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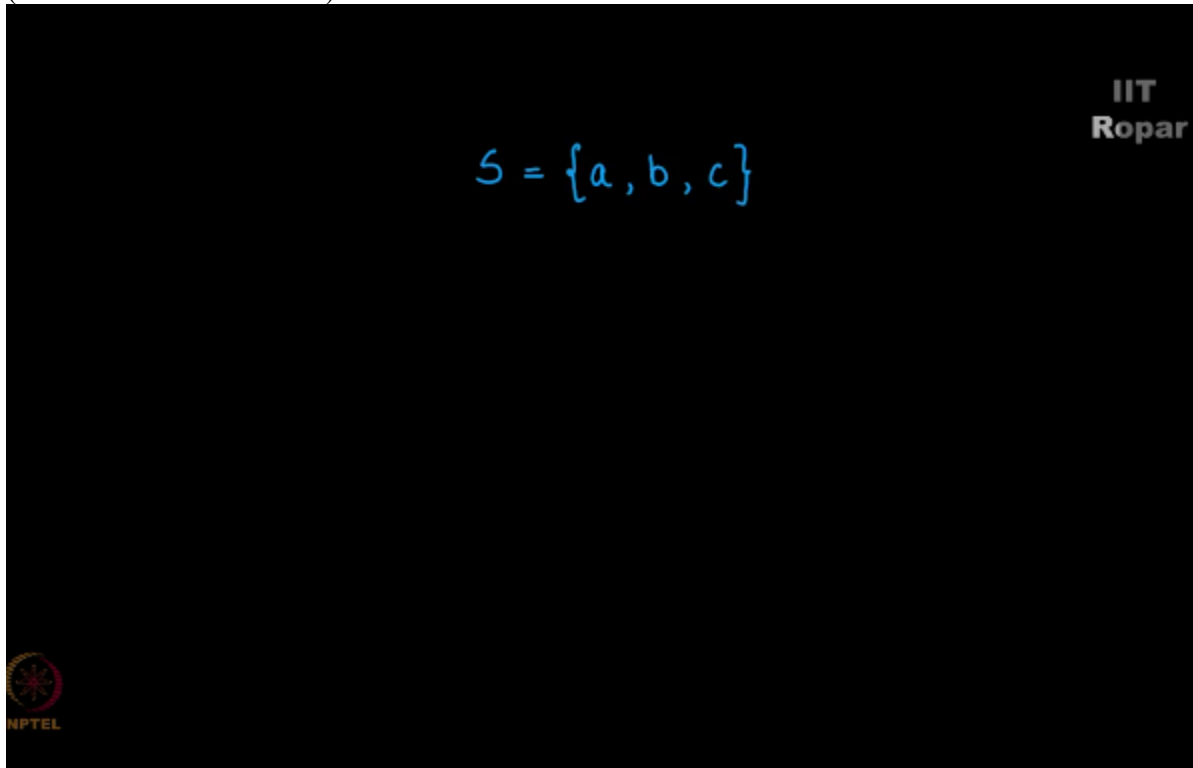
Discrete Mathematics  
Graph Theory – 3 &  
Generating Functions

Generating functions - Problem 3

By

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Consider this set  $S = A, B, C$ , so we have 3 elements in the set  $S$ ,  
(Refer Slide Time: 00:10)



now consider this function  $F(x)$  as  $(1+AX)(1+BX)(1+CX)$ , so the product of these three expressions  $(1+AX)(1+BX)(1+CX)$  is my function  $F(x)$ ,  
(Refer Slide Time: 00:32)

$$S = \{a, b, c\}$$

$$f(x) = (1+ax)(1+bx)(1+cx)$$



now what is the product of these polynomials? It is going to be  $1+AX+BX+CX+ABX$  square +  $ACX$  square +  $BCX$  square +  $ABCX$  cube, on simplification this becomes  $1+A+B+C$  into  $X$  square +  $AB + AC + BC$  into  $X$  square +  $ABCX$  cube,  
(Refer Slide Time: 00:50)

$$S = \{a, b, c\}$$

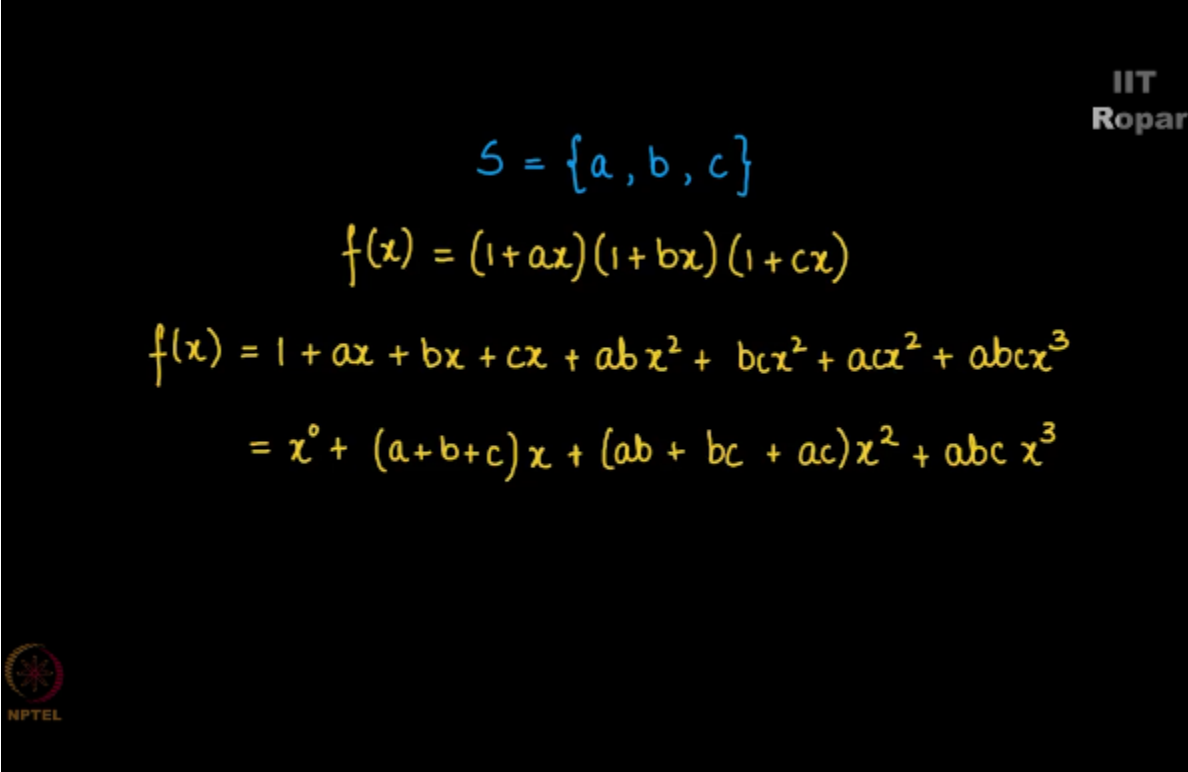
$$f(x) = (1+ax)(1+bx)(1+cx)$$

$$f(x) = 1 + ax + bx + cx + abx^2 + bcx^2 + acx^2 + abcx^3$$



on simplification we get it as  $1 + (A+B+C)x + (AB+AC+BC)x^2 + ABCx^3$ , now do you see that 1 is actually  $x^0$  here, right,

(Refer Slide Time: 01:23)



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$$S = \{a, b, c\}$$
$$f(x) = (1+ax)(1+bx)(1+cx)$$
$$f(x) = 1 + ax + bx + cx + abx^2 + bcx^2 + acx^2 + abcx^3$$
$$= x^0 + (a+b+c)x + (ab + bc + ac)x^2 + abc x^3$$

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so we have  $x^0 + (A+B+C)x + (AB+AC+BC)x^2 + ABCx^3$ , now keep this aside for a while, we have the set  $S$  as  $ABC$ , I'm going to consider all the subsets of  $ABC$ , I'm going to take the power set of  $S$  that  $S$ , what are the elements in the power set of  $S$ ? I have singleton  $A$ , singleton  $B$ , singleton  $C$ ,  $A,B$ ,  $A,C$  and  $B,C$  and the last one as  $ABC$ , I have missed out one that is the empty set.

(Refer Slide Time: 02:04)

$$S = \{a, b, c\}$$

$$P(S) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$



So these are the elements in my power set, I'm going to take the polynomial pack now, I have the function, the coefficient of  $X$  to the 0 is 1, do you see that? Now observe the beautiful representation or the relation between the coefficients and the power set, the coefficient of  $X$  to the 0 is 1, it represents the subset  $\phi$  of  $S$ , you have only 1  $\phi$  as a subset of  $S$ .  
(Refer Slide Time: 02:39)

$$S = \{a, b, c\}$$

$$P(S) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \emptyset\}$$

$$f(x) = 1x^0 + (a+b+c)x + (ab + bc + ac)x^2 + abc x^3$$

↓  
 $\emptyset$



Now the coefficient of  $X$  to the 1 or  $X$  is  $A+B+C$  that represents the singleton  $A$ , singleton  $B$  and singleton  $C$ , these 3 subsets of  $S$ , and a coefficient of  $X$  square is  $AB + BC + AC$ , now this represent the sets  $A,B$ ,  $A,C$  and  $B,C$ , and the last one the coefficient of  $X$  cube is  $ABC$ , now this represents the set  $A, B, C$ , did you observe the beautiful relation between the coefficients of the function and the subsets of the set  $S$ ,

(Refer Slide Time: 03:25)

$$S = \{a, b, c\}$$

$$P(S) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \emptyset\}$$

$$f(x) = 1x^0 + (a+b+c)x + (ab + bc + ac)x^2 + abc x^3$$

↓                    ↓                    ↓                    ↓

$\emptyset$      $\{a\}, \{b\}, \{c\}$      $\{a, b\}, \{b, c\},$      $\{a, b, c\}$

$\{a, c\}$



well the generating function for the subsets of S is given by  $F(x) =$  the product of  $(1+AX)$   $(1+BX)$   $(1+CX)$  where these elements ABC are the elements of the set S.  
(Refer Slide Time: 03:45)

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Generating function for subsets of S is

$$(1+ax)(1+bx)(1+cx)$$
$$S = \{a, b, c\}$$

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