NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

Planar graphs - Inequality 1

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Given a graph being planar, we have observed that V - E + R is always equal to 2,
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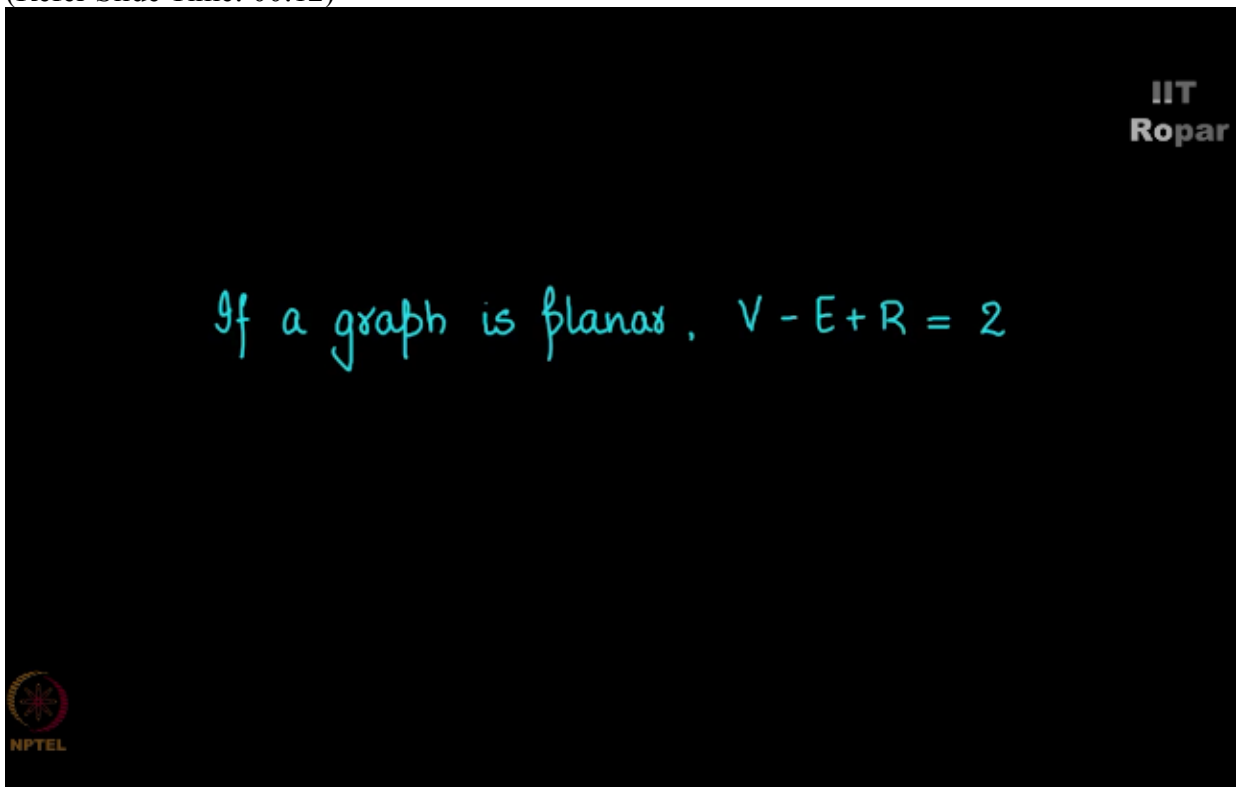

so this is something like a regularity, that the physics of it is sort of showing us that something is very regular here, no matter what kind of a graph you can take in the world there are infinitely many of course, if it is planar you observe that V - E + R will always be equal to 2.

Now let us look at this inequality, 3R is less than or equal to 2E, I say this is true always, is it true? Let's verify, look at this graph here with over you count, you will observe that there are 12 vertices, 14 edges, and 4 regions, V - E + R is indeed true here, 12 - 14 + 4 = 2,
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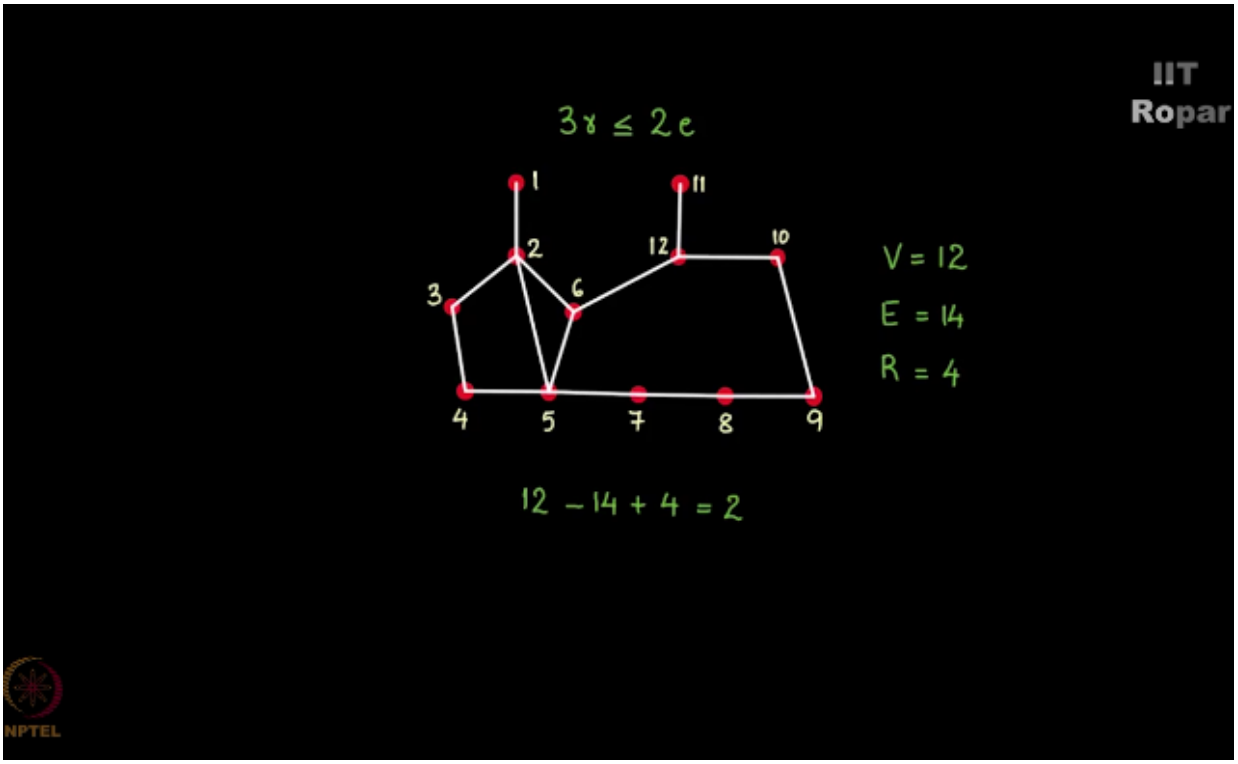

but then do you think 3R is being less than or equal to 2E here, what is this? I say this holds good in any planar graph, R here is 4, 4 times 3 is 12, and 2 times 14 is 28, and hence 12 is definitely less than or equal to 28 and this holds good for any planar graph,

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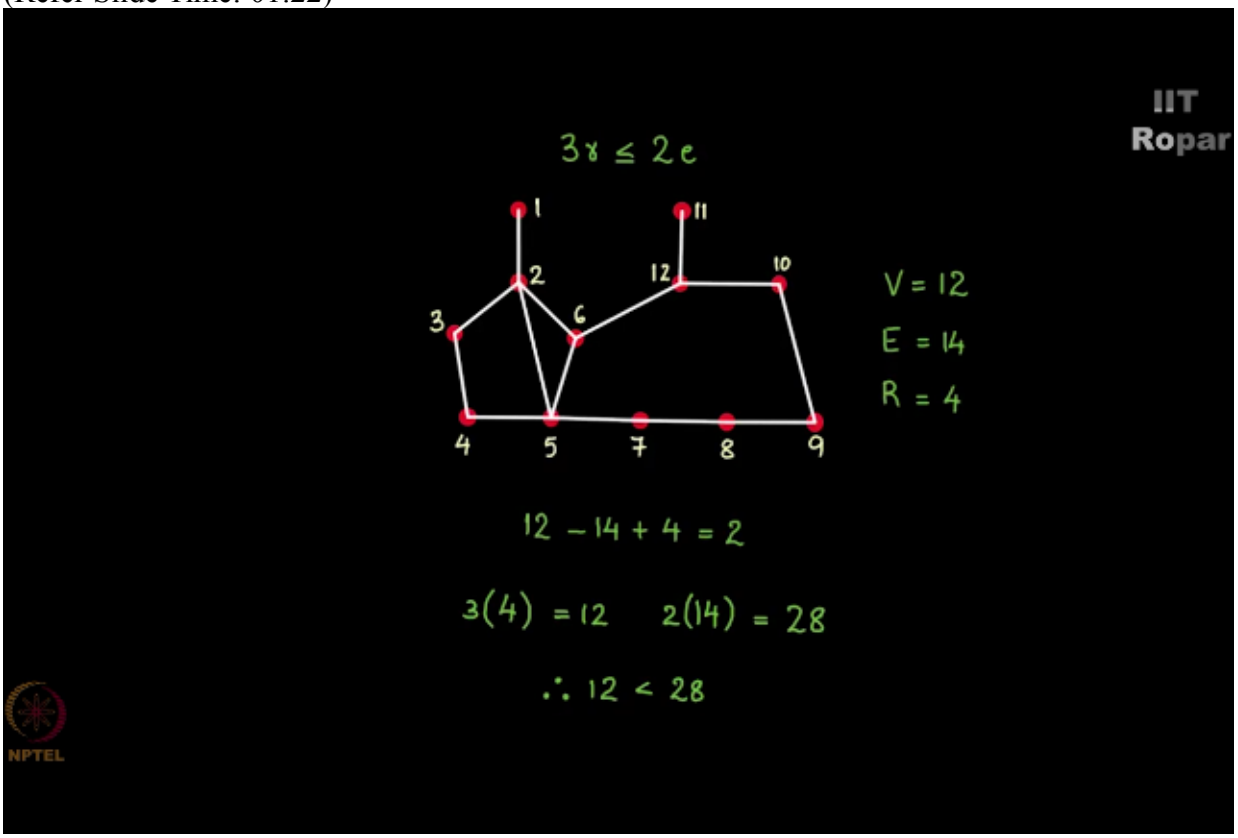

why is this happening? This is a very important inequality, it will help you solve something that we have been talking from some time, okay, I'll reveal the climax soon but in its own right we'll try to solve this problem, so how is 3R less than or equal to 2E here? Look at this, look at the first region,
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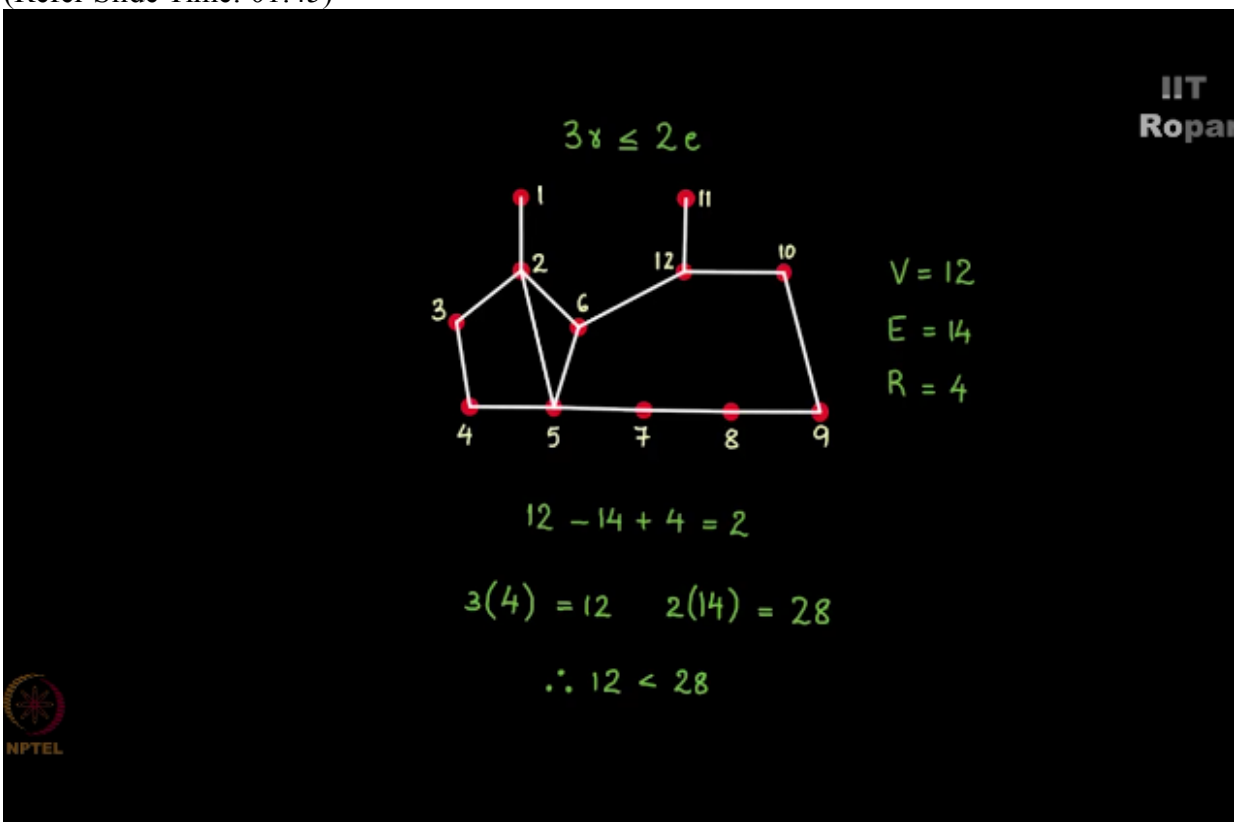

first region has at least 4 edges, second region has at least 3 edges, let me write that down R1 has 4 edges, R2 has 3 edges, R3 has 7 edges, and R4 let me count 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, has 12 edges,
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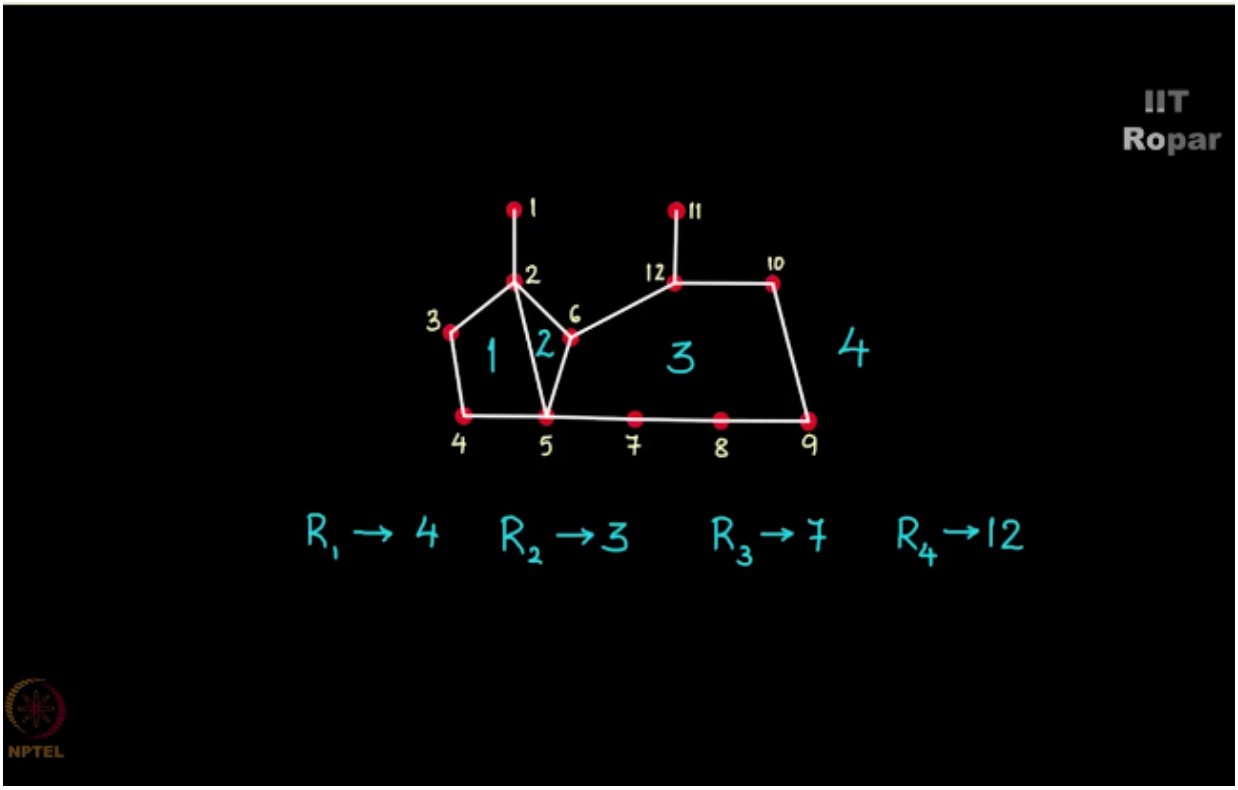

now do you see each region has at least 3 edges, so the total number of regions if it is R, you will have at least 3R number of edges that come in this regions, so let me say 3R is at least is less than or equal to the cumulative sum of all the edges that you write down corresponding to the regions, so 3R is less than or equal to the sum total here, what is that 4 + 3 + 7 + 12, correct, you add all of them, what is that 14, 15,16, 26 correct, the total is 26,

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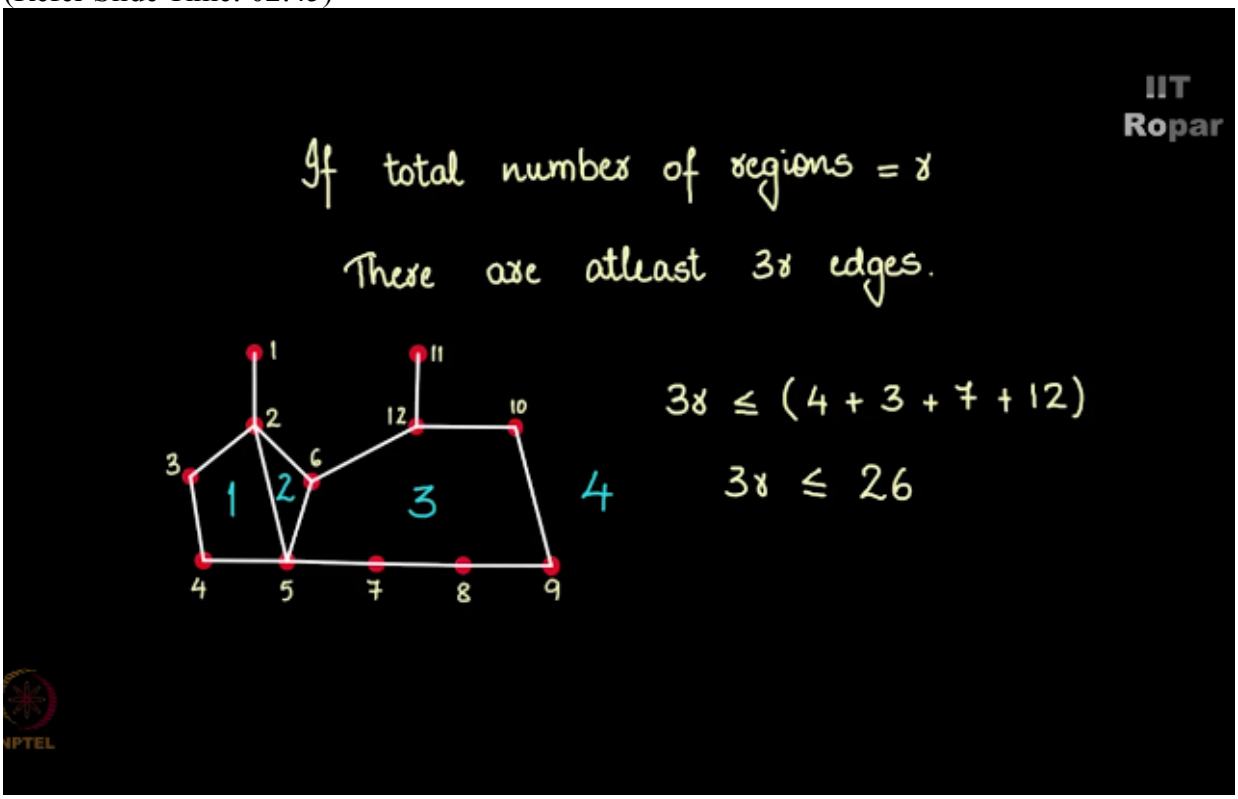

so that is my cumulative sum, so 3R is less than equal to 26 which is my sum of all the edges that appear as part of a region, but then there's something when I observe here every edge comes at most across 2 regions, look at this edge here 2, 5,
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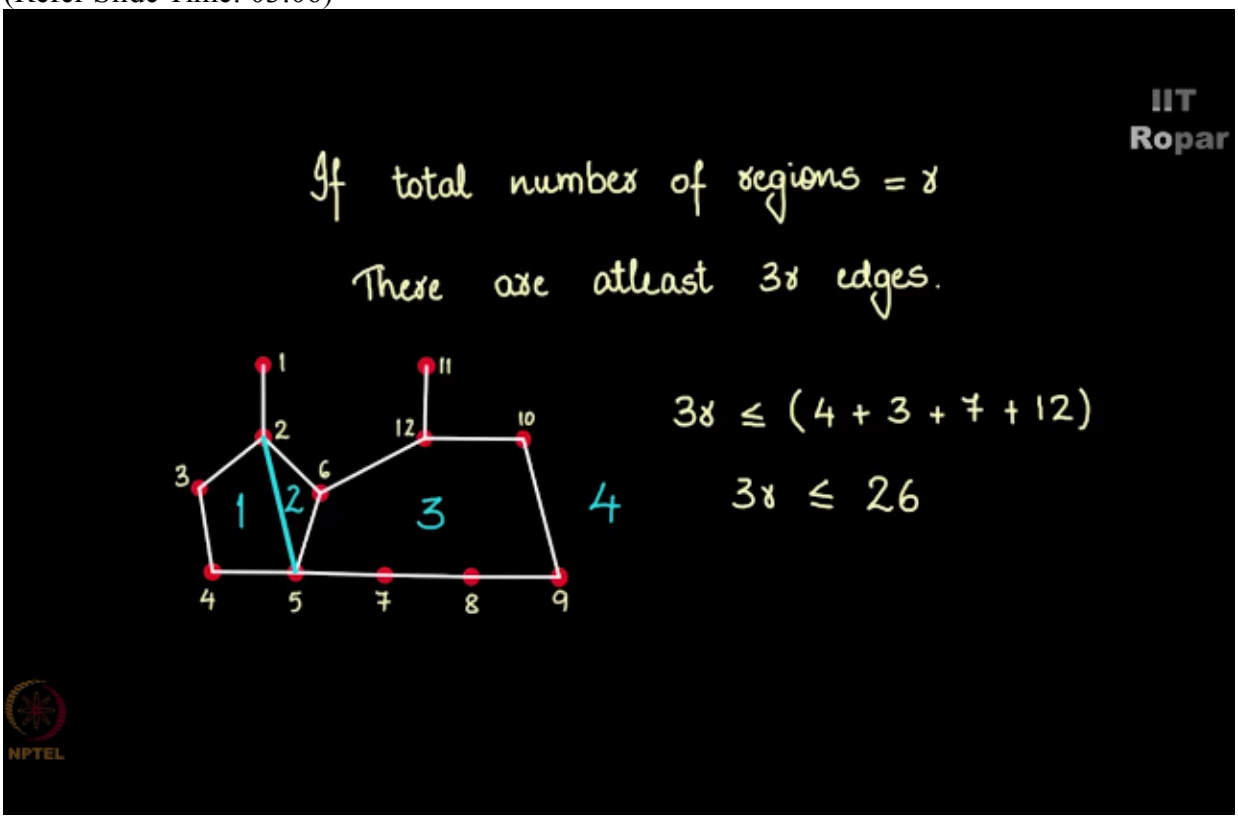

it is an edge that is counted both in region 1 and region 2, so every edge is counted at most twice, why do I say at most twice? Look at this edge 1, 2 or 11,12,
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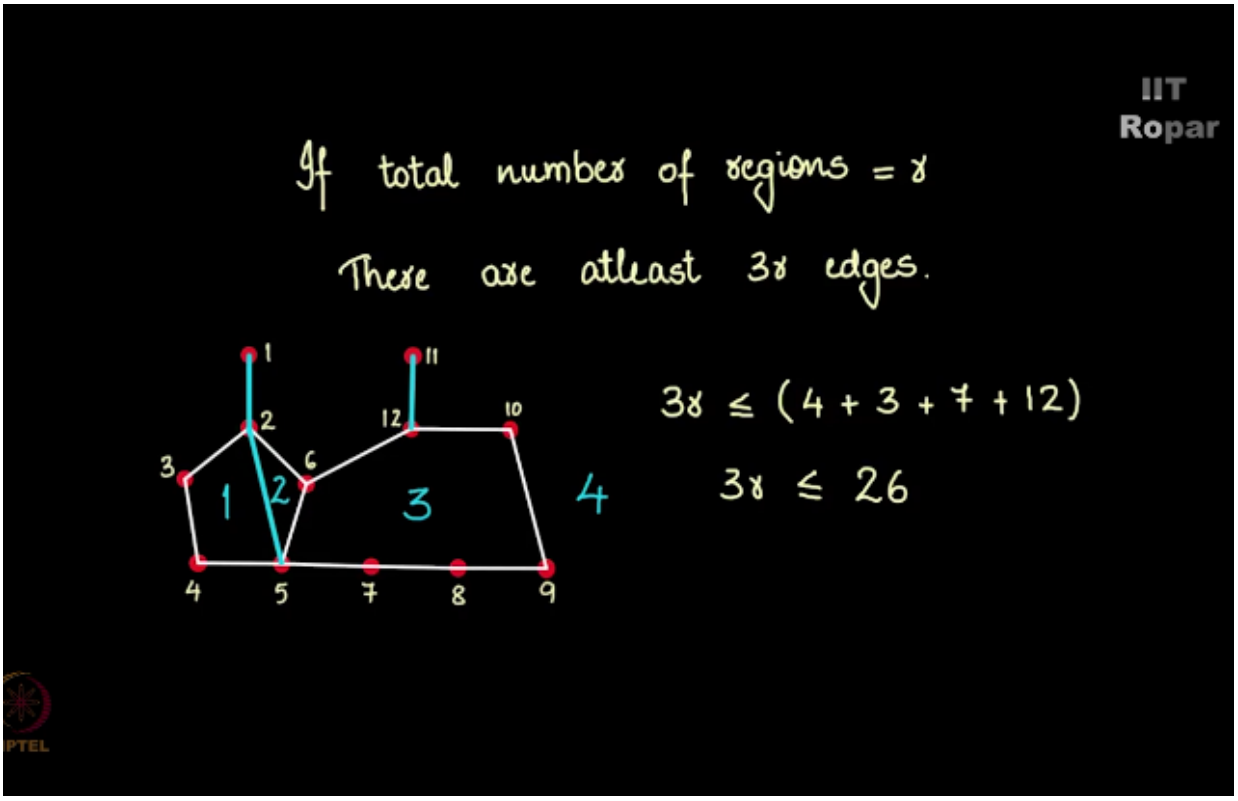

it is counted only once in region 4 so it's counted at least once and at most twice, so my cumulative sum this 26 is definitely less than or equal to 2 times the edges, an edge at most is counted twice and not more, so this should be less than or equal to 2E and hence 3R is less than or equal to 2E,

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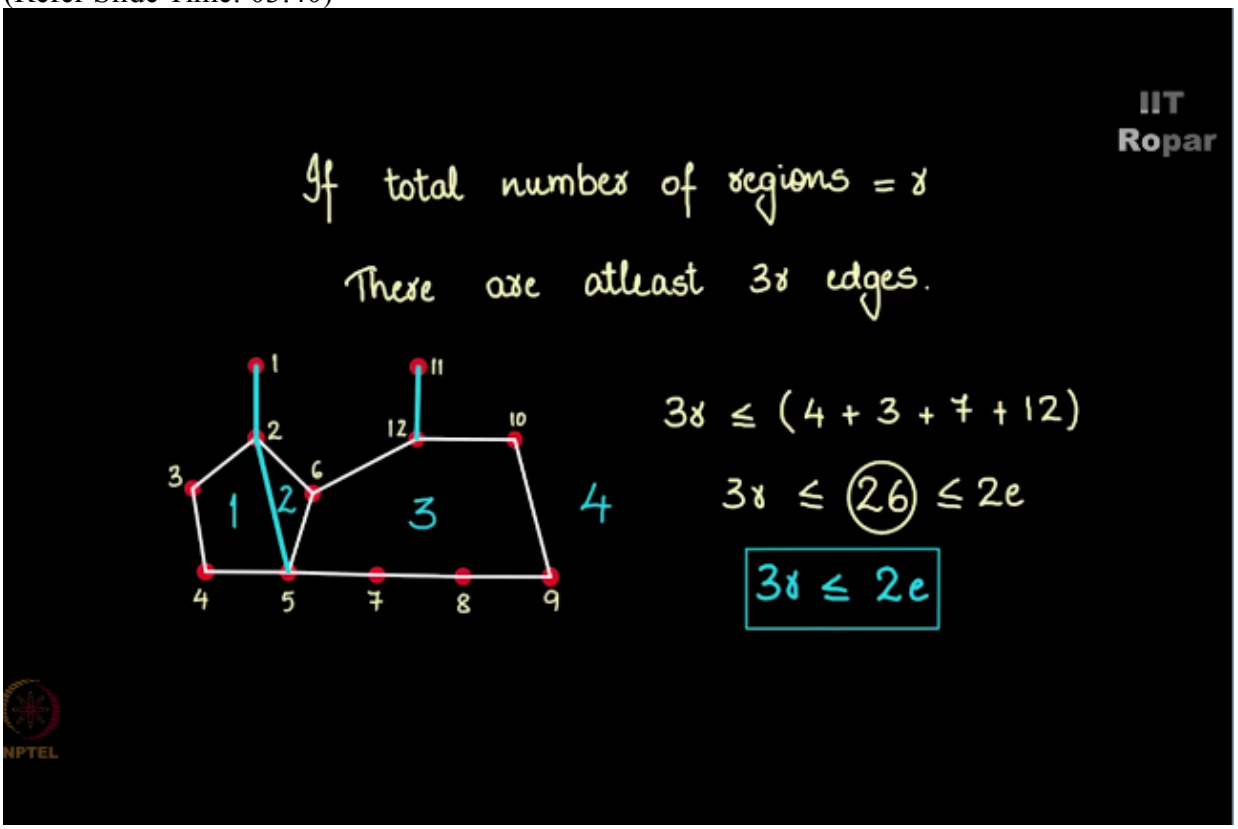

a very important result let us keep it aside and meanwhile I am going to challenge you people with this question, can you tell me how can you prove this 3R less than or equal to 2e using induction, just the way we solved that V – E + R = 2 using induction, can you show that 3R is less than or equal to 2E using induction.

Let us look at another inequality now, I also show you that E is always less than or equal to 3V – 6, it sounds like crazy you see I'm just going on giving you some inequalities you probably are wondering why are we even doing this, yes even I would wonder the same if I were in your place, but you will see some magic happen very soon, E is less than or equal to 3V - 6 always is that true? Let us try this in our graph here,

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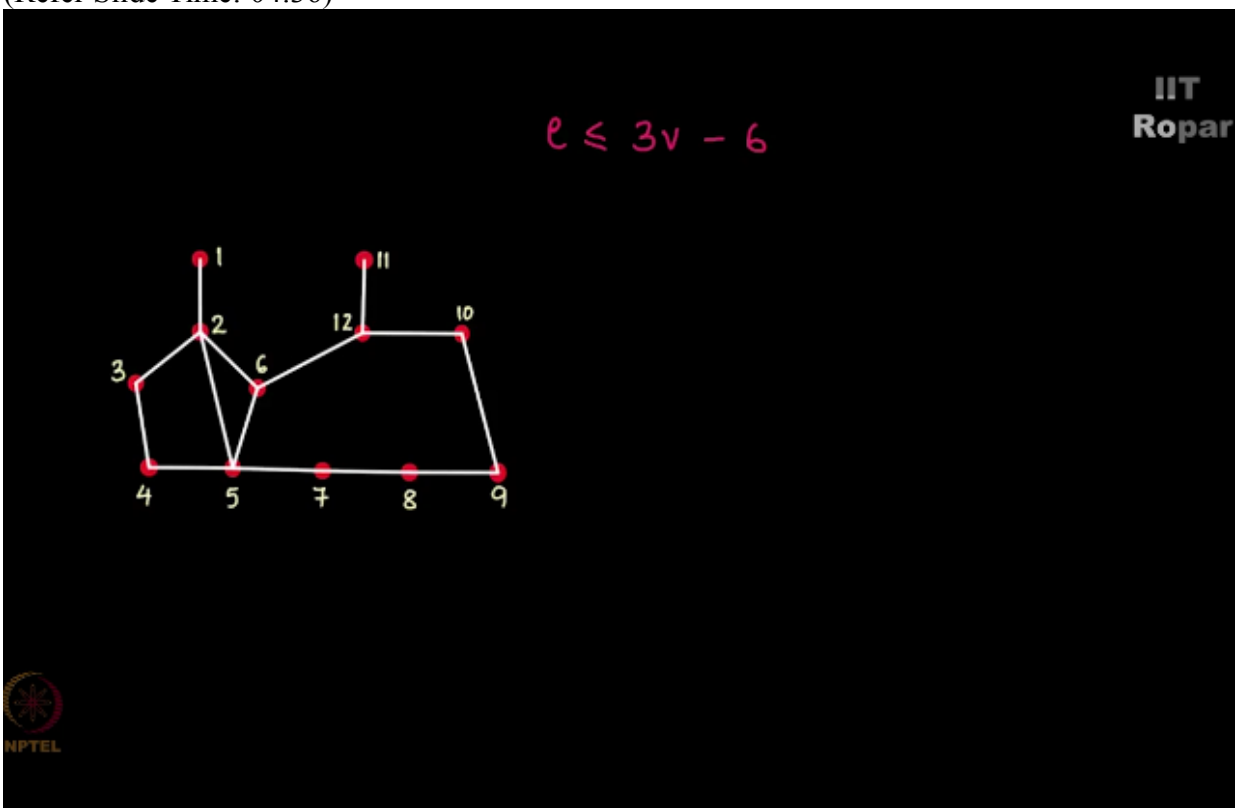

E is 14, so 14 E is less than or equal to 3 times 12, that is 36 - 6 which is 30, of course yes 14 E is less than or equal to 30 which means this inequality seems to be true at least in this case,

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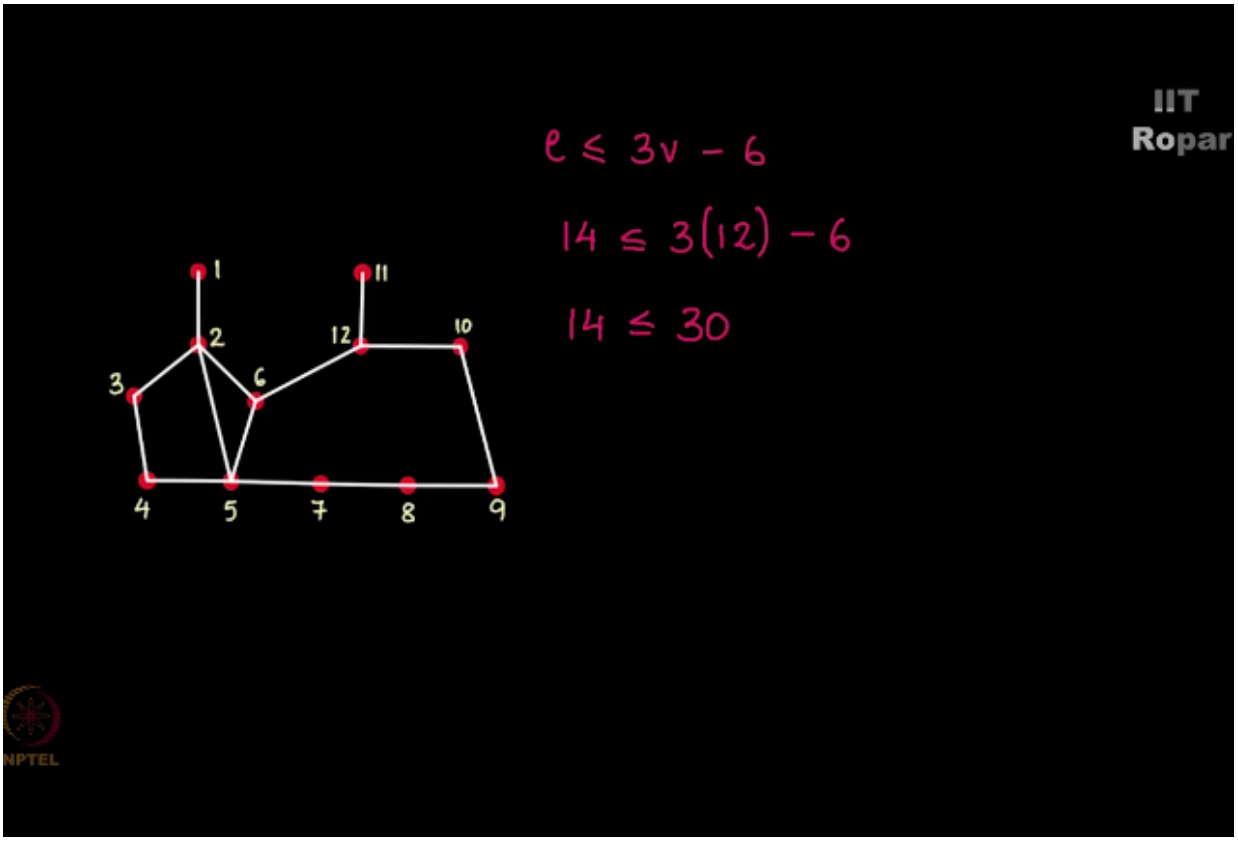

why is it true? Let's observe, look at this 2 = V - E + R, that's right, correct,

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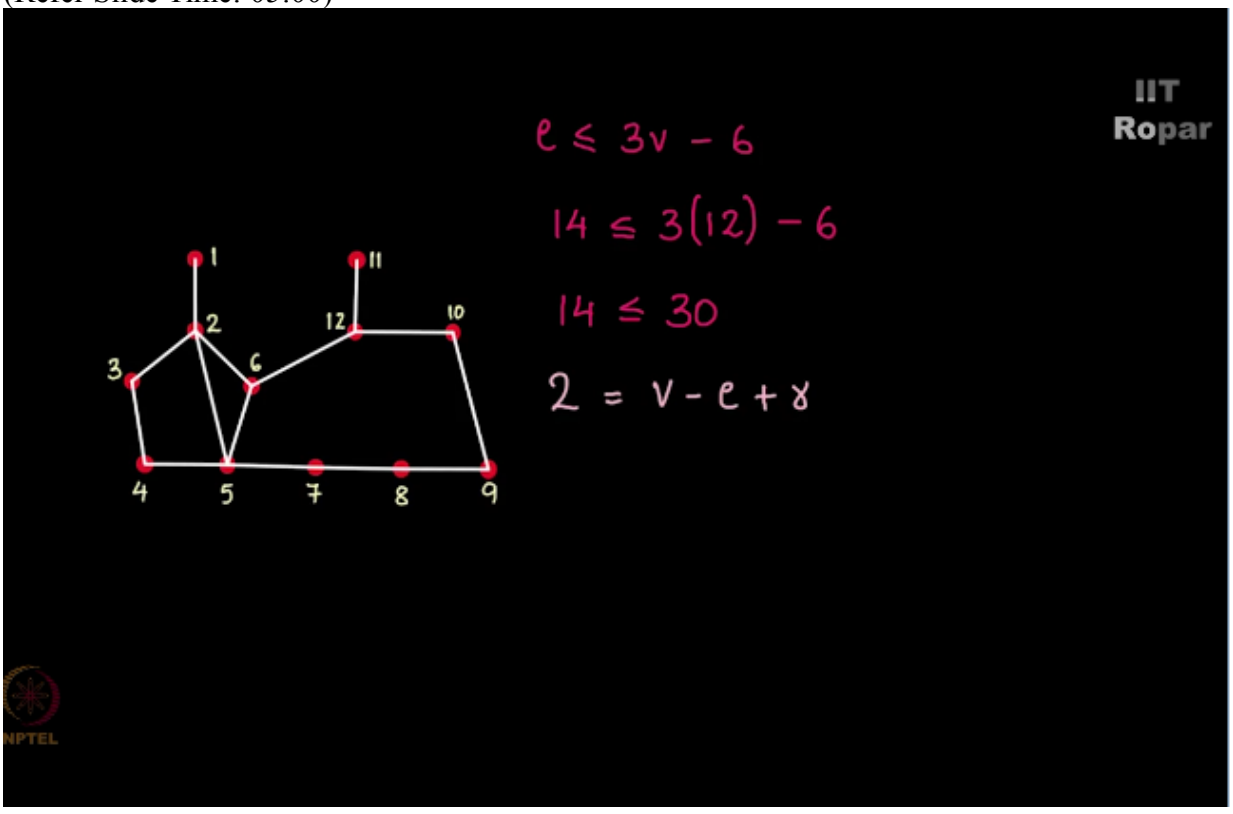

but then R is less than or equal to 2E/3, why? Because we saw that 3R is less than or equal to 2E, we prove it just now, so 2 = V - E + R which is less than or equal to V - E + 2E/3, because R is less than or equal to 2E/3, and then the next step I get 2 is less than or equal to V – E/3,

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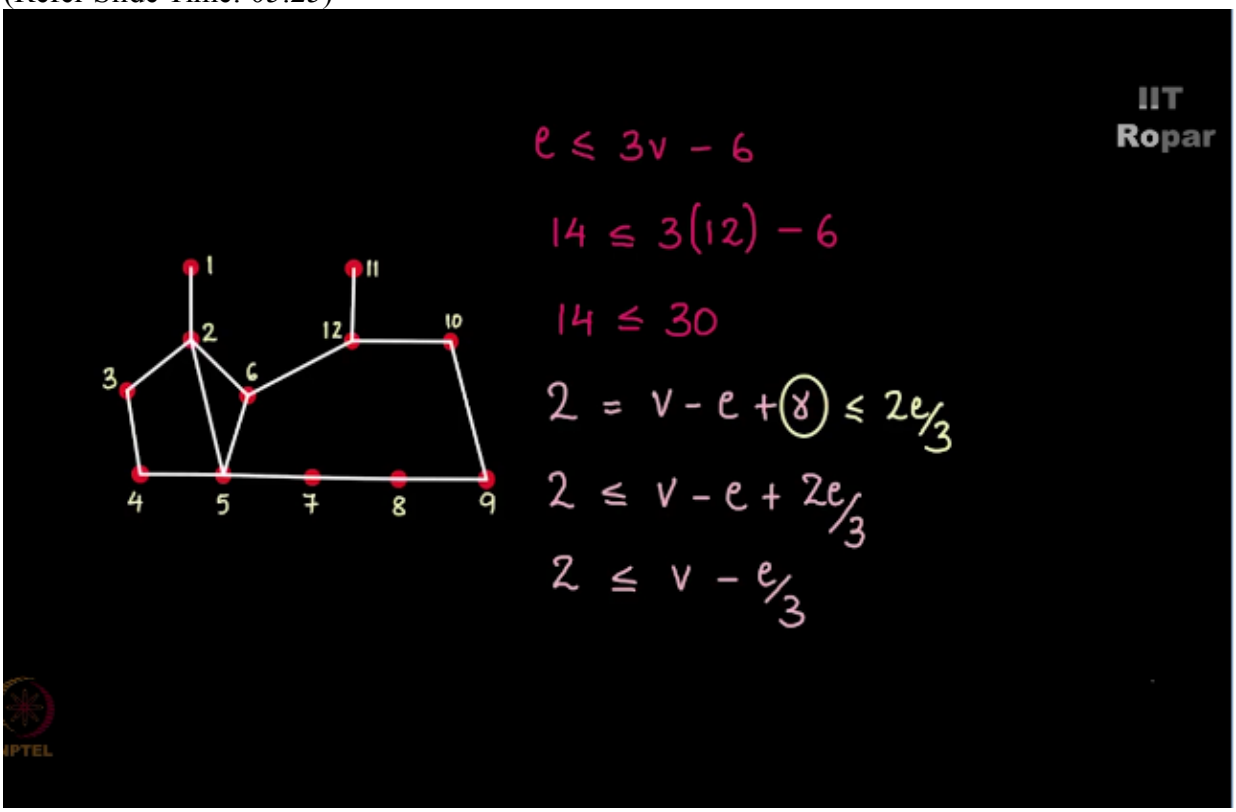

a little bit of jugglery here and there gives me 6 is less than or equal to 3V – E multiplying 3 throughout and then putting E on the left side and putting 6 on the right side you get the answer which is E is less than or equal to 3V – 6.

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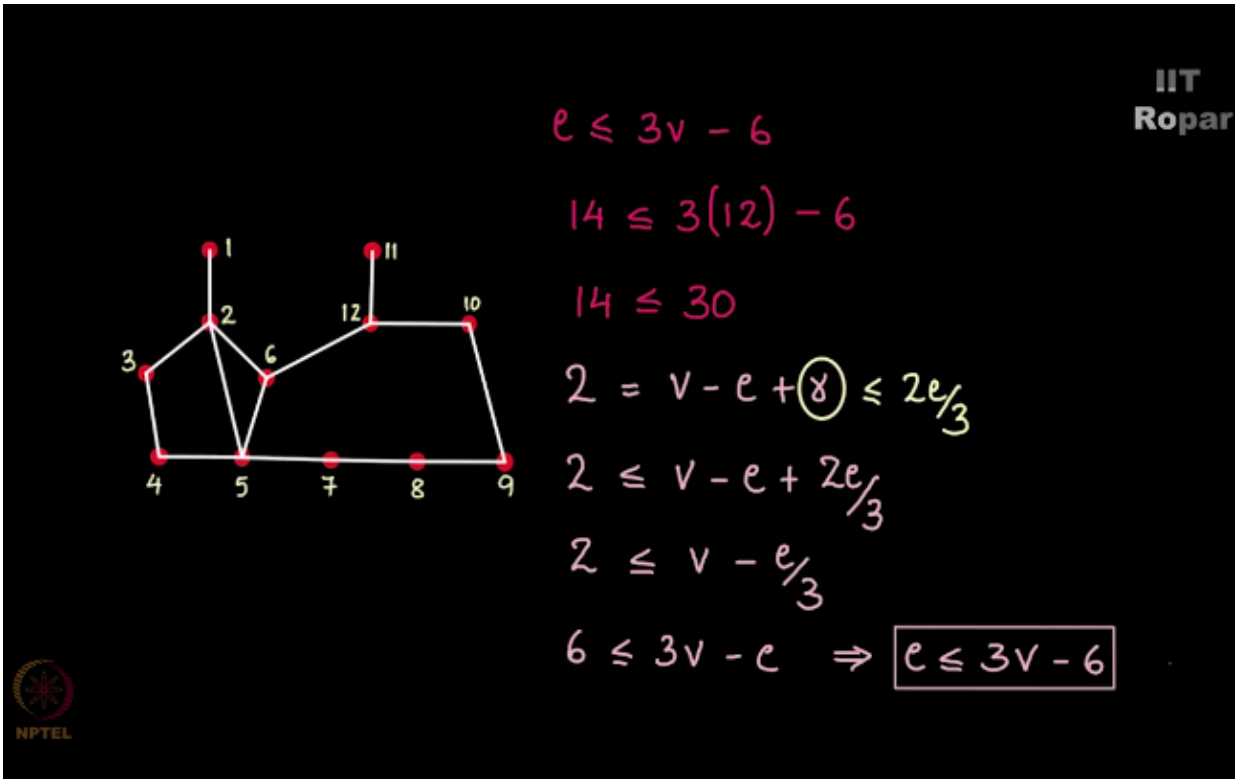

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