Earlier it was 2 ways husband wife, wife husband for one couple, now this is true for couple 1 they can sit either HW or WH this way, but it is true for couple 2 as well,

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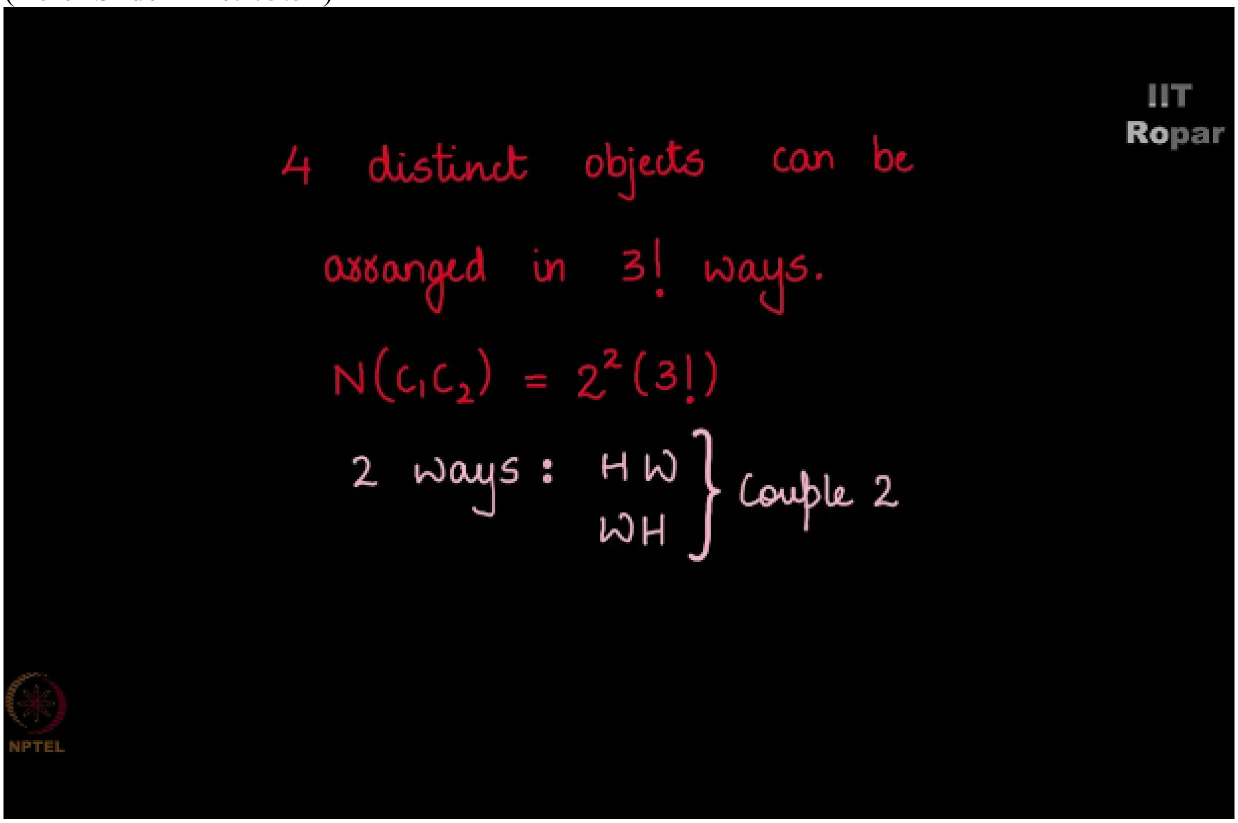

and by rule of product we have it as 2 square into 3 factorial, right, you can probably enumerate the different possibilities and check it yourself HW, WH, the two couples can sit like this or HW-HW they can sit like this, WH-WH this is one possibility, and WH, HW this is another possibility, so in these 4 ways this is how we get it as 2 square, right.

So now for N(C1,C2) it was 2 square x 3 factorial, same holds true for N(C2,C3) and N(C1,C3) it is 2 square x 3 factorial, in these many ways they can sit.

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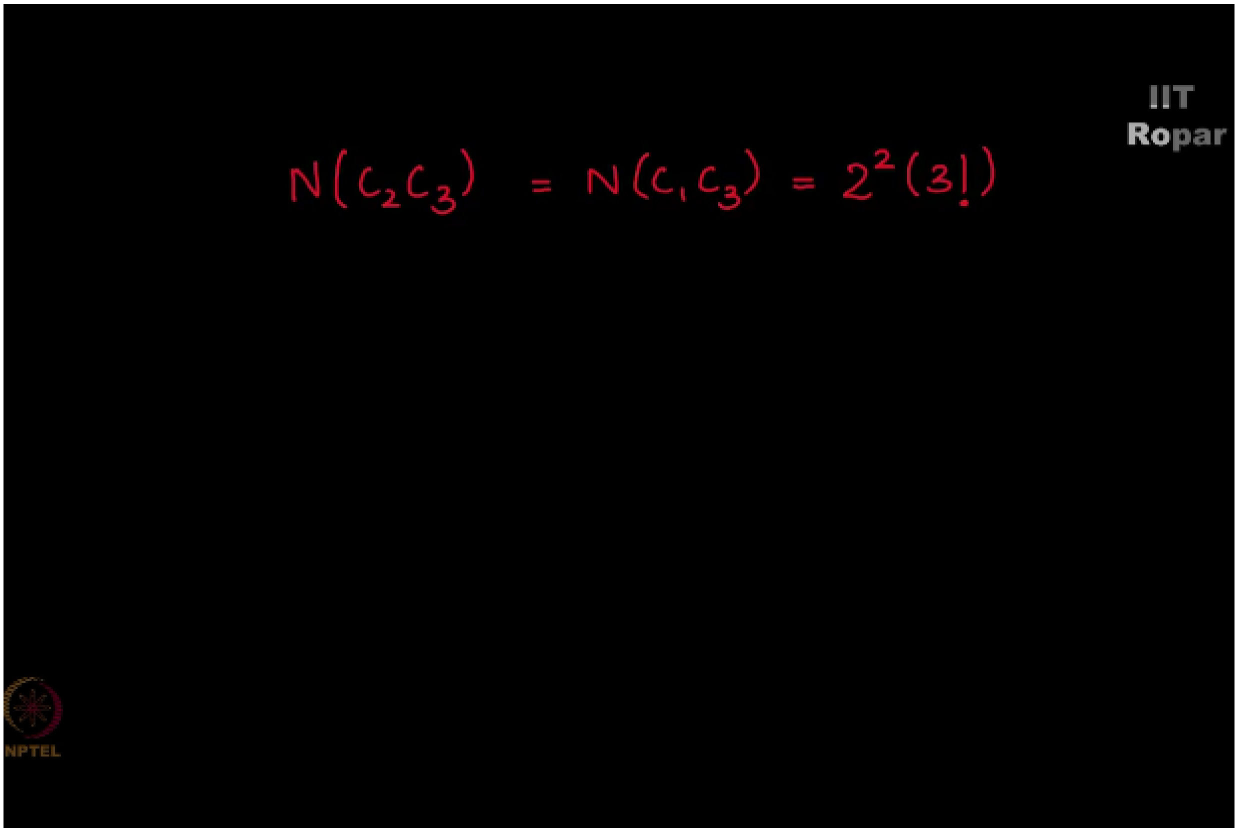

Now the next question would be N(C1,C2,C3), in how many ways can these 3 happen like this? Consider this table, C1 is locked, C2 is locked, C3 is locked,

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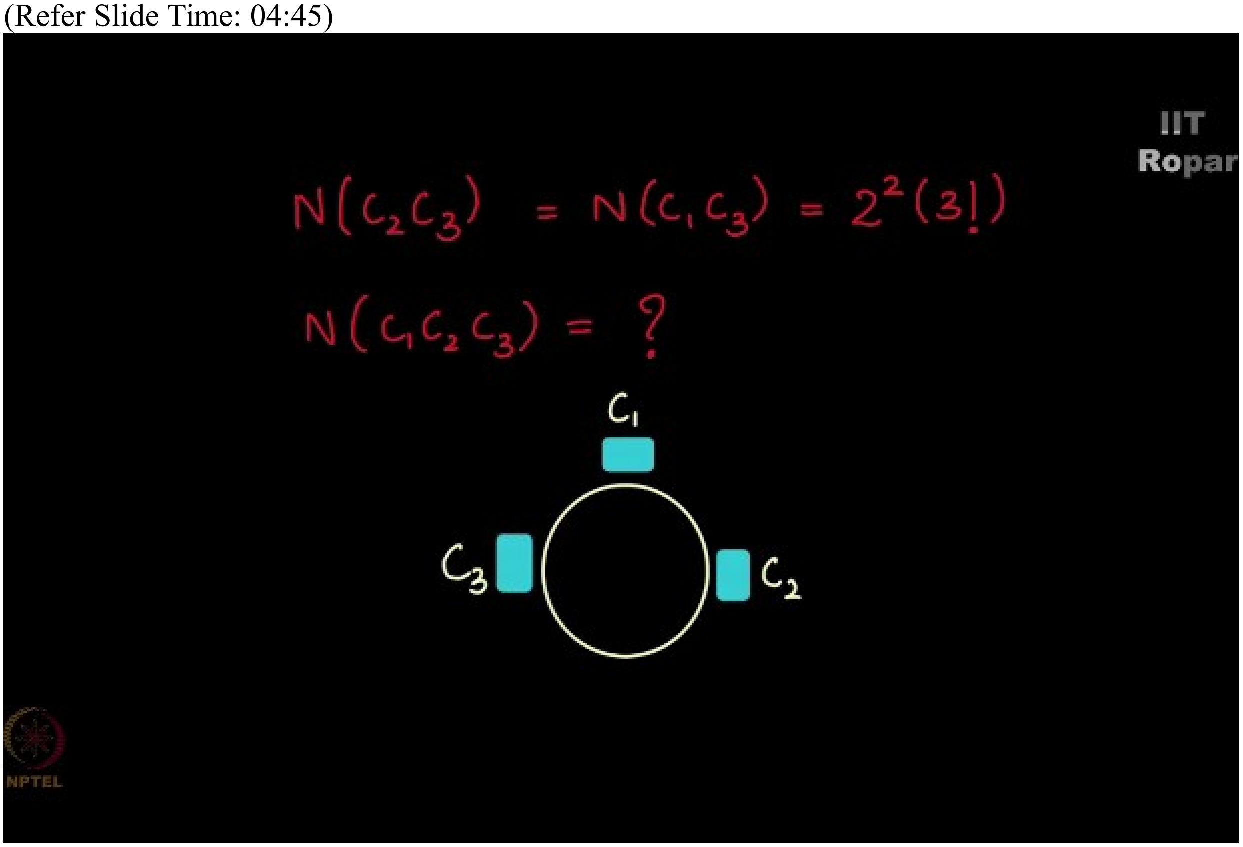

so we have 3 distinct objects here couple 1 is one object, couple 2 is another object, and couple 3 is yet another object, so what are the different permutations possible? The different permutations possible can go something like this C1 C2 C3, C1 C3 C2 and so on, now from the formula we see that it is 2 factorial, why? 3 distinct objects can be arranged around a circular table in 3 - 1 factorial which is 2 factorial ways, right, but we have 3 couples here and 1 couple can sit in 2 ways, right HW, WH, so for 3 couples it will be into 2 cube, did you get that? So N(C1,C2, C3)happens to be 2 factorial x 2 cube.
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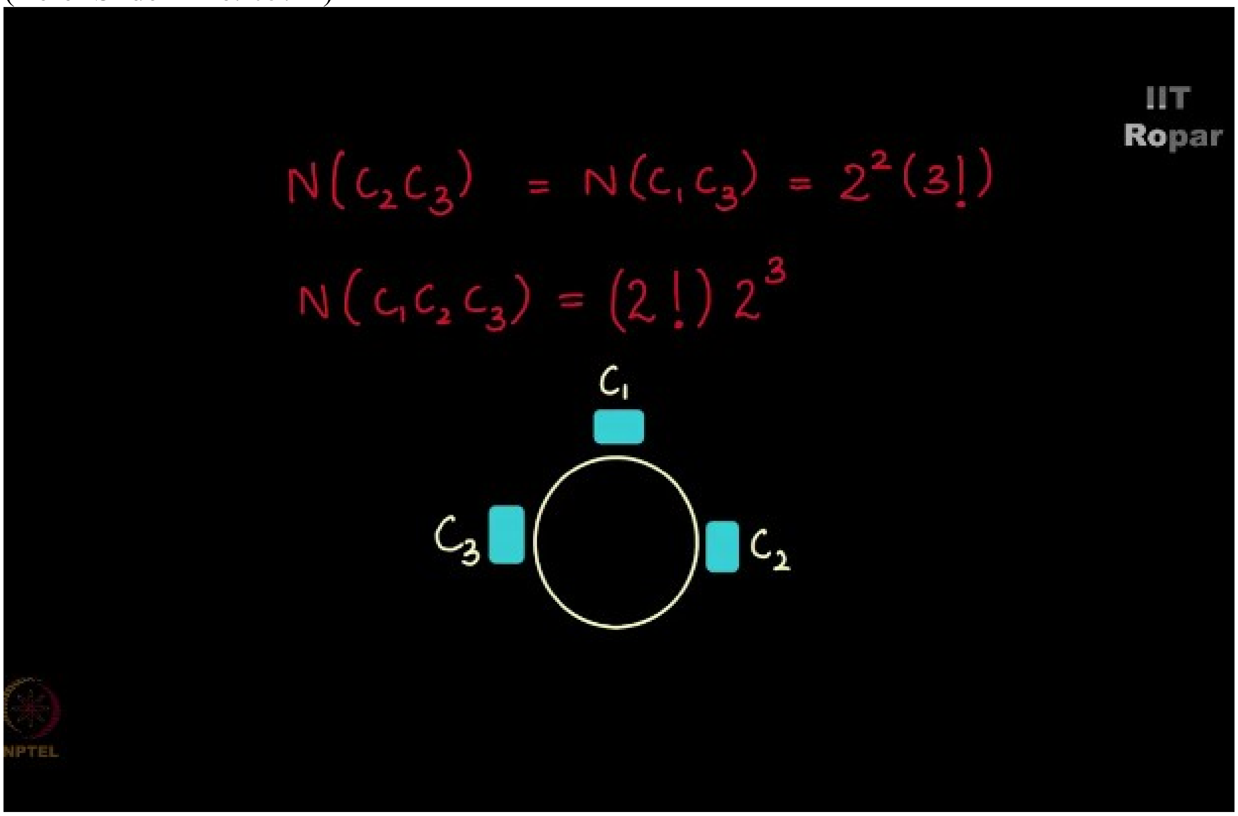

And now we are done with whatever we wanted to find out the answer, what is the final answer? So N(C1 bar, C2 bar, C3 bar) happens to be S naught -S1 + S2 - S3 this was the formula which we have learnt, so what is S naught?
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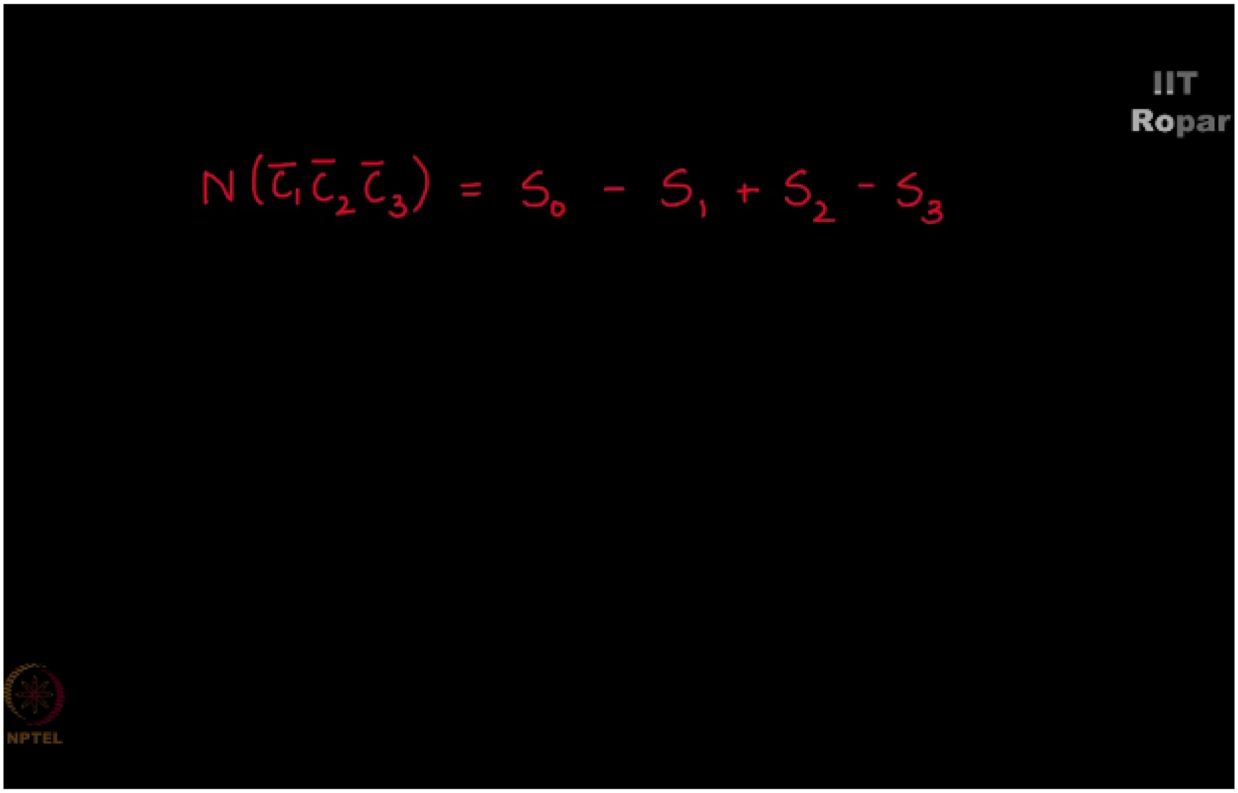

S naught is all possible permutations of 6 people, why 6 people? Initially we started with 3 couples which means 6 people all possible permutations is how many? 6 people sitting around a round table, in how many ways can they do that? 6 objects can be arranged around a round table in 5 factorial ways, and hence it is not 6 factorial, it is 5 factorial, please do not get confused it is just a matter of your pen and paper writing and you will understand things very clearly.

Now what is S1? S1 is N(C1) + N(C2) + N(C3), so -S1 happens to be -3 x 2 factorial, right 3 times it is the same thing 2 x 4 factorial, S2 is N(C1,C2) + N(C1,C3) + N(C2,C3) which is 3 x 2 square times 3 factorial –S3 is 2 cube into 2 factorial.

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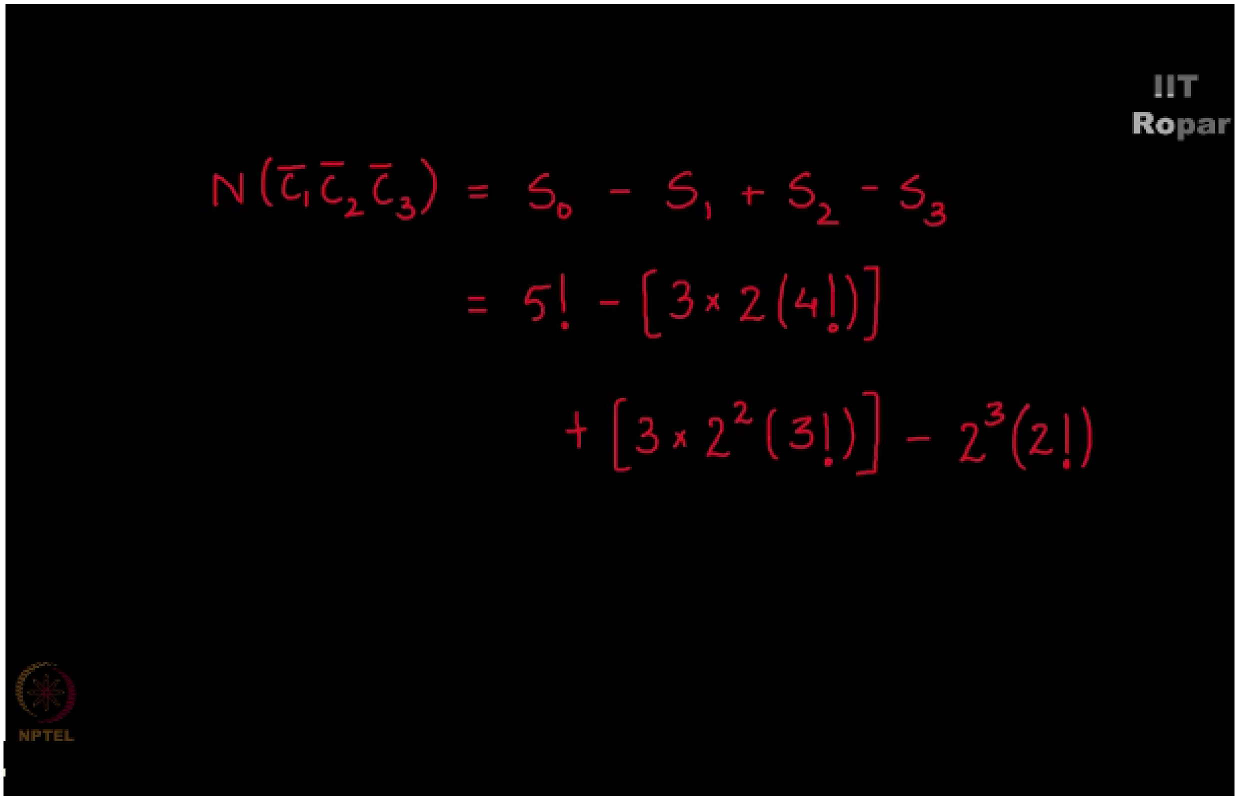

Now this big expression will give you the formula or it is the answer for your question that in how many ways can these 3 couples be seated at a round table such that no 2 like, no husband and wife will sit beside each other, so if you want for your better understanding or for completeness sake you can solve it yourself and find out the answer.

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