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Discrete Mathematics

Let Us Count

Catalan Numbers - Part 3

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So I have tried all the possibilities manually for the total number of paths without crossing the diagonal from 0,0 to 5,5. Let me show you. This is one possibility. This is another one. This is yet another possibility and so on. It might surprise you that there are 42 such possibilities to go from 0,0 to 5,5 without crossing the diagonal.

We saw that the answer is not necessarily 10C5 which is all possibilities divided by 2. It was something else. So a straightforward intuition of seeing that crossing and not crossing would be of the same numbers and hence it's going to be 10C5 by 2 may not work. Something to note here. Correct.

So what else will work? Let's see what works here. Before going any further I will try recollecting a small tip that I have been giving you when it comes to counting, correct. When you want to count a set count the number of elements in a set what you do is you try creating a one-to-one correspondence to another set where counting is very easy and then show that this set has the same number of elements as this set and counting in this set is very easy and hence the number of elements in this set is now known, right. I'm going to use this technique right now in a very subtle way.

The concept is slightly deep. You may want to watch this video multiple times to understand what we are saying. So now let's get started. What was the question? What are the total number of ways in which you can reach 5,5 from 0,0 without trespassing the diagram? Alright. Let me now observe an instance where I actually trespassed the diagonal and I will count all those

possibilities which crosses the diagonal. And then I will subtract that from my all possibilities which is 10C5 and I am done, correct. So all that remains right now is to count the number of ways in which you can trespass the fence.

I will start with just one such instance. Look at this. RUURURRURU. When you take a first step that is R you are well below the fence and next when you take U, you touch the fence and another U you trespass the fence. Right? And then you take a few more steps. Let me put a vertical line immediately after that state when you trespass the fence, and what I do is a small trick here. I take the right side of this vertical line and take the compliment of it. So an R becomes U and a U becomes R. I will take a compliment of it like this. Alright. Why am I doing this? Call me crazy. You will get to know why I am doing this in some time but as of now you don't worry much about why exactly is he doing this compliment business. Right.

Okay. Observe. On the left side of the vertical line you have one U more than the R that is when you put a vertical line the rule is not just you put a vertical line after to use it is whenever you trespass trespassing is when you have one U more than the number of Rs correct. There are two Us here, one R here that is when you trespass the diagonal. When you are on the diagonal the number of Rs and Us is the same, correct. When you trespass one there is one U more than the number of Rs. Put a vertical line and on the right side when you have one R, one U more than an R on the left side of the vertical line obviously there should be one R more than the number of Us in the right side that is when you will get equal number of Rs Us. Pause the video and think about it. On the left side you have one U more than the number of Rs; on the right side you have one R more than a number of Us always whenever you put a vertical line on the left side you will have this on the right side you will have this and what I do is I take the compliment of the right side which had one R more than a number of Us. In the compliment you will get one U more than the number of Rs. On the left side you already had one U more than the number of Rs and a net total right now will be 2 Us more than the number of Rs which means you now have four Rs and six Us. What on earth does this denote? This doesn't denote away from 0,0 to 5,5. This simply denotes a combination of four Rs and six Us. Okay.

Now what exactly happened so far? I took an invalid move and I created a unique mapping to a set containing elements with four Rs and 6 Us and my claim right now is this mapping is unique. Unique in what sense? Every element on this side, on which side? Of the left side what does that denote? It denotes all possible trespassing cases on the left side and on the right side all possible four Rs and six Us and its combinations. How many such things are there? Pretty straightforward, isn't it? 10C6 correct. There are 10C6 such possibilities and now what do we observe, for any 4 Rs and 6 Us possibilities you can actually put a vertical line after one U more than the R possibility and then take the complement you will get a unique element in the left side and vice-versa, correct, for an element to the left side you have a unique element on the right side. For every element in the right side you have a unique element on the left side. Right? And

hence the total number of possibilities that are trespassing are precisely 10C6 and I want to subtract this from 10C5 which is total number of possibilities. So my final answer of all those possibilities that doesn't trespass the line is 10C5 minus all the invalid ones trespassing 10C6. 10C5 minus 10C6 and that's going to be my answer. Observe. We can even generalize this and we get 2nCn minus 2nCn minus 1 as the answer for the question when you want to move from 0,0 to n,n without trespassing the line.

I believe this lecture was quick. It was intended to be that way. I request you all to please pause here and there and watch the video multiple times so that you understand every piece of what is being told. There are actually several pieces here, right. There's a beautiful one-to-one correspondence idea that we use to count the total number of trespassing instances and you use that to subtract from all possibilities and hence finally you will get those possibilities where you do not trespass.

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