Discrete Mathematics
Principle of Inclusion and Exclusion

Example 7 - A dog nor a cat

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Consider a community of 50 people, where 30 people own a pet dog, and 25 people own a pet cat, 10 people own a dog and a cat as a pet, now how many of them do not own anything is the question, well let me write this as a community of 50 people,
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now let this represent let me call it as set B where people have dog as a pet and 30 of them are there, and this is a set C where they own cat as a pet and 25 people are there here, so what does this represent?
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It represents those 10 people who have both dog and cat as a pet.
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The question is how many of them do not have anything as a pet? So basically we should be finding out this area right, how many of them are there here?
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Consider a community of 50 people, where 30 people own a pet dog, 25 people own a pet cat and 10 people own both pet dog and cat. How many of them do not own anything?
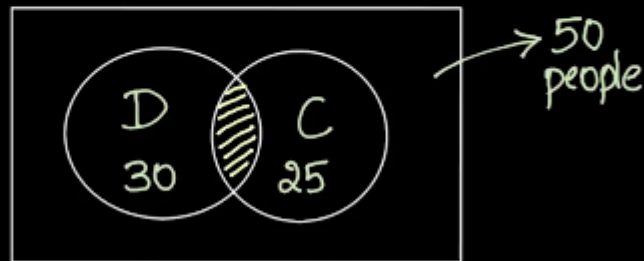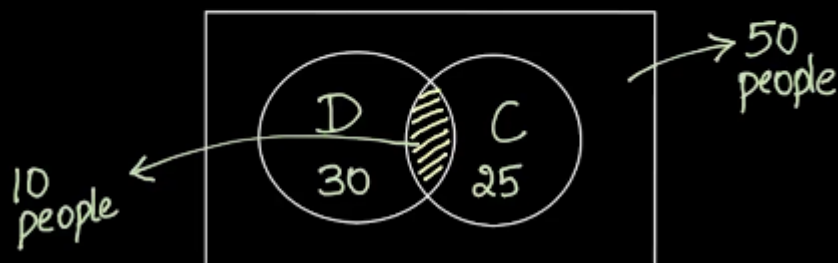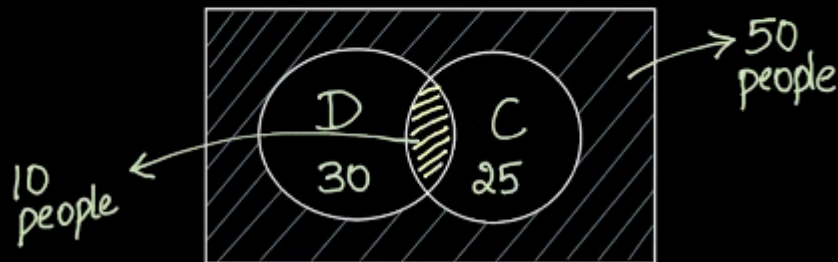

So N(C1) as you know is 30 here because we have 30 who have dog as pet, N(C2) is 25, so I did not mention what is C1 and C2 here, C1 is having that condition having dog as a pet and C2 is the condition having cat as a pet,

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C_1: People having dog as a pet.

C_2: People having cat as a pet.

$$N(C_1) = 30$$

$$N(C_2) = 25$$

so now C1, C2 is those people who are having both cat and dog, now which is this position or which is this region right, so we have to calculate this region, how many fall under this region?

So N(C1 bar, C2 bar) is N − N(C1) + N(C2) − N(C1,C2),

$$N(\bar{C_1}\bar{C_2}) = N - [N(C_1) + N(C_2)] + N(C_1 C_2)$$

so if I have to explain in a diagram it will be like this,

N(C1 bar, C2 bar) if you want you have to first find out N which is 50 here, now I'm going to keep subtracting things you can see this it is the shades will go on getting removed then you can find out the answer then, so the entire thing is shaded now which means that I am going to consider N which is 50 the total one, -N (C1), so N(C1) I am going to remove this area, this portion, right so that is 30.

Now N(C2) also has to be removed, I'm going to remove this area too, do you observe that this position has got removed twice,
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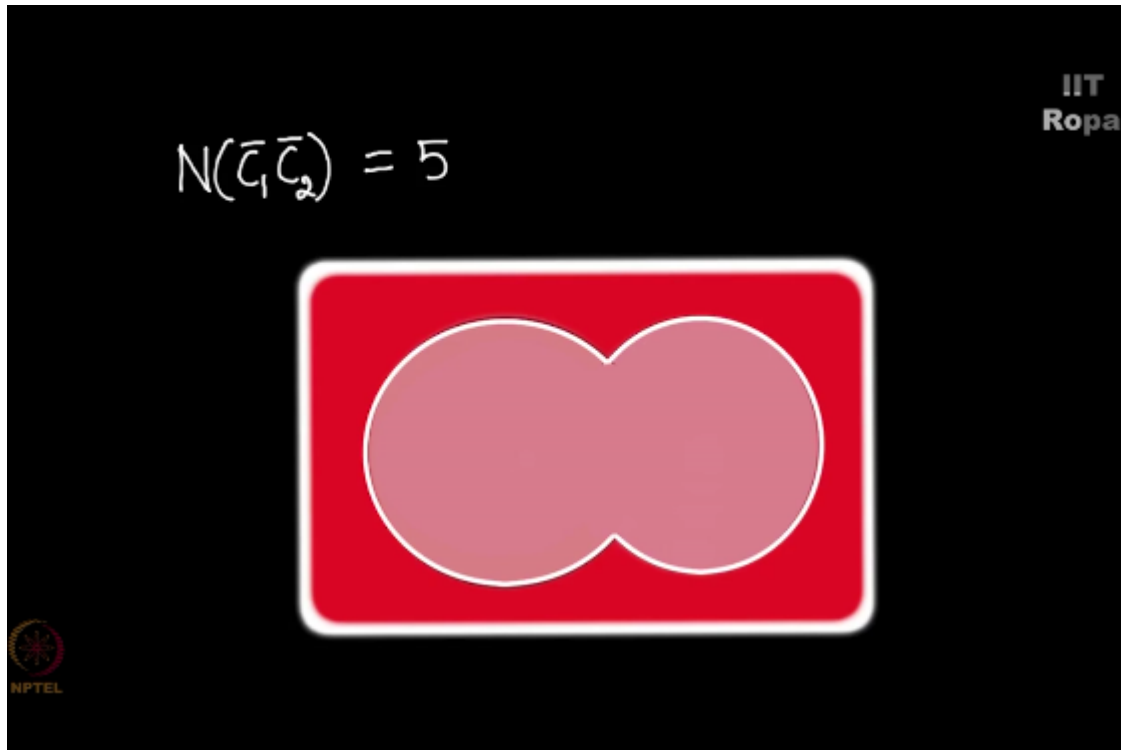
$$N - N(C_1) - N(C_2)$$
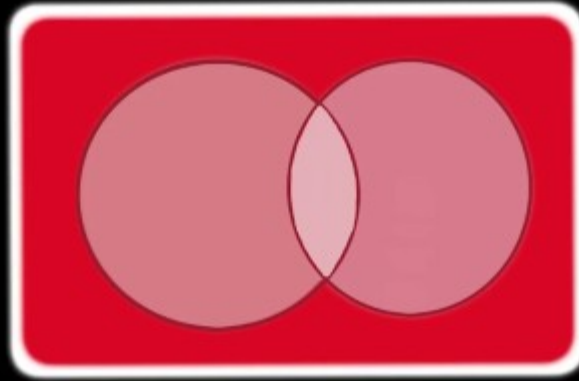
we are over subtracting and hence we should add it once, so this becomes I am going to add it once, so it is $+ N(C1,C2)$ right, so I have removed this area twice so I'm going to add it back once which means that I have just removed it once, so this area is no more there so we do not have this entire portion what remains is the area outside the two circles,

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$$N - N(C_1) - N(C_2) + N(C_1, C_2)$$
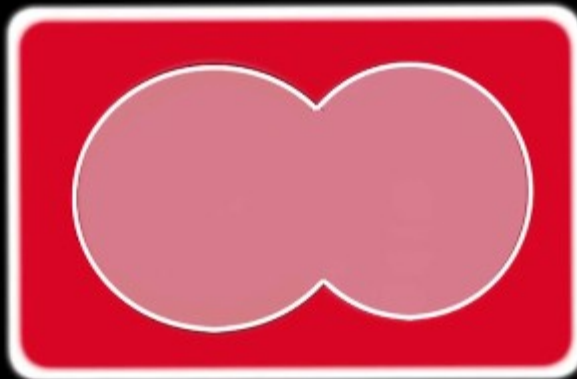
so how do we do that? N (C1 bar, C2 bar) is N 50 - 30 + 25 which is 55 + 10, so 50 – 55 is -5 + 10 happens to be 5, so 5 of the people in the community do not have dog or a cat as pet.
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