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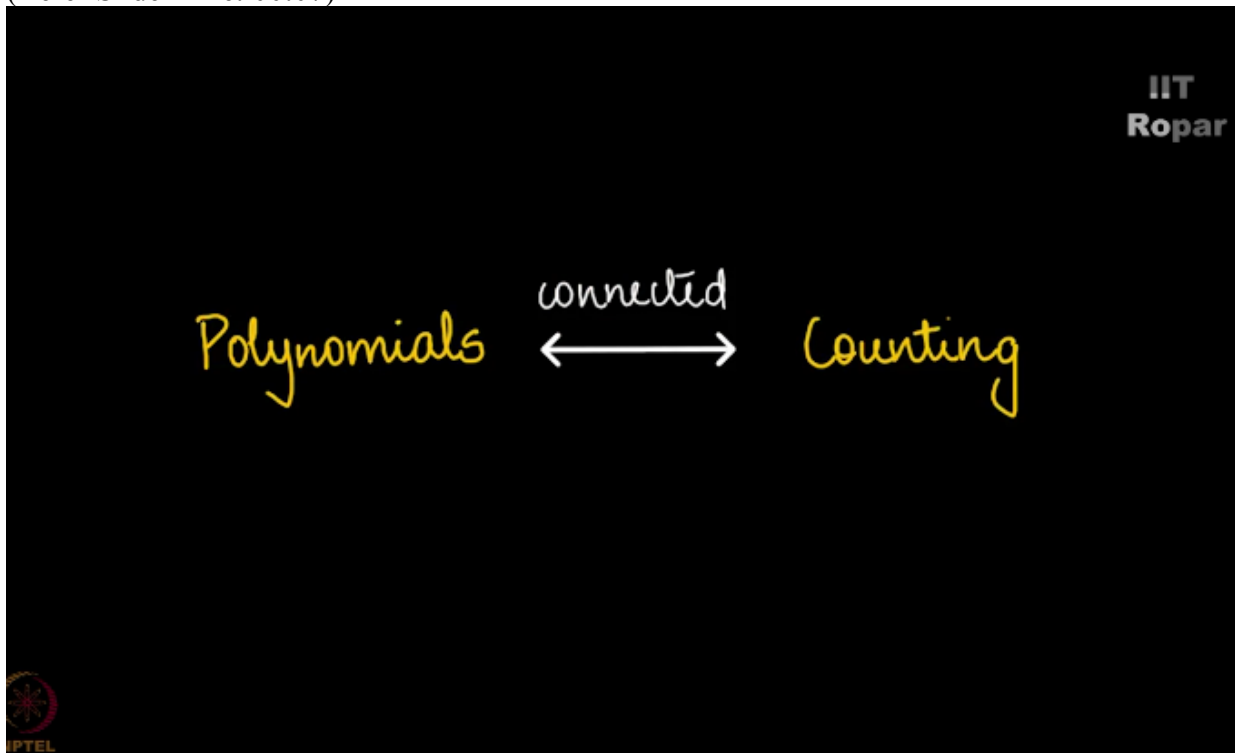
NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics  
Graph Theory – 3 &  
Generating Functions

Definition of Generating function

By  
Prof. S.R.S Iyengar  
Department of Computer Science  
IIT Ropar

We saw that polynomials are surprisingly connected to counting,  
(Refer Slide Time: 00:07)



let me go ahead and tell you some of the popular definitions related to polynomials, look at this  $1 + X + X^2 + X^3 + X^4 + \dots$  up to infinity, this is not a polynomial actually, but this looks like a polynomial,  
(Refer Slide Time: 00:24)

$$1 + x + x^2 + x^3 + x^4 + \dots$$



if it stops at a finite stage, finite position it's called a polynomial, right, a polynomial basically is the form  $A_0 + A_1X + A_2X^2 + \dots + A_N X^N$ , and degree polynomial, (Refer Slide Time: 00:40)

$$1 + x + x^2 + x^3 + x^4 + \dots$$

$$\text{Polynomial} : a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$



right, but this stuff whatever I'm showing you right now is going up to infinity, I'm sure you all know that this is actually  $1/(1-x)$ ,

(Refer Slide Time: 00:49)

The slide features a black background with white and green text. In the top right corner, the text "IIT Ropar" is visible. A green rectangular box contains the series  $1 + x + x^2 + x^3 + x^4 + \dots$ . Below this, the general form of a polynomial is written as  $\text{Polynomial} : a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ . The series is then equated to a fraction:  $1 + x + x^2 + x^3 + x^4 + \dots = \frac{1}{1-x}$ . In the bottom left corner, there is a circular logo with a star-like pattern and the text "NPTEL" below it.

so called the infinite geometric series, and this is true only when  $x$  is less than 1, not otherwise if  $x$  is even equal to 1 as you can see there is a 0 in the denominator and this infinite series becomes  $1 + 1 + 1$  and goes to infinity,

(Refer Slide Time: 01:10)

$$1 + x + x^2 + x^3 + x^4 + \dots$$

$$\text{Polynomial} : a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

$$1 + x + x^2 + x^3 + x^4 + \dots = \frac{1}{1-x}, \quad |x| < 1$$



let me not bother you with all those things, all that you got to know is 1 over 1 - X is equal to 1 + X + X square up to infinity.

So let me now differentiate both the sides, I then get derivative of 1 over 1 - X, some simple calculus tells me it is 1 over 1 - X the whole square,  
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$$\frac{d}{dx} \left( \frac{1}{1-x} \right) = 1 + x + x^2 + x^3 + x^4 + \dots$$



but then derivative of 1 happens to be 0, derivative of X will be 1, X square will be 2X, X cube will be 3X square, and so on, you see look at the coefficients of this particular thing  $1 + X + X$  square it was 1, 1, 1, 1, 1, 1, 1,  
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$$\frac{d}{dx} \left( \frac{1}{1-x} \right) = 1 + 1x + 1x^2 + 1x^3 + 1x^4 + \dots$$

$$\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + \dots$$



so the language we use is  $1/(1-x)$  generates, what we say, generates 1, 1, 1, 1, 1, okay,  $1/(1-x)^2$  generates 1, 2, 3, 4, and so on, correct, so this is called a generating function of this infinite sequence, 1, 2, 3, 4, etcetera.

Now can you people think and tell me what would be the generating function which generates 1 square, 2 square, 3 square, 4 square and so on?

(Refer Slide Time: 02:27)

$\frac{1}{1-x}$  generates  $1, 1, 1, 1, \dots$

$\frac{1}{(1-x)^2}$  generates  $1, 2, 3, 4, \dots$

Generating function of  $1, 2, 3, 4, \dots$

What is the generating function which  
generates  $1^2, 2^2, 3^2, 4^2, \dots$ ?



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