NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

Prisoners example and Proper coloring

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Let us look at the question about prisoners in a brand new light, look at this example here A, B, C, D, E, F, G, H are the 8 prisoners,
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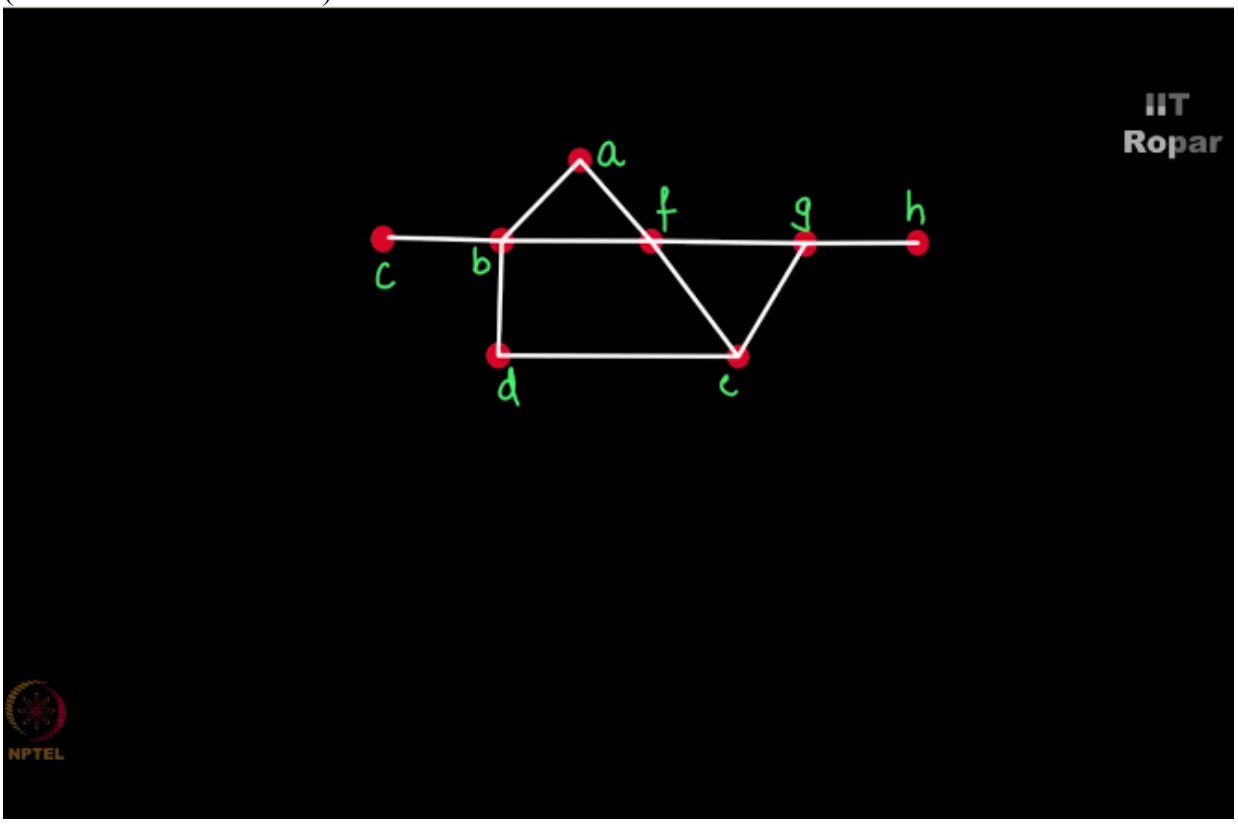

A hurts B, you cannot put A and B in the same cell, they'll kill each other, A hurts F and so on, one less commonsensical way is to simply put each one of them in a different prison, but you see A and E can actually be in the same prison, you can reduce a prison cell right, so pause the video and try to figure out what is the minimum number of prison cells that you can think of, so that you can put these prisoners in the prison cell so that no 2 enemies belong to the same prison cell, is this even possible?

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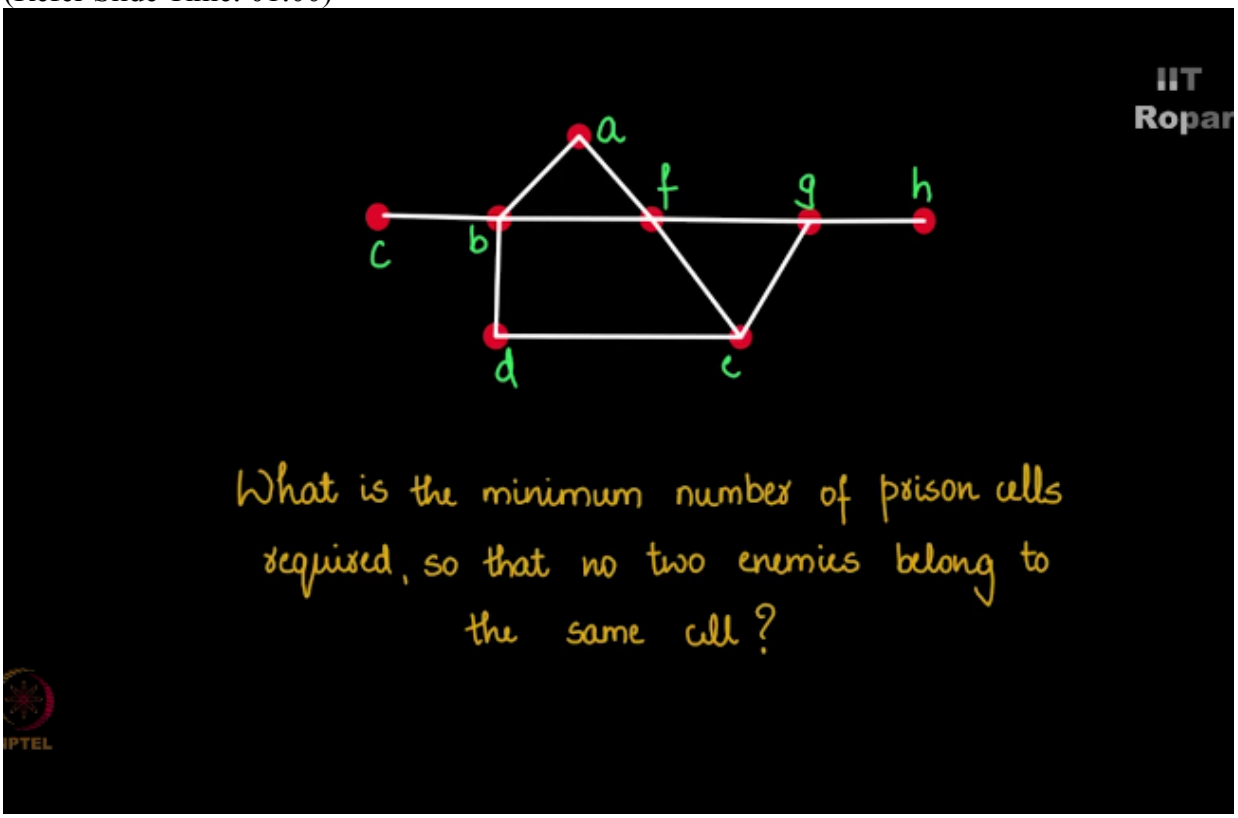

Yes of course I showed you if you had 8 prison cells it's possible for you to assign one per prison cell, can you reduce it? How less can you manage with? Think, okay I think you all gave it a thought, here goes the answer look at this 1, 2, 3 is the naming convention I have used to denote the prison cells,

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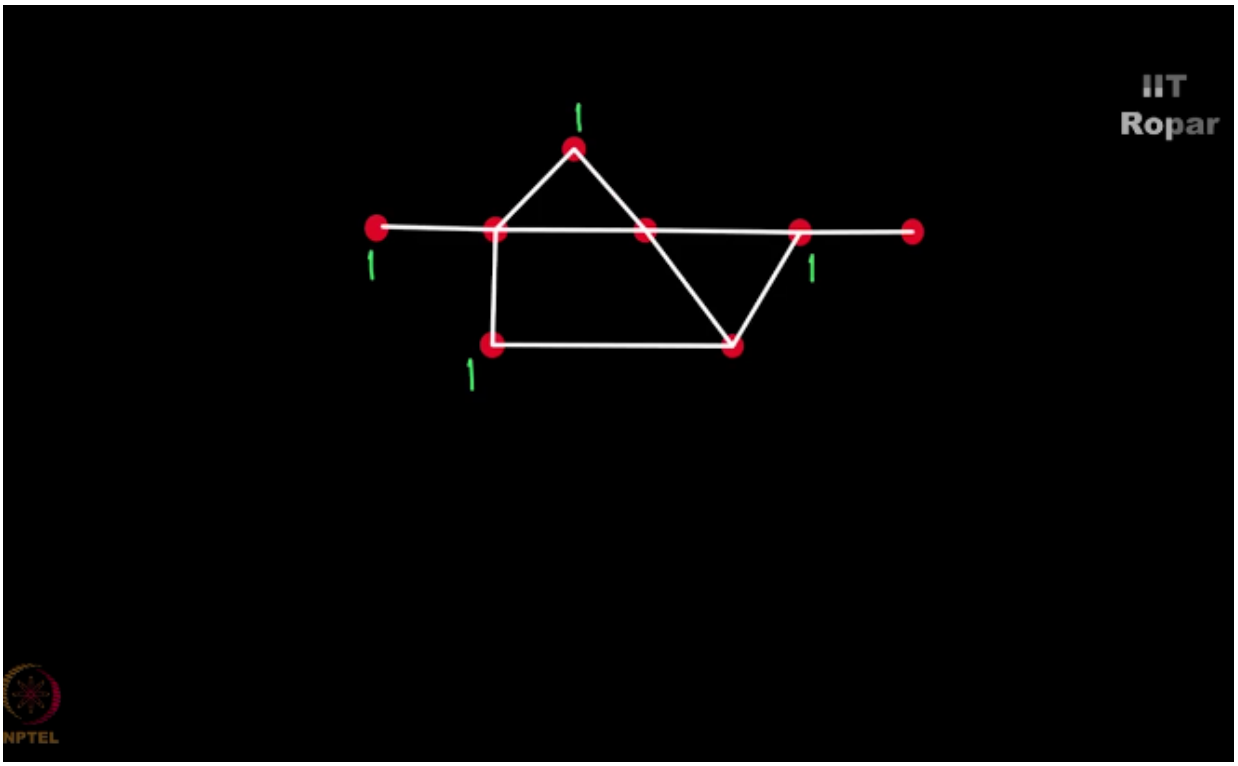

so this fellow goes to prison 1, 1, 1, 1, and these 3 people go to prison two, and only one person goes to prison 3, basically A, C, G goes to cell 1, B, E, H goes to cell 2, and F goes to cell 3,

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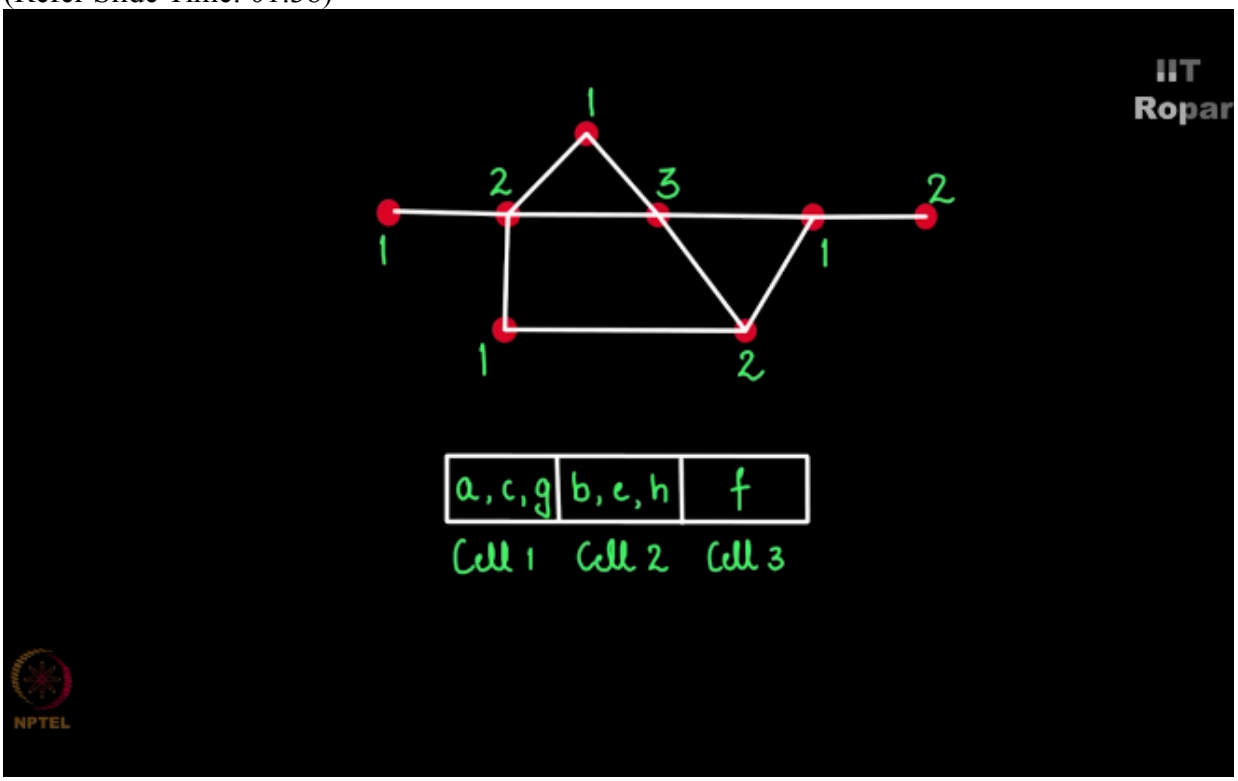

correct, this is one way of doing it. The other way of doing it is simply this A, D, G, B, E, C, F, G there may be many such ways,
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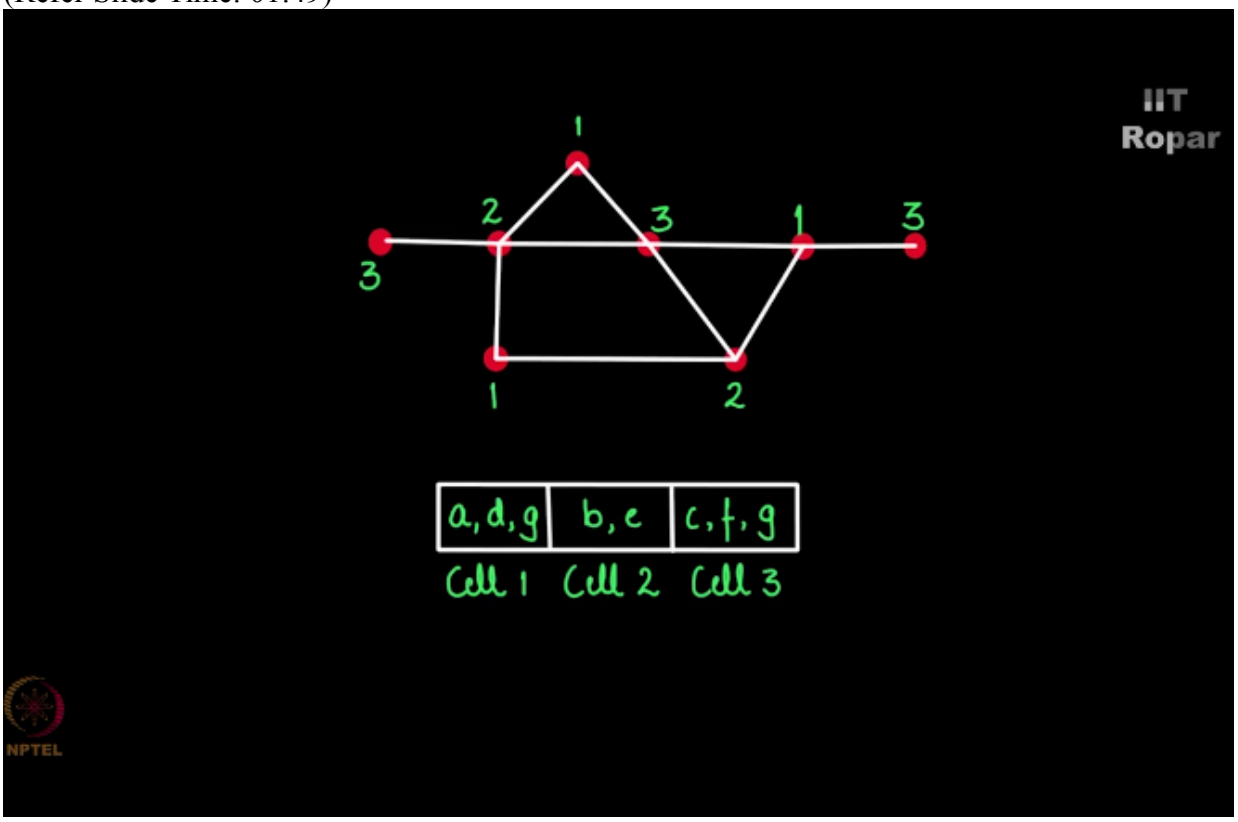

right, okay, think about what we just did.

Now let us cut the example from where we started and look at the mathematical definition for which the prisoners question was a motivation, the definition goes by the name proper coloring it means given a graph G can you color the nodes in such a way that no two adjacent nodes have the same color, don't you think this is precisely what we have been discussing with the prisoners question, it is exactly the same as you can see, correct, so I repeat given these prisoners and given the enemy network you must ensure that no 2 enemies belong to the same prison cell, can you minimize the number of cells? I'm trying to figure out who should be assigned to what prisoner number so that no 2 enemies come in the same prison, this question is same as can you assign with minimum number of colors, assign colors to all the vertices such that no 2 vertices that are adjacent have the same color, this is called a proper coloring of a graph.
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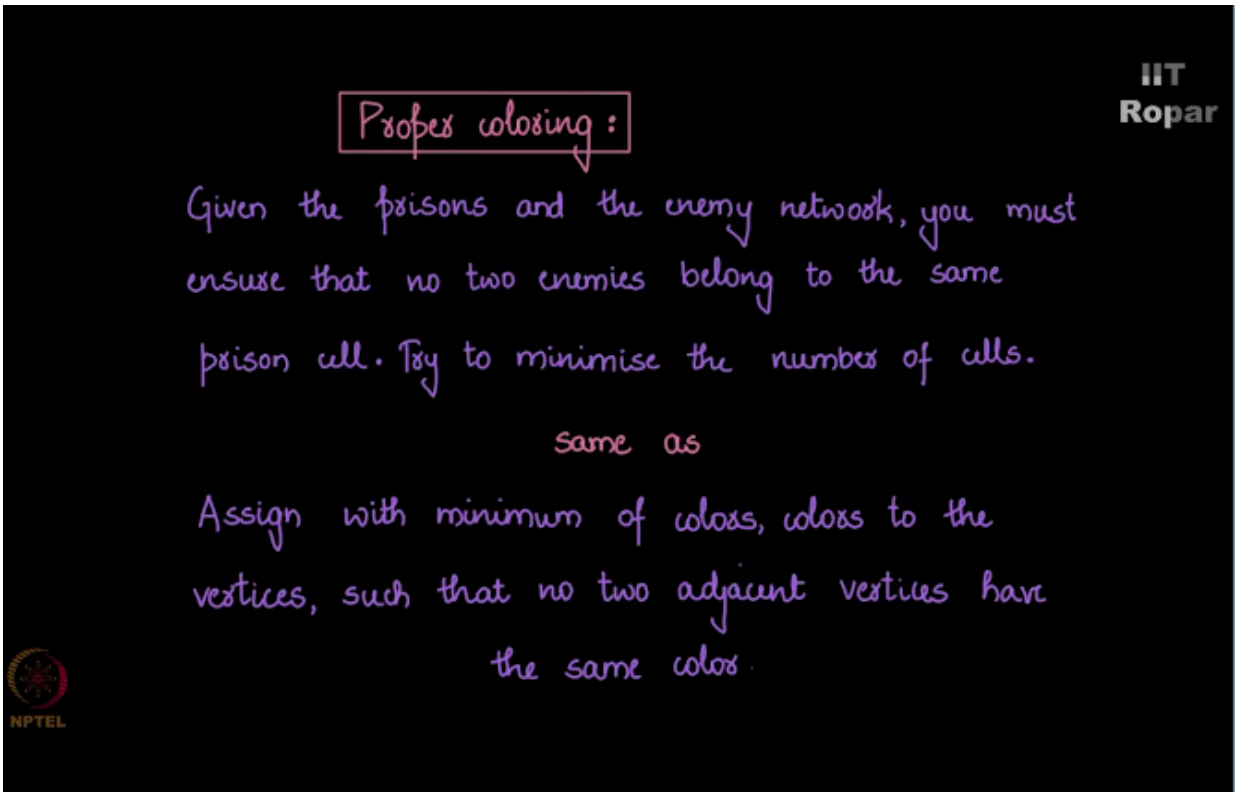

A proper coloring of a graph is simply assignment of colors to vertices such that no 2 adjacent vertices get the same color, but the question here is obviously if you color every single node with different colors it's a proper coloring, but what I want is the minimum number of colors, that minimum number of colors is called the chromatic number, right, we will see this more right now,

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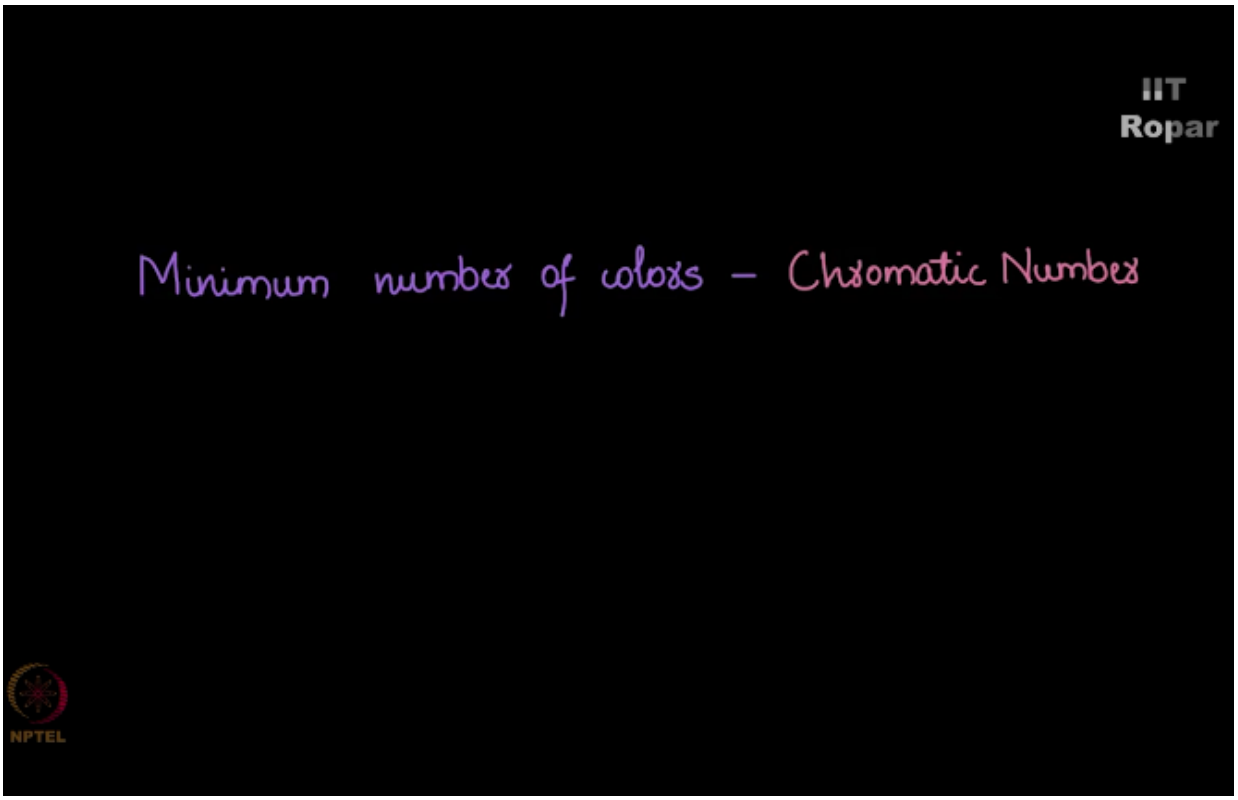

we have already seen it, but I'm just telling you that connection between simply proper coloring and the minimum number of colors required and that is called chromatic number, you saw the example of prisoners, you just saw the raw definition equivalent to that of prisoners and that is proper coloring.