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NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics  
Graph Theory – 3 &  
Generating Functions

Generating function examples - Part 2

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We are going to see a few other generating functions they're closed forms and the sequence which they give, we know that  $1/1-X$  is  $1 + X + X^2 + X^3 + \dots$  and so on,  
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The slide displays the mathematical equation  $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$  in green text. The background is black. In the top right corner, the text "IIT Ropar" is visible. In the bottom left corner, the NPTEL logo is present.

right, now in place of X I am going to substitute 2Y, so what does it become?  $1/1-2Y = 1 + 2Y + 2^2 Y^2 + 2^3 Y^3 + 2^4 Y^4 + \dots$  and so on,  
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$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$x = 2y$$

$$\frac{1}{1-2y} = 1 + 2y + 2^2y^2 + 2^3y^3 + 2^4y^4 + \dots$$

now it's very easy for your people to observe here that  $1 - 2Y$  is the generating function for the sequence or rather it gives the sequence 1, 2, 2 square, 2 cube, 2 to the 4 and so on, 1 here if you observe was 2 to the 0 actually, so it is 2 to the 0, 2 to the 1, 2 square 2 cube, 2 to the 4 and so on.

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$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$x = 2y$$

$$\frac{1}{1-2y} = 1 + 2y + 2^2 y^2 + 2^3 y^3 + 2^4 y^4 + \dots$$

$$\frac{1}{1-2y} \text{ generates } 1, 2, 2^2, 2^3, 2^4, \dots$$



Now in place of 2Y what if I substitute 3Y, I will get it as  $1/1-3Y$  is  $1 + 3Y + 3 \text{ square } Y \text{ square} + 3 \text{ cube } Y \text{ cube} + 3 \text{ to the } 4, Y \text{ to the } 4 + \text{ so on,}$   
(Refer Slide Time: 01:24)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$x = 2y$$

$$\frac{1}{1-2y} = 1 + 2y + 2^2 y^2 + 2^3 y^3 + 2^4 y^4 + \dots$$

$$\frac{1}{1-2y} \text{ generates } 1, 2, 2^2, 2^3, 2^4, \dots$$



now do you observe that  $1/1-3Y$  has given us the sequence 3 to the 0, 3 to the 1, 3 square, 3 cube and so on,  
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$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$
$$x = 3y$$
$$\frac{1}{1-3y} = 1 + 3y + 3^2 y^2 + 3^3 y^3 + 3^4 y^4 + \dots$$

$\frac{1}{1-3y}$  generates  $3^0, 3^1, 3^2, 3^3, 3^4, \dots$

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you must be guessing my obvious next question, can we substitute in place of X any KY, or any constant into Y, so in general I can write  $1/1-AX$  in place of Y I am just writing X here,  $1/1-AX$ , A is any constant, I will get it as  $1 + AX + A$  square  $X$  square + A cube  $X$  cube + A to the 4  $X$  to the 4 + so on, so  $1/1-AX$  is the closed form of the generating function for the sequence A to the 0, A, A square, A cube, A to the 4 and so on,  
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$$\frac{1}{1-ax} = 1 + ax + a^2x + a^3x^3 + a^4x^4 + \dots$$

$$\frac{1}{1-ax} \text{ generates } a^0, a^1, a^2, a^3, a^4, \dots$$

take some time and go through all of these.

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