NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 3 &
Generating Functions

Generating function examples Part 3
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We are going to see another interesting generating function 1/1-X as we know is 1 + X + X square + X cube and so on,
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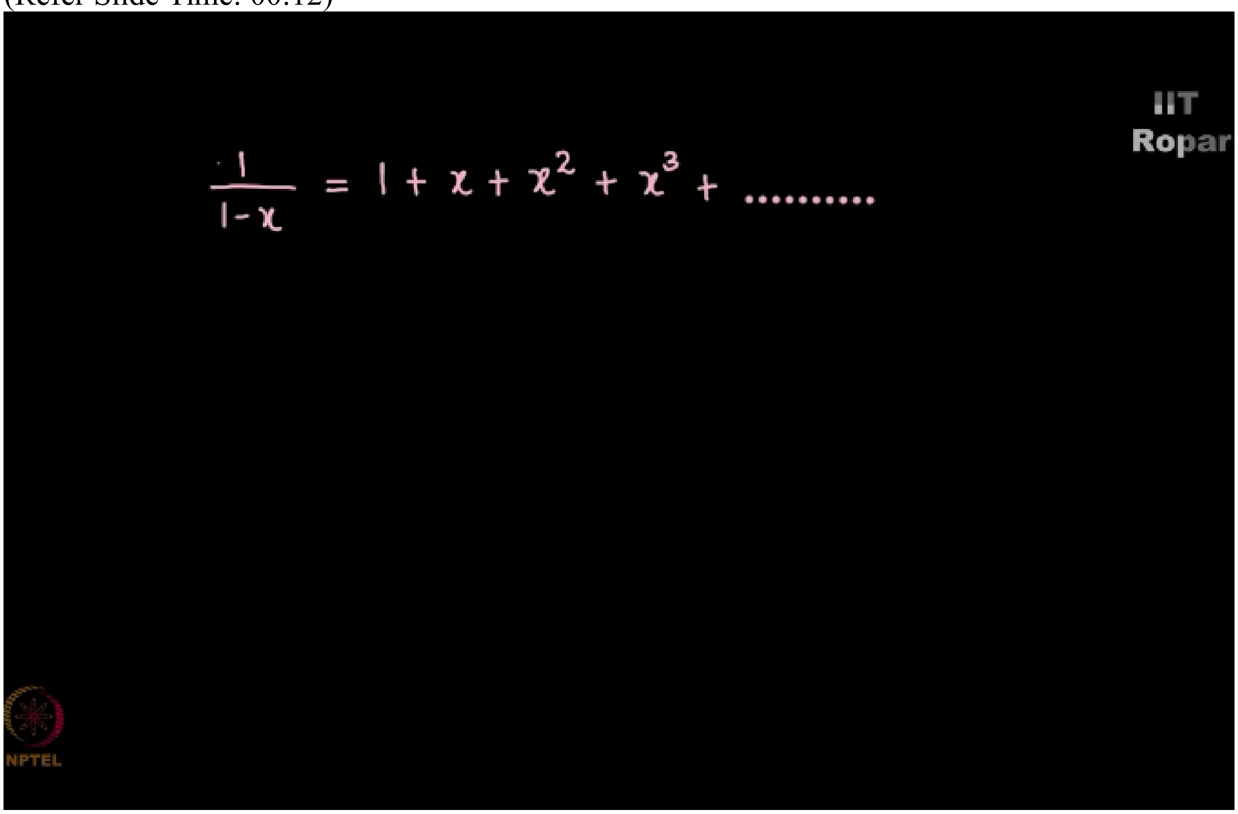

you must be wondering I always started this point, why is it so? Well, you can do some jugglery on this generating function and obtain several other generating functions, now if I multiply 2 into 1/1-X, I'm just multiplying 2 on both the sides, what do I get? I get it as 2 + 2X + 2X square + 2X cube + 2X to the 4 and so on.

Now do you see that 2/1-X is the closed form for the sequence 2, 2, 2, 2 and so on,
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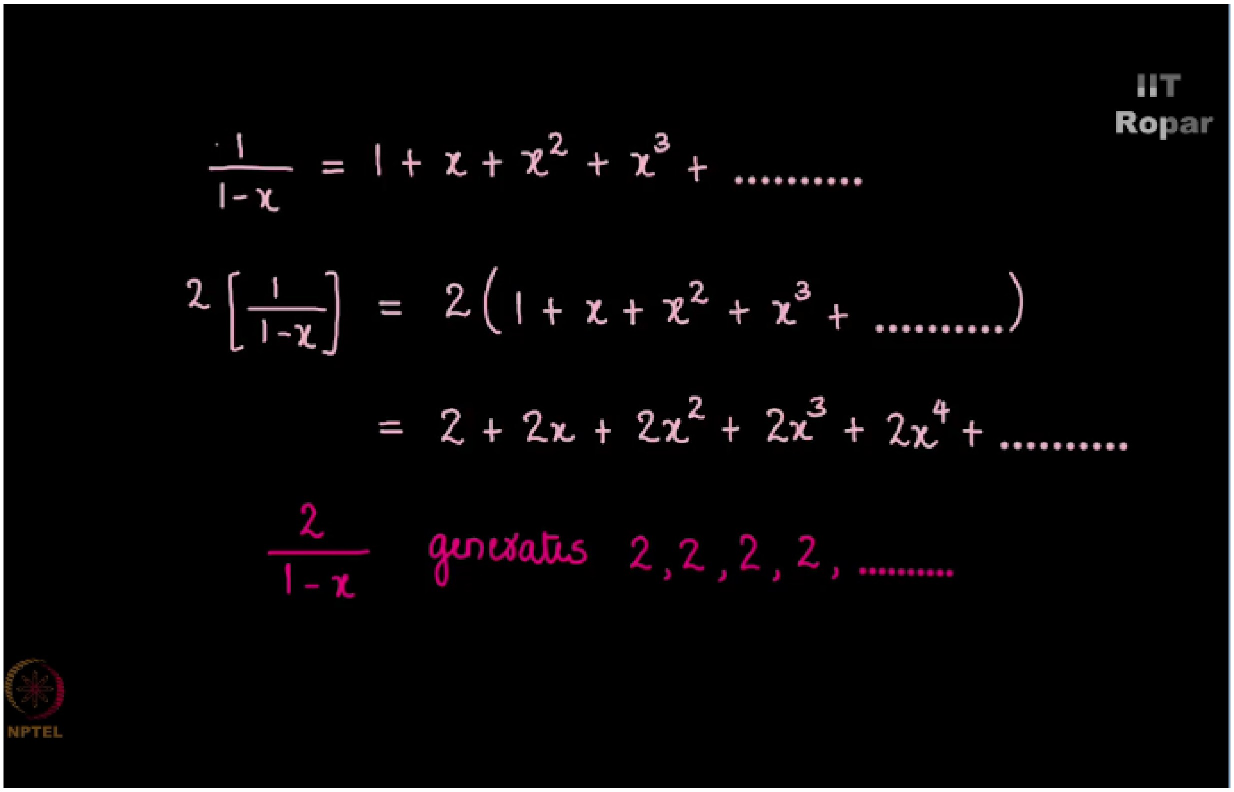

do you observe that? Now what if I multiplied by 3 instead of 2, I'll get the sequence 3, 3, 3, 3 from the closed form 3/1-X,

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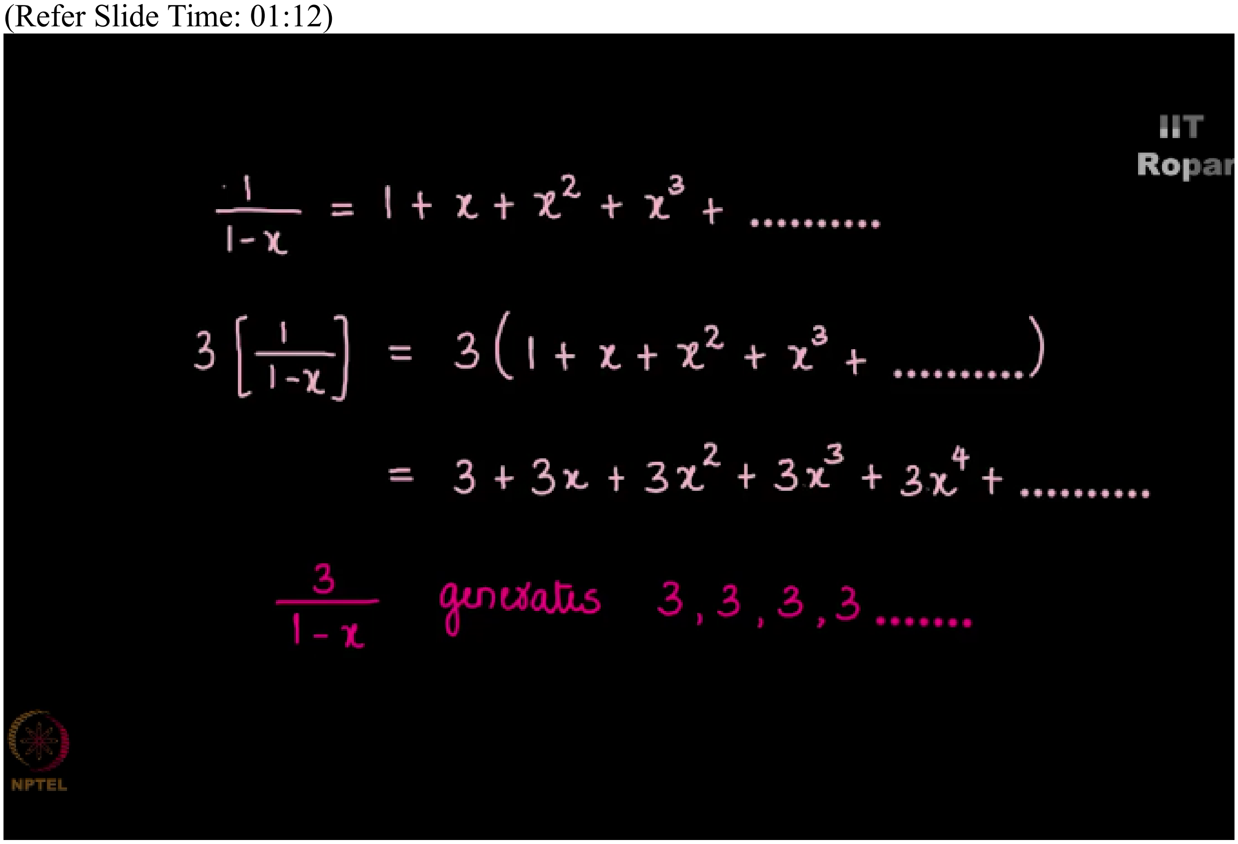

you must jump and tell me that this is indeed true for any K, so K/1-X or any constant A/1-X is the closed form of the generating function A + AX + AX square + AX cube + AX to the 4 and so on, now this is the generating function of the sequence A, A, A, A, A and so on.

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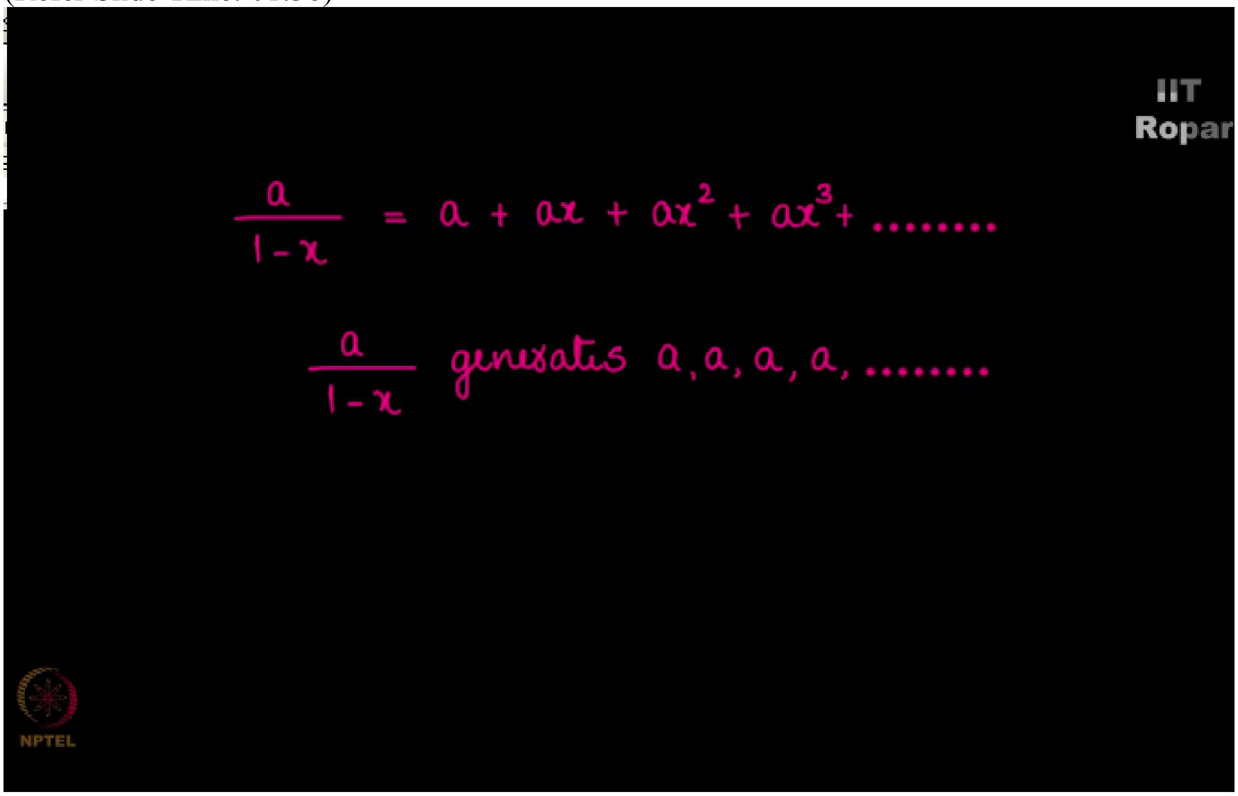

Now observe something? We know that 1/1-X is the generating function for the sequence, 1 + X + X square + X cube + X to the 4 and so on, and I'm giving it to you that 1/1+X is the closed form of the generating function 1-X + X square –X cube + X to the 4 – X to the 5 and so on, we have alternating positive and negative signs here.

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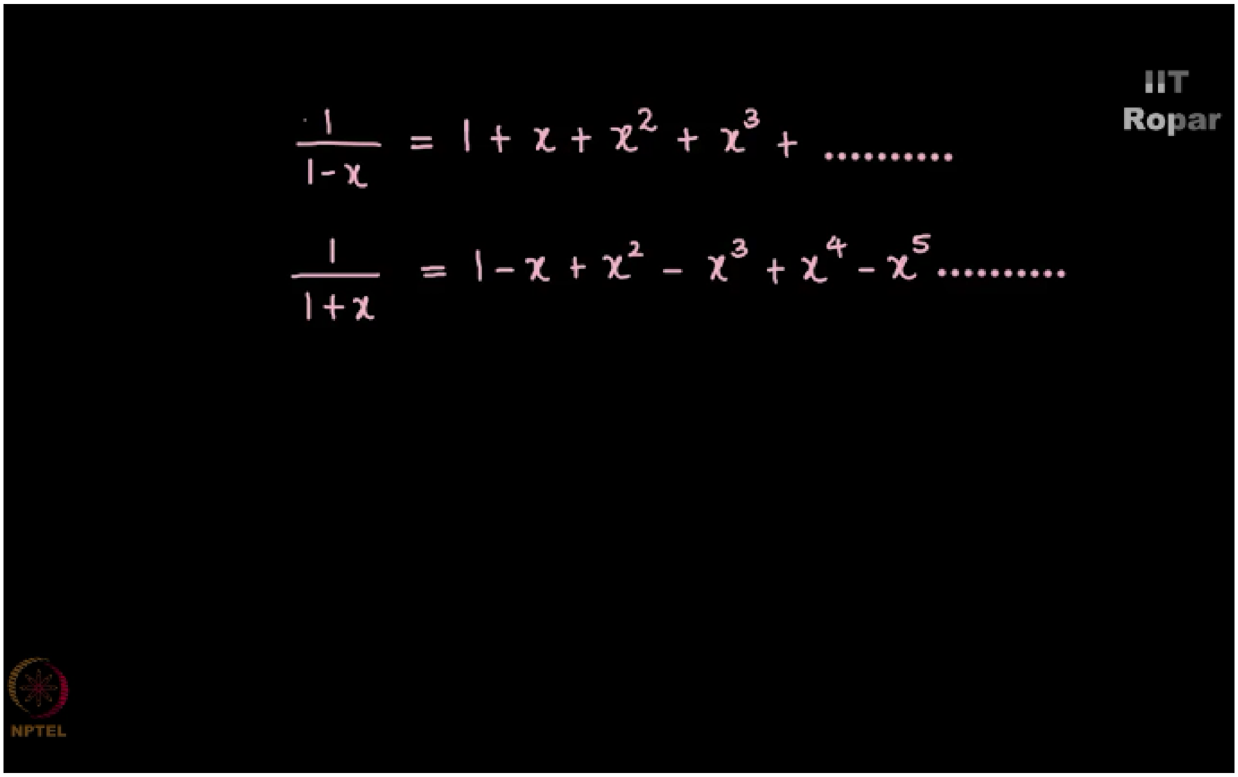

Now observe these two equations, I'm going to add both of them, so 1/1+X + 1/1-X gives me after adding both of the right hand sides, what do I get? Do you see that some terms get cancelled, -X, +X, -X cube, +X cube, so such alternative terms get cancelled and what remains is 2 + 2X square + 2X to the 4 + 2X to the 6 + so on.

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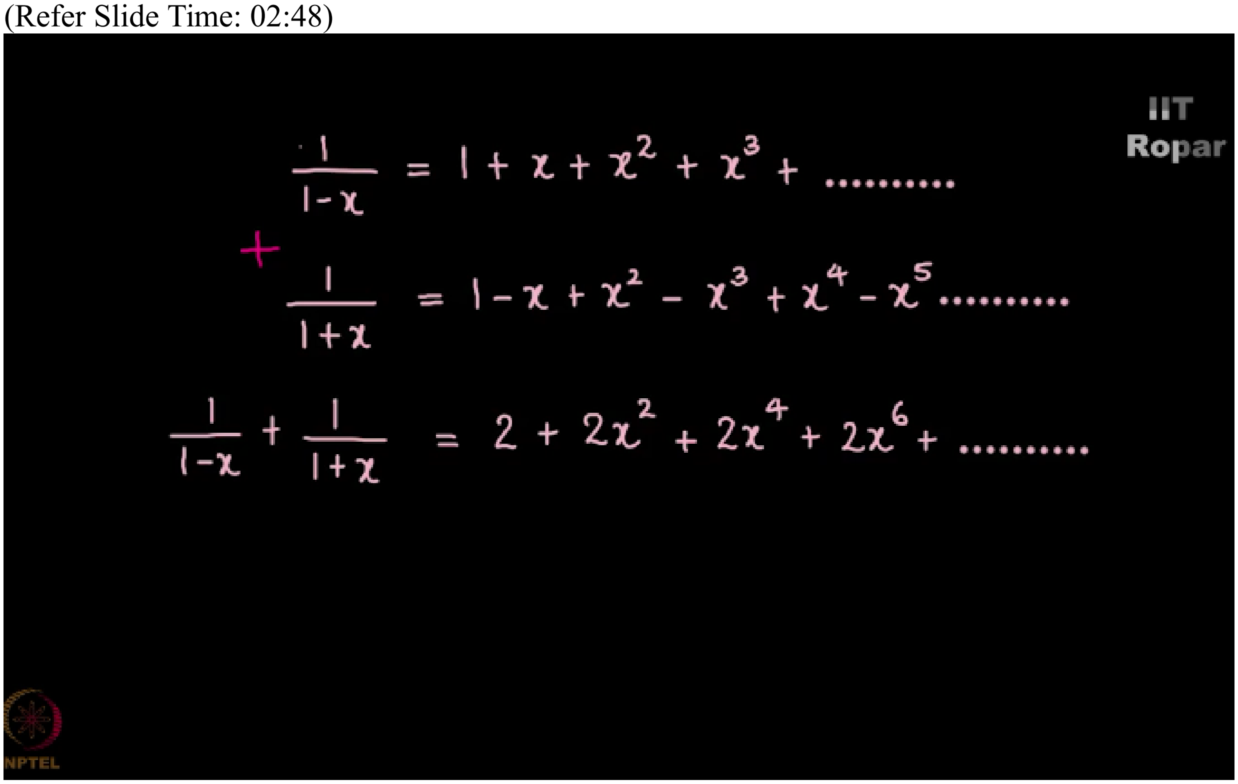

Now by simple calculation we see that the sum of these two is 1/1-X square, 1/1-X square = 1 + X square, by simple calculation we see that the left hand side here is equal to 2/1-X square, and this is equal to 2+2X square + 2X to the 4 + 2X to the 6 and so on,

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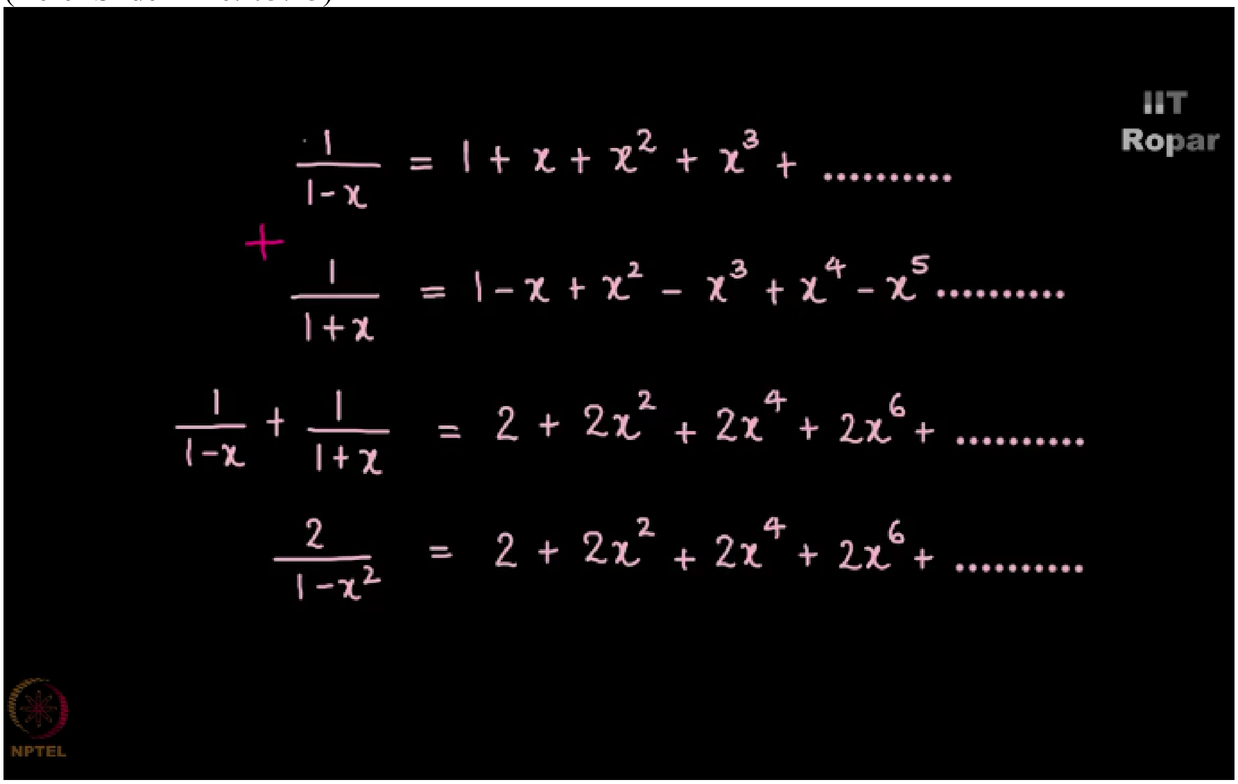

cancelling 2 on both the sides, what do we get, I have a 2 on the left hand side and I take out 2 on the right hand side common, so we can cancel both of them and we get 1/1-X square = 1 + X square + X to the 4 + X to the 6 and so on, do you see that we have all even powers here, 1 is X to the 0, so we have X to the 0, X square, X4, X to the 6 and so on, and the closed form for this is 1/1-X square.

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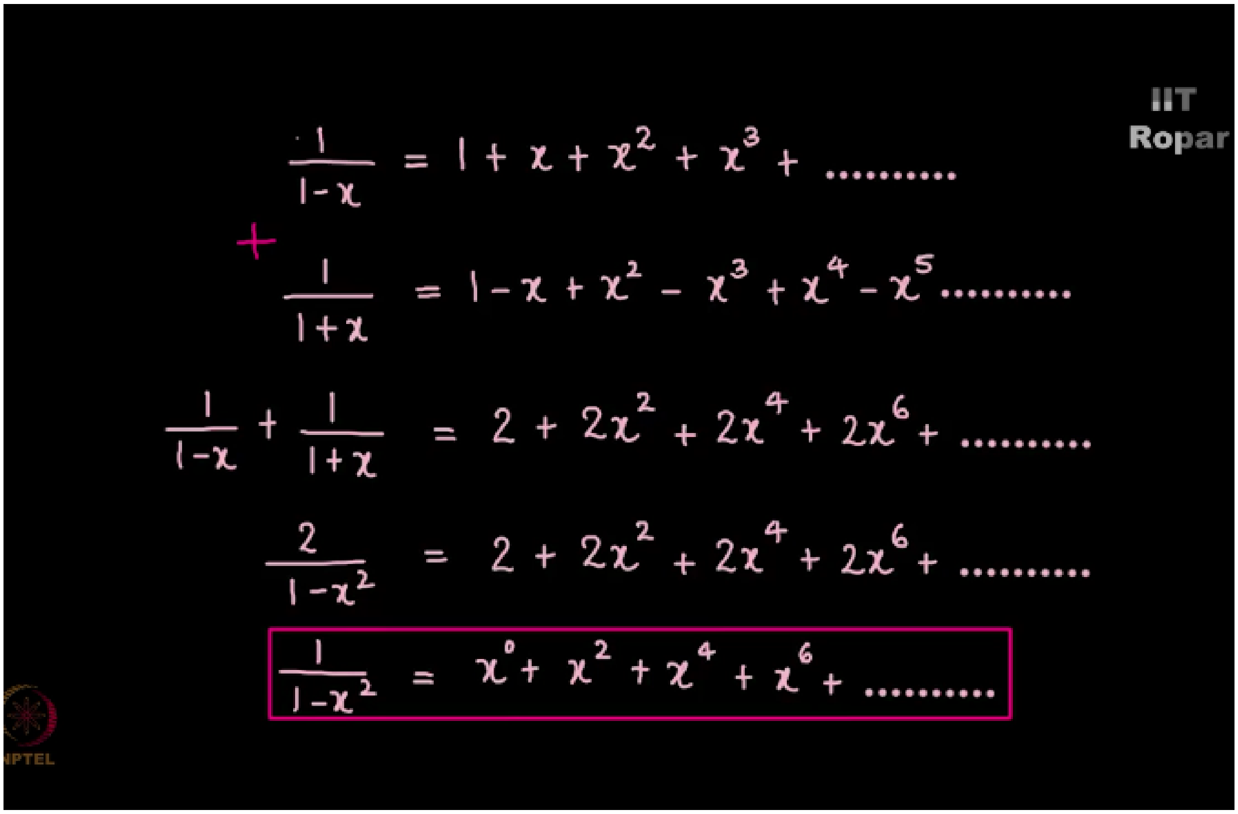

Now the question for you all, a challenge is how do we obtain X + X cube + X to the 5 + X to the 7 and so on, I have showed it for all the even powers, can you try it for all the odd powers?

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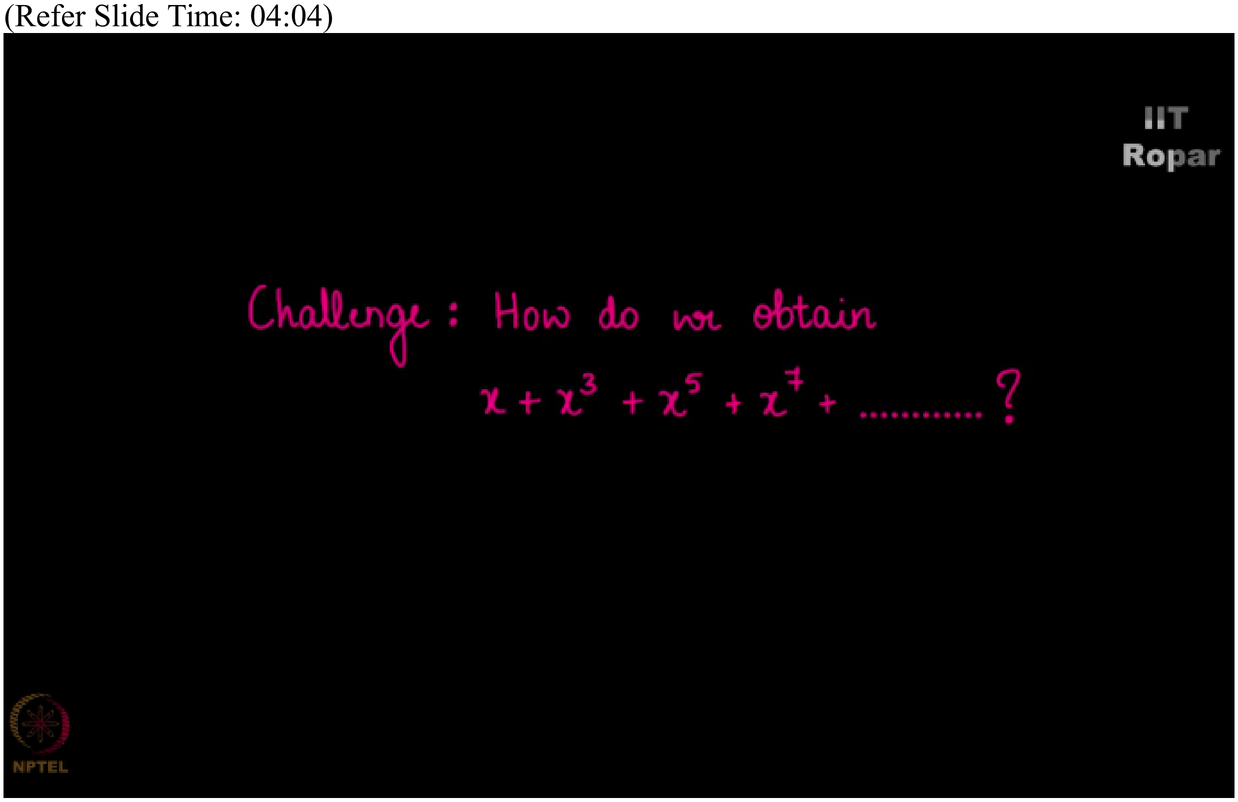

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