NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

Complete graph on 5 vertices is non-planar - Proof

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Now, let's move to this structure called K5, the complete graph with 5 vertices,
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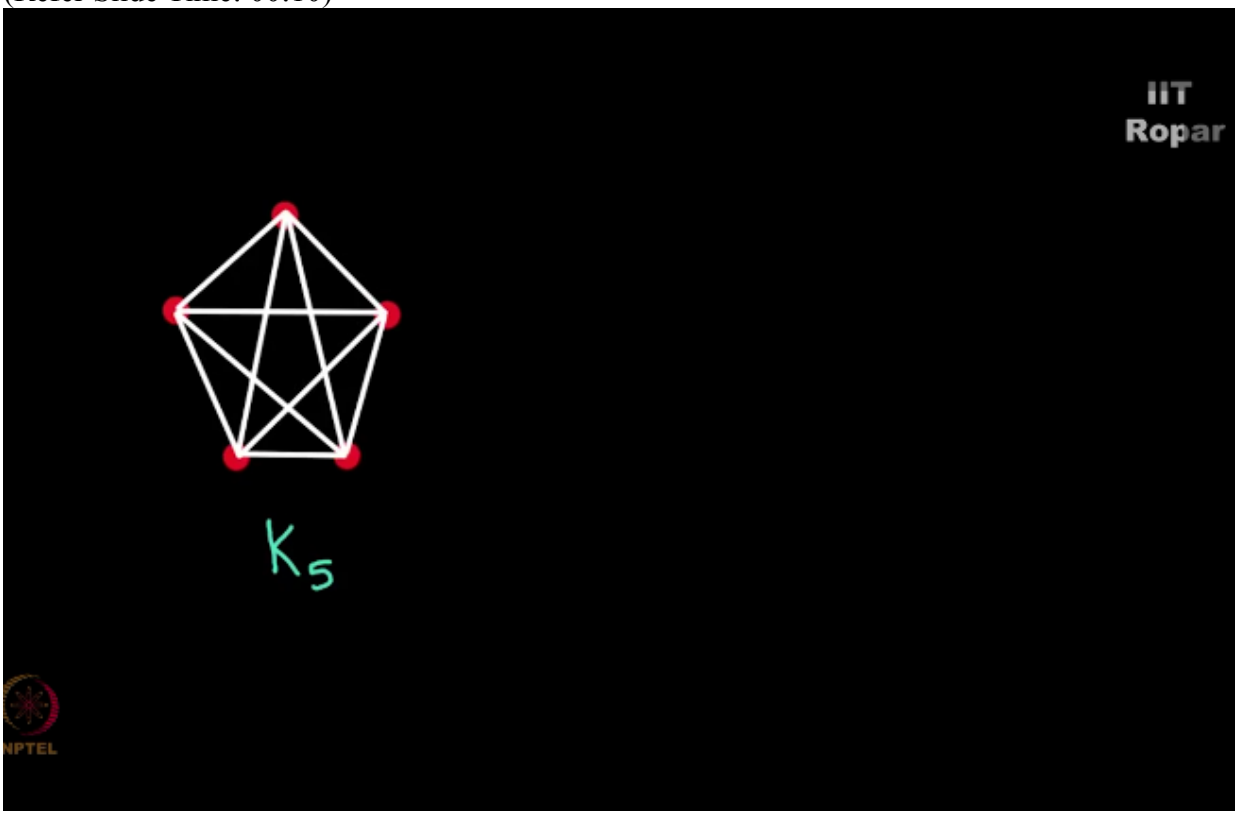

so complete this graph, what is the number of, what is the degree of every vertex? 4, 4, 4, 4, and 4, so the sum total of degree will be 20 and hence the number of edges E will be 10, the number of vertices V will be, how much? 5, what is R?
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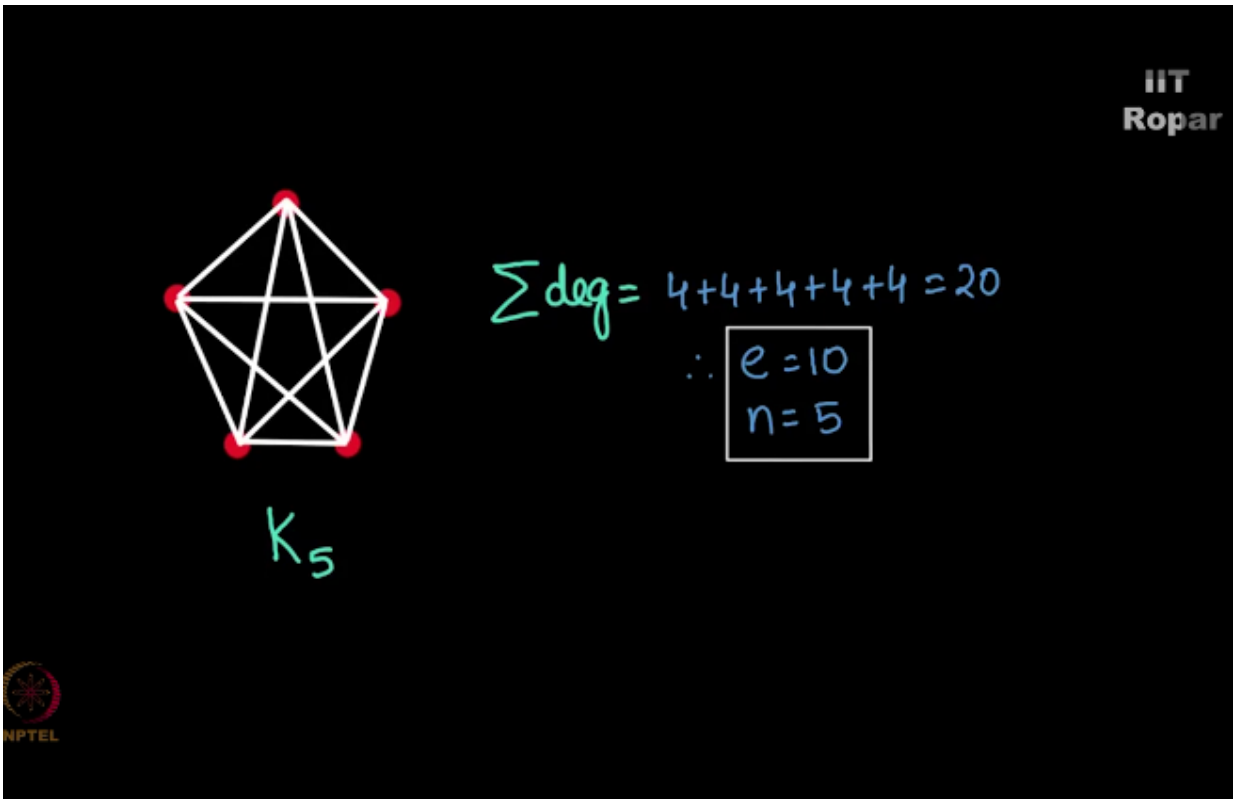

Now our point is K5 is it planar? Let us assume it is planar, okay, if it is planar V – E + R will be equal to 2, which means 5 – 10 + R = 2, at R = as you can see 2 - 5 + 10 which is 7, so R turns out to be 7,
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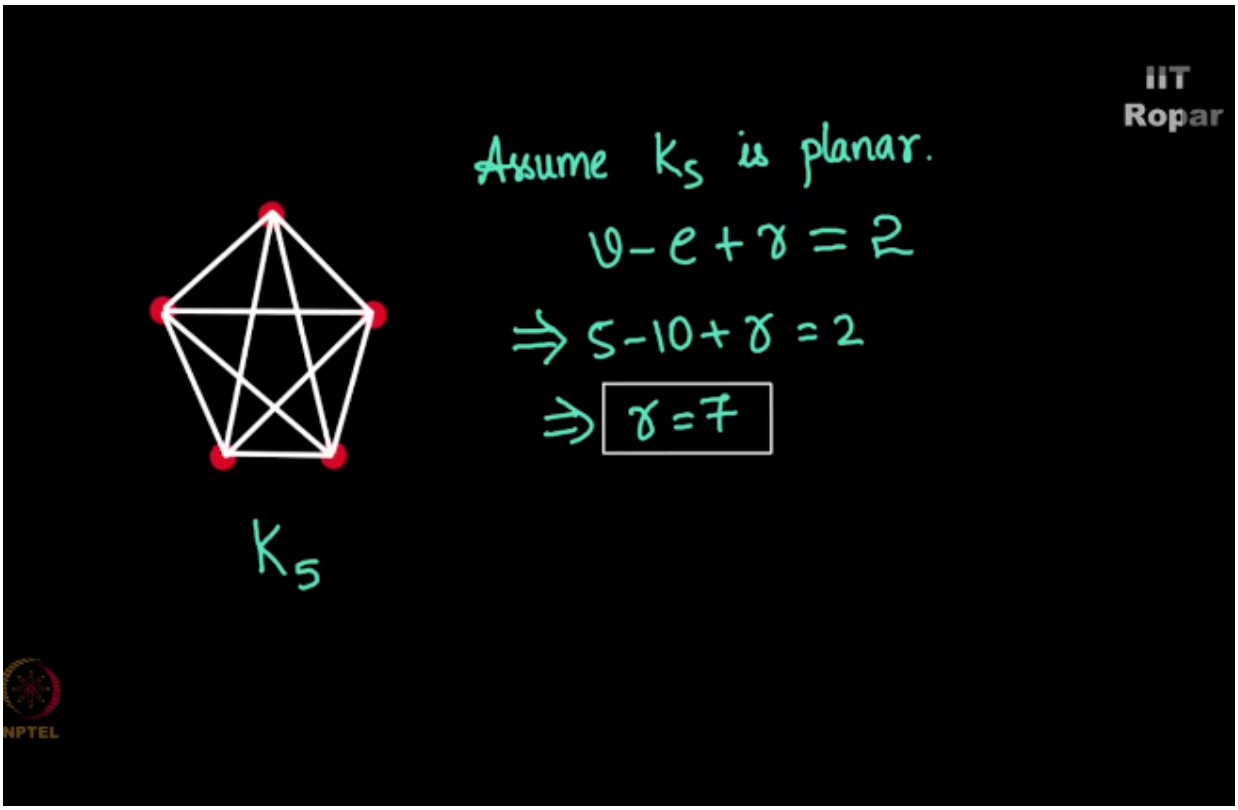

but then if the graph is planar we know 3R is less than or equal to 2E, 3 times what is R? 7, it is less than or equal to 2 times E which is 10, so 21 is less than or equal to 20, again absurd, absurd and hence our assumption that K5 is planar, is not true,

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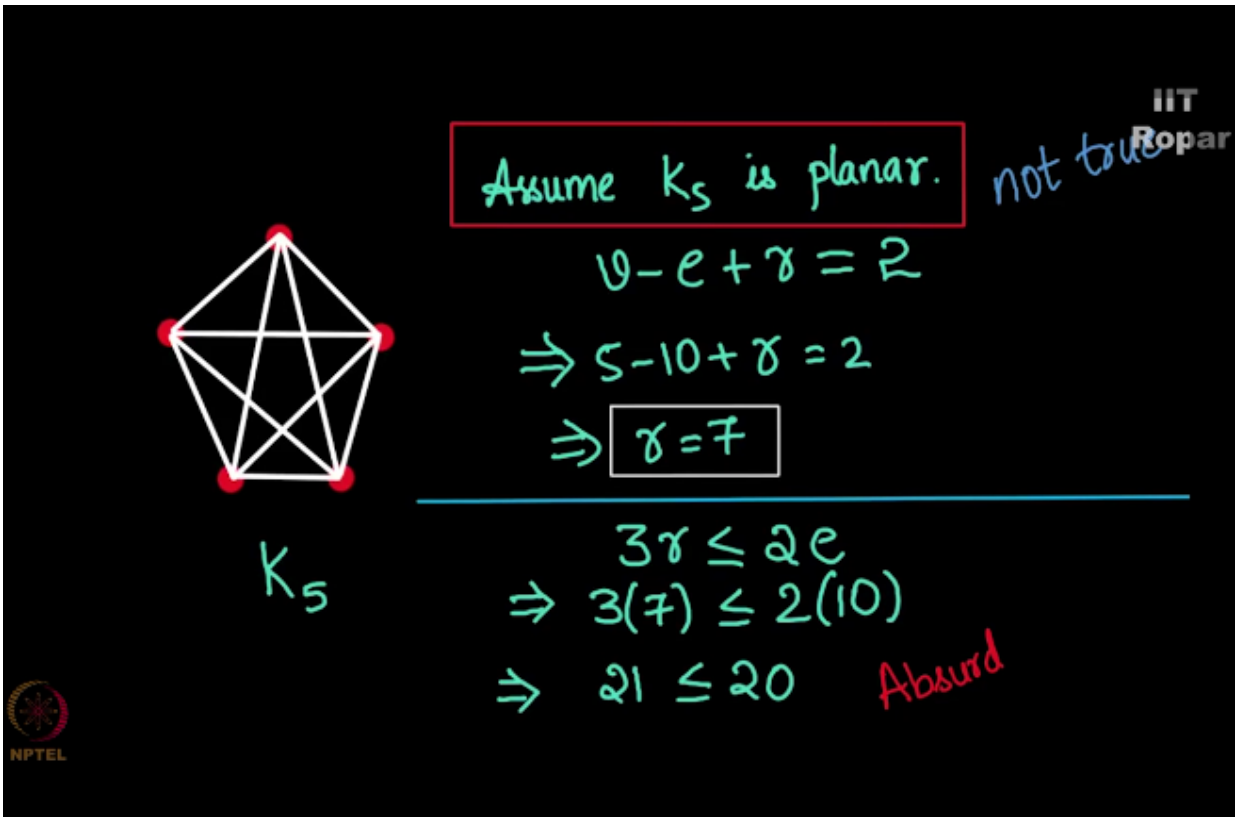

and hence K5 is non-planar, right,

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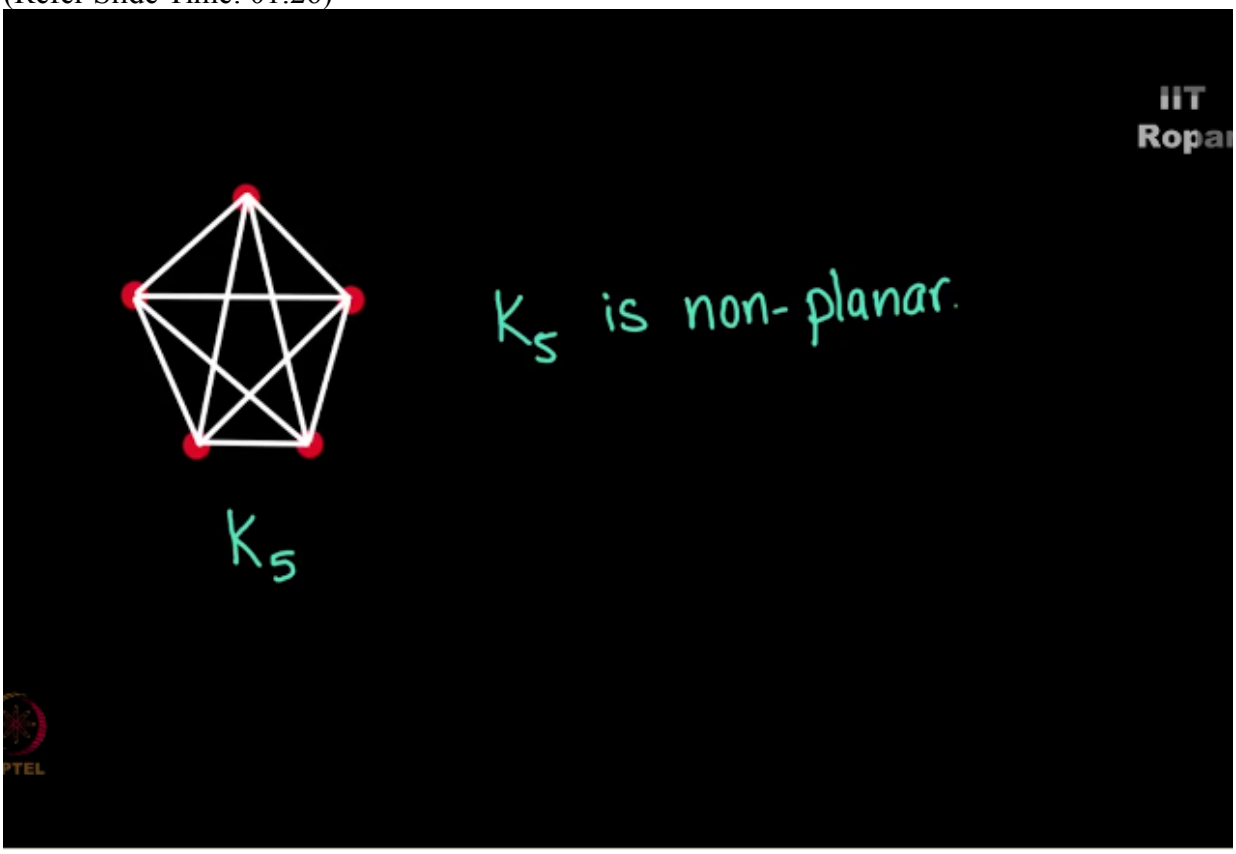

a same old story, the same old elegance that we saw in K3, 3 is found here, I'll not further exaggerate, you see, you start with the statement you are at absurdity and your statement is false, so K5 is non-planar, remember the example, we spoke about 5 friends having their houses and they plan to construct personal roads to every other friends house such that no 2 roads intersect, this is plane impossible, right, we just now proved, this is plane impossible provided, provided you construct flyovers and we are not going to allow that in the example, so if you were to just use roads on the ground, this is plane impossible and you just now saw the proof K5 is non-planar.