NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

3 Utilities problem - Revisited

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Remember the question that we all started our discussions from 3 houses and 3 utilities, and we say it is not possible for us to construct roads from these 3 houses to these 3 utilities such that roads do not cross each other you keep trying, you keep trying, you keep trying, you don't succeed is not the reason, why this doesn't happen,

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if you think this doesn't happen you need a solid proof to say that this particular thing doesn't really happen, how do you do that? Let me first write down what is this graph, it has 3 dots here and 3 dots here, and every dot connects every other dot, correct,

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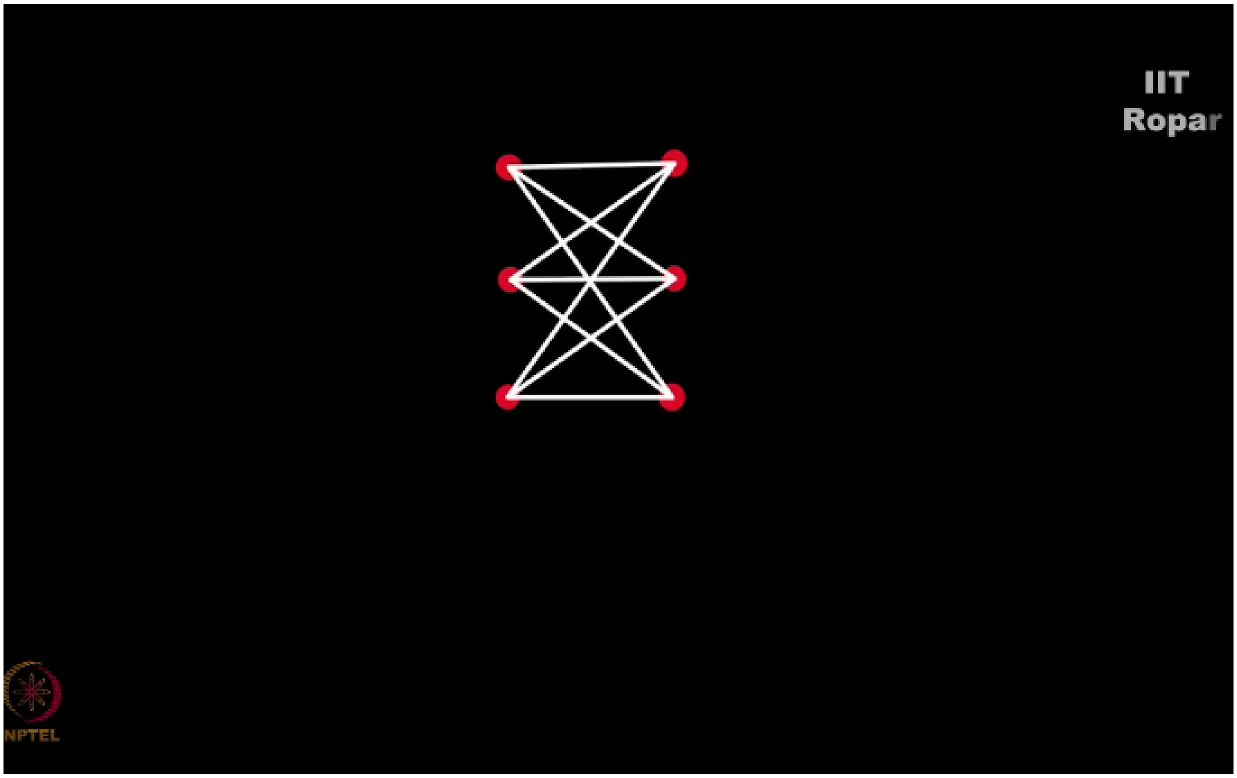

this is the structure and it's a bipartite graph as you know, it's a complete bipartite graph, that's what you mean by K3, 3.

Now look at this we know a small result that a bipartite graph can never have an odd cycle, and in particular can never have a cycle with 3 vertices, a triangle is never visible which means the same old result 3R is less than or equal to 2E might hold good here, if it is planar,
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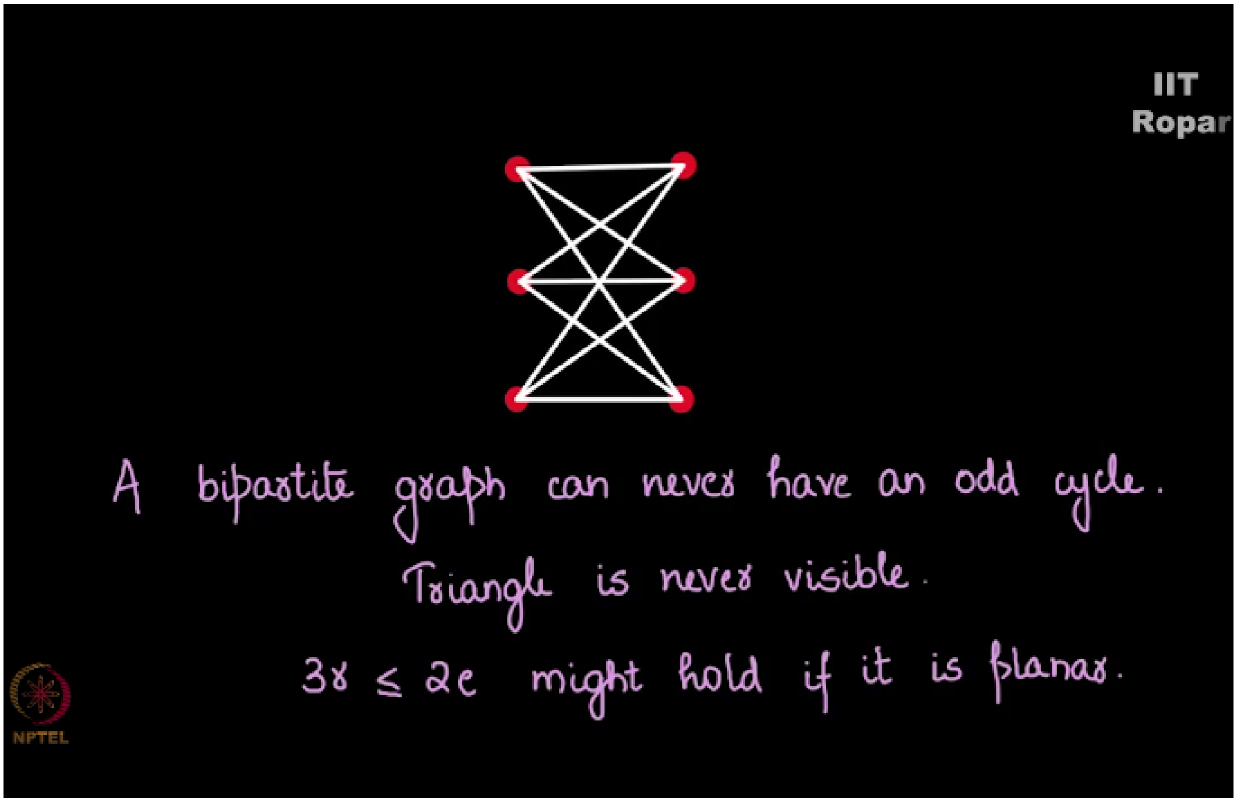
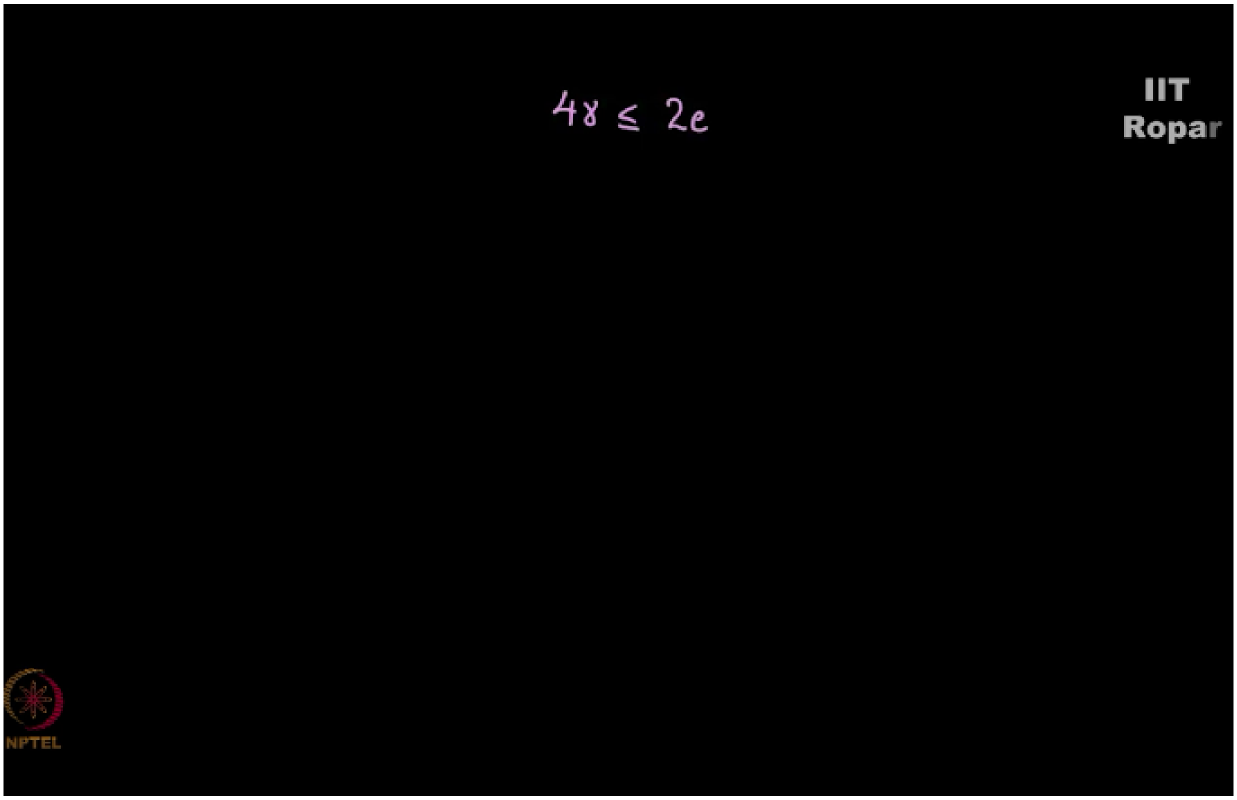


but then 3R you see how we got this in the previous proof, that's because every region was bounded by at least 3 edges, but in this case we are sure that it is not just 3 but 4, which means the entire theorem holds good but we can indeed write this as 4R is less than or equal to 2E, (Refer Slide Time: 01:47)



pause and think if it's not clear why this is true, if you have followed the proof of 3R less than or equal to 2E you will know why given a bipartite graph it is always 4R less than or equal to 2E provided the graph is planar, is the graph planar?

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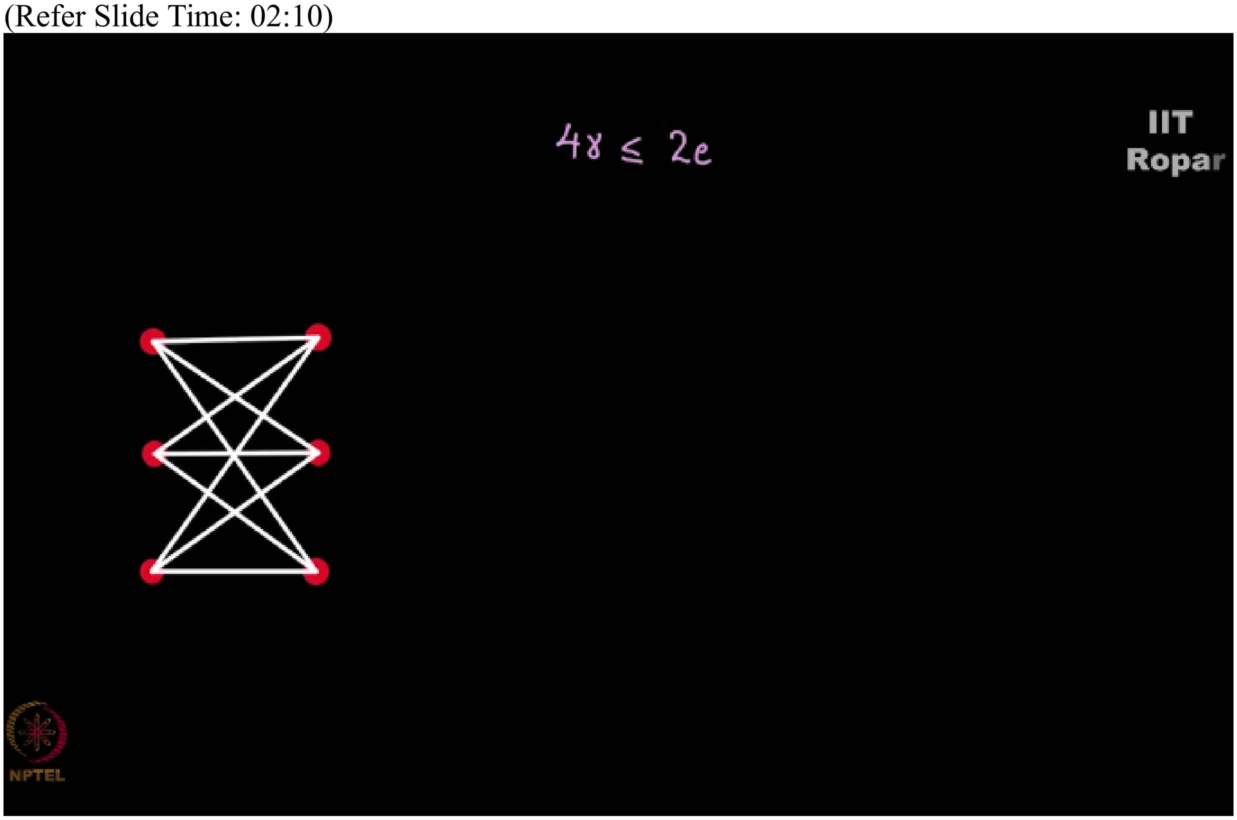

If the graph is planar it implies that 4R is less than or equal to 2E, let us write this down, if the graph is planar, then it implies that 4R is less than or equal to 2E, well so if the graph is planar we know that V - E + R = 2,

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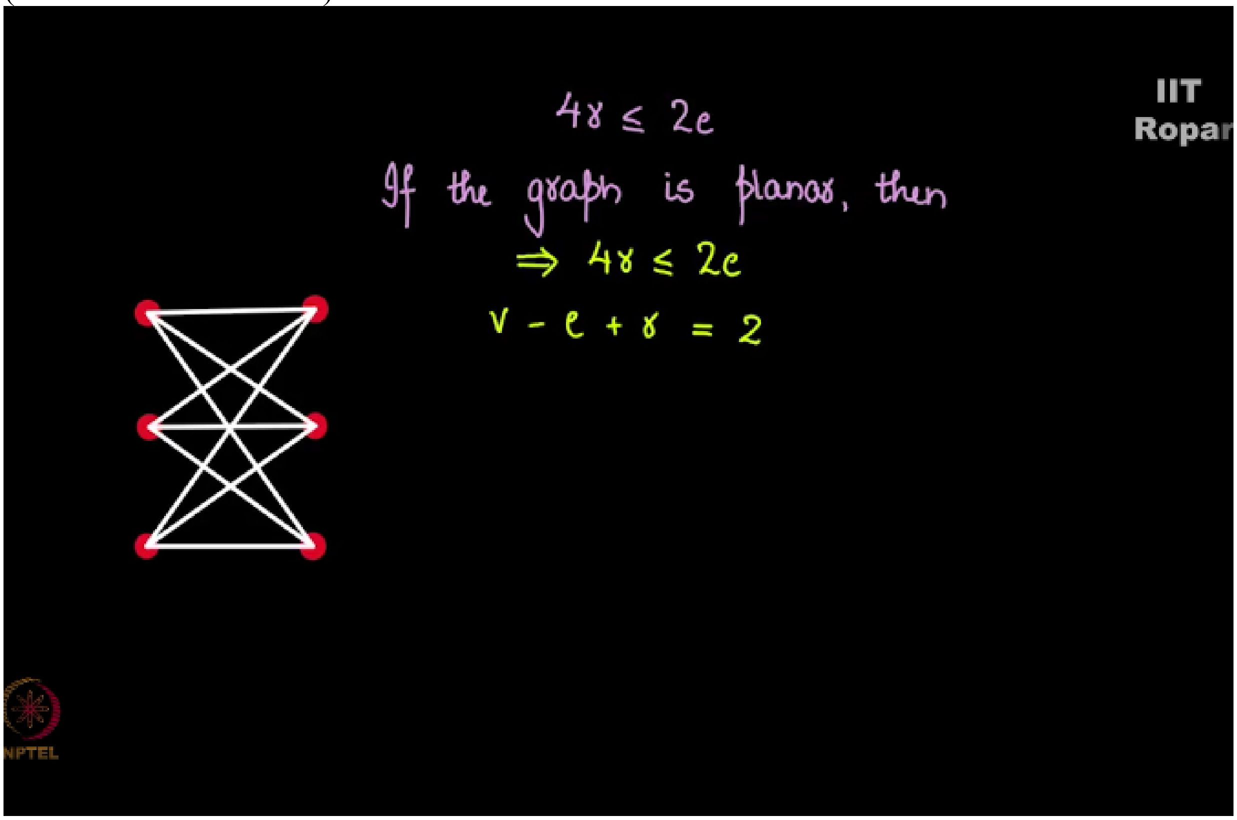

I'm also saying that 4R is less than or equal to 2E, let us club these two things together and see what we get, now R = 2 + E – V, I'm just rewriting V - E + R = 2,

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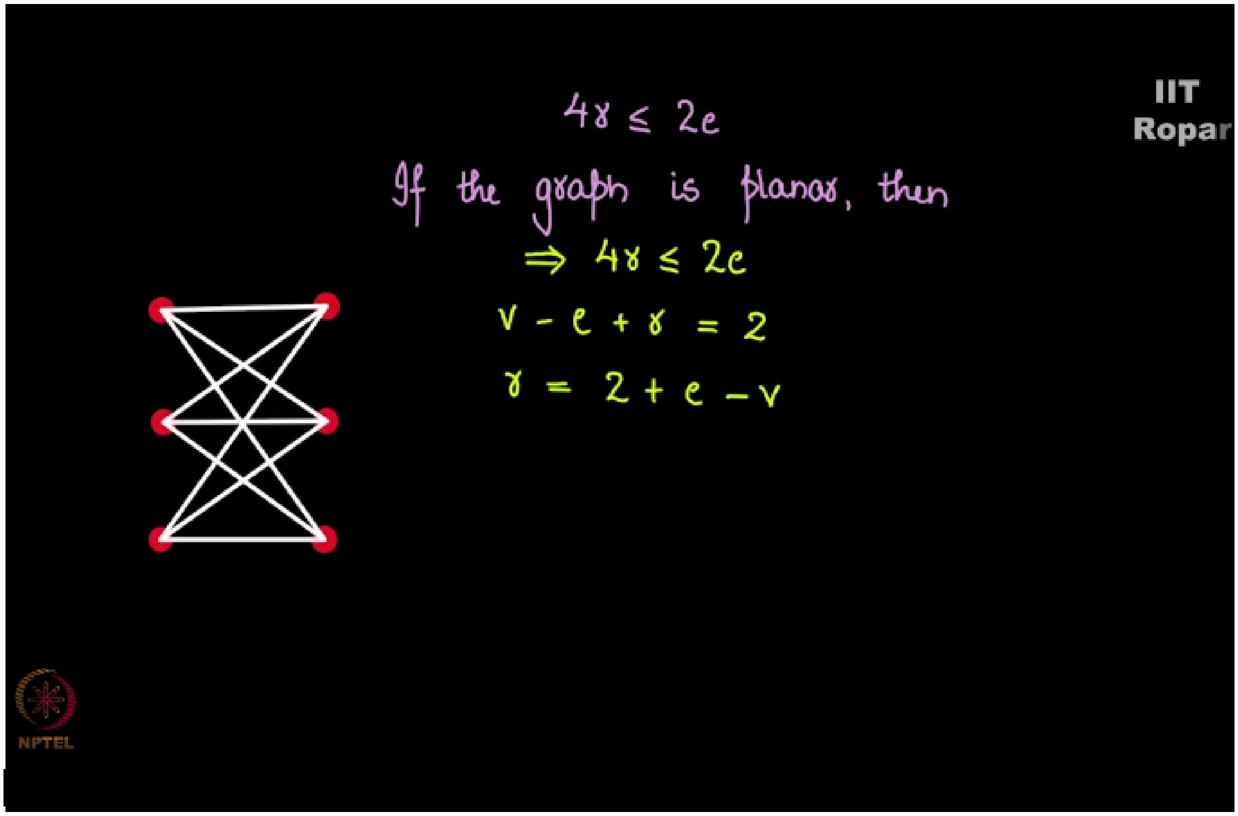

so instead of R we can write 4 times 2 + E – V, this has to be less than or equal to 2E,

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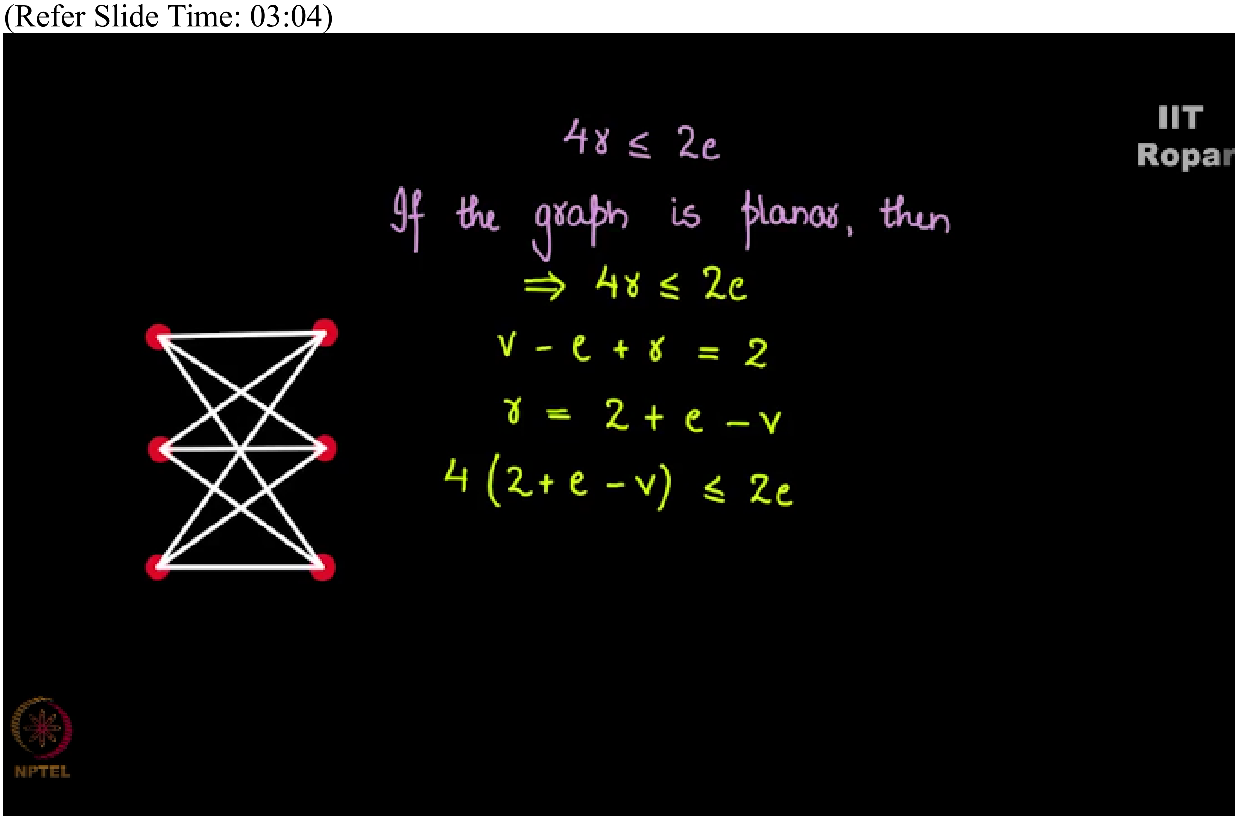

let's see if this is true, 4 times, so what I did was I just got rid of R because I don't know what is R in a K3, 3, but I know what is V and E, correct, that's the trick, if you didn't follow don't very much, the point is we have to somehow show that if the graph is planar then it implies something, it implies something, and finally a contradiction, right, so 2 + the number of edges here is let us count 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 18/2, the sum of degree is 18/2 is the number of edges so you can even count the number of edges, manually the number of edges happen to be 9, the number of vertices happened to be 6,

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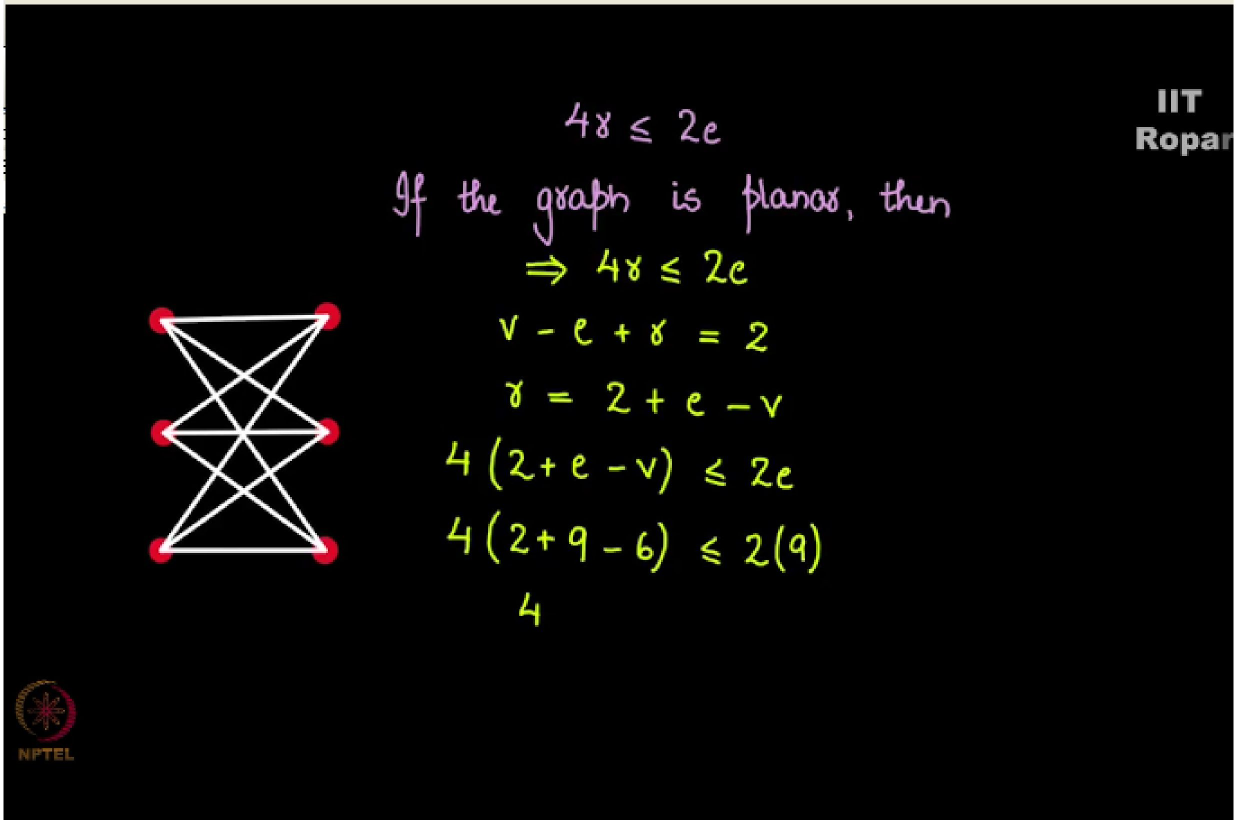

so you see what you're getting here, whatever is the number less than or equal to 2 times the number of edges which is again 9, is this true? Let's see, 4 times 11 - 6 is 5, and this happens to be 20 is less than or equal to 18,

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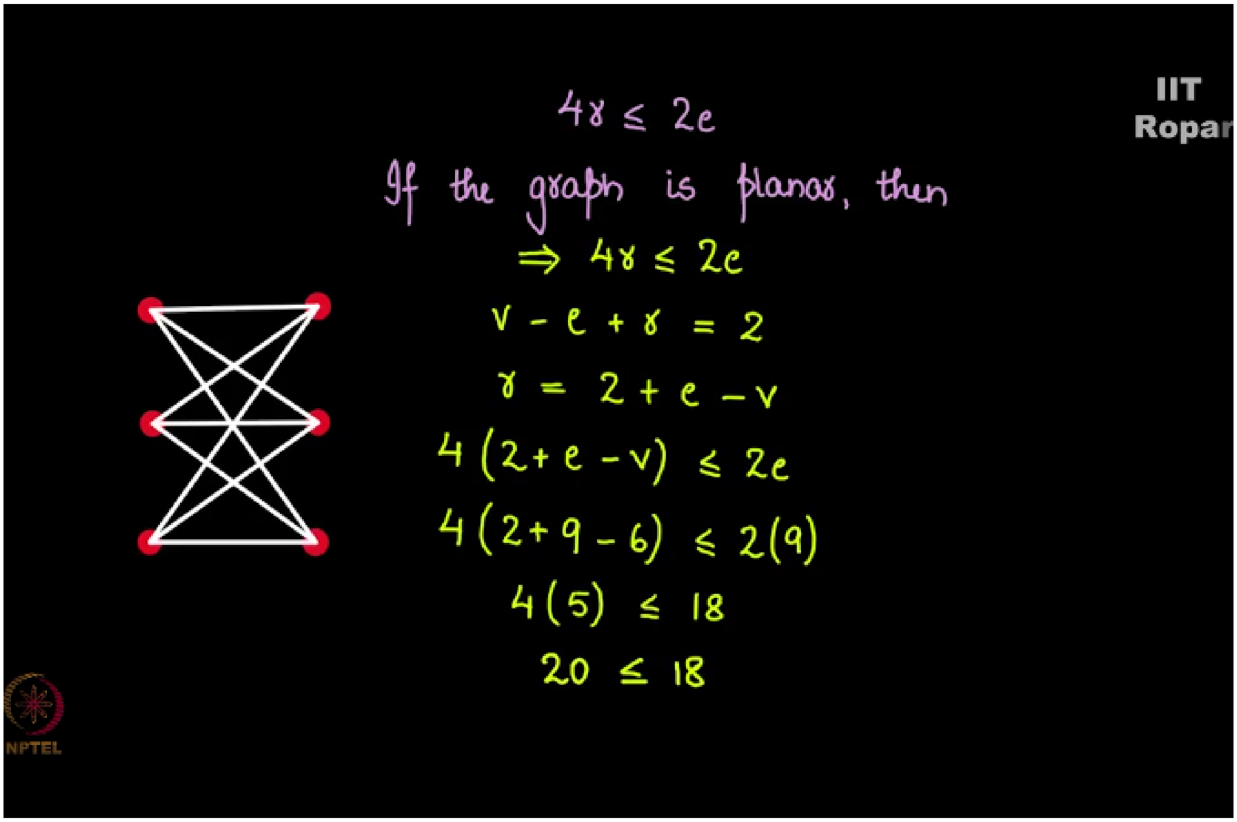
I mean what on earth



is this where in the universe is this inequality true, do you think 20 is less than or equal to 18, it's absurd let's write that down it is ABSURD, absurd, what is absurd?

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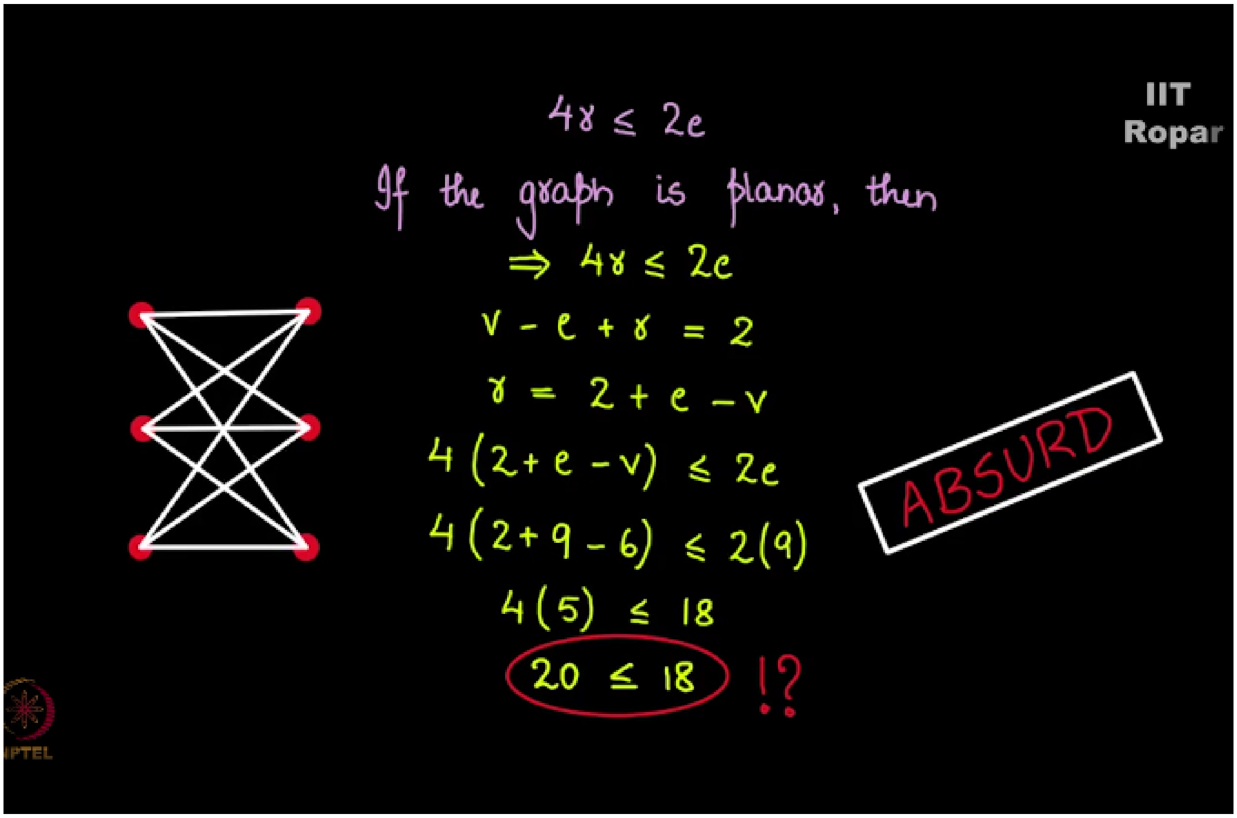

The fact that 20 is less than or equal to 18, so what? So what? You trace back and see how you started, you started with the fact that the graph is planar and that implies absurdity which means the fact that the graph is planar is not in fact true which means the graph is not planar, which means K3, 3 is non-planar which means you cannot in the lifetime of this universe try to construct 3 houses to 3 utilities all possible roads such that roads do not intersect each other don't even try to do it you will not succeed, remember the story sounds familiar in Konigsberg bridge we told you that don't even try to traverse this path, you will not succeed, and we gave you a mathematical proof.

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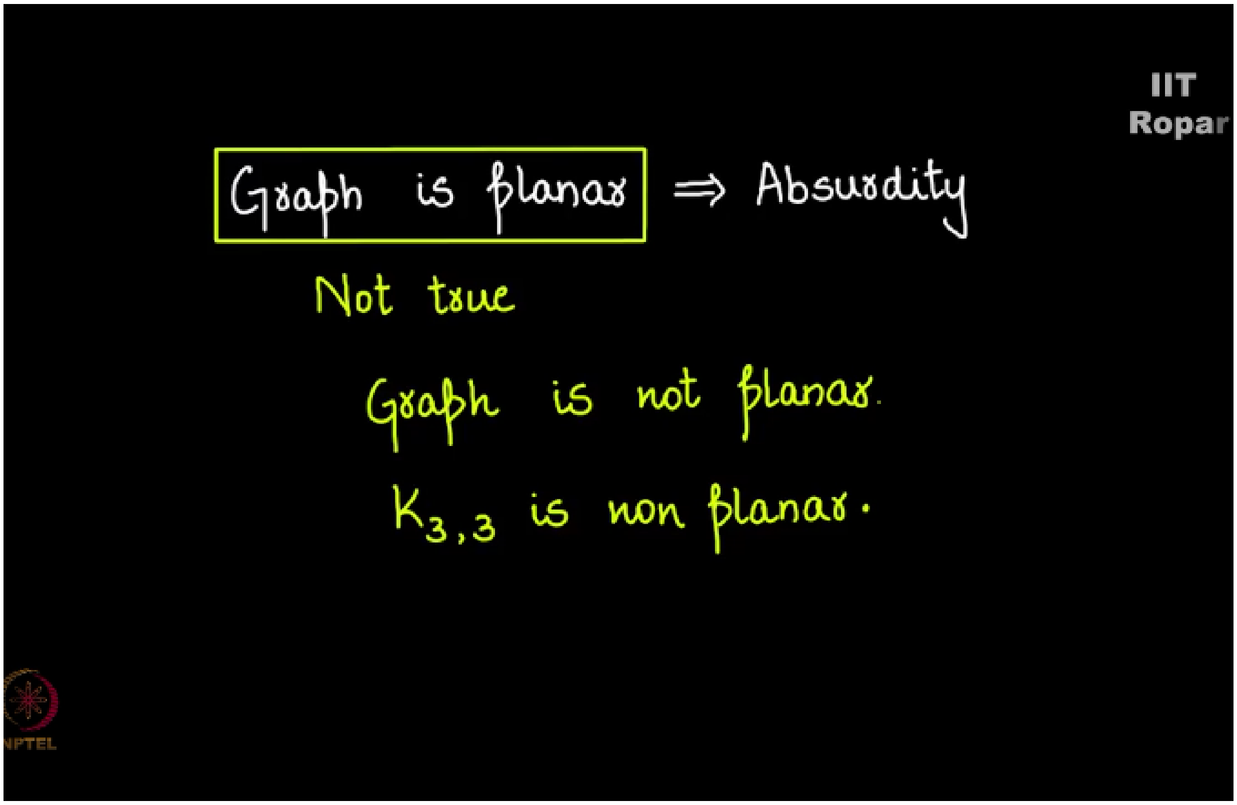

Just summarize look at the beauty of this result you don't have to keep trying, keep trying, keep trying, and then go breathless and say this is not possible, all you got to do is analyze mathematically you will get some absurdity and hence the graph is not planar.

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