NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

Proof of V - E + R = 2

By
Prof. S.R.S Iyengar
Department of Computer Science

Now let us see how V - E + R = 2 using induction, let us write our own graph, a planar graph of our choice, look at this graph,
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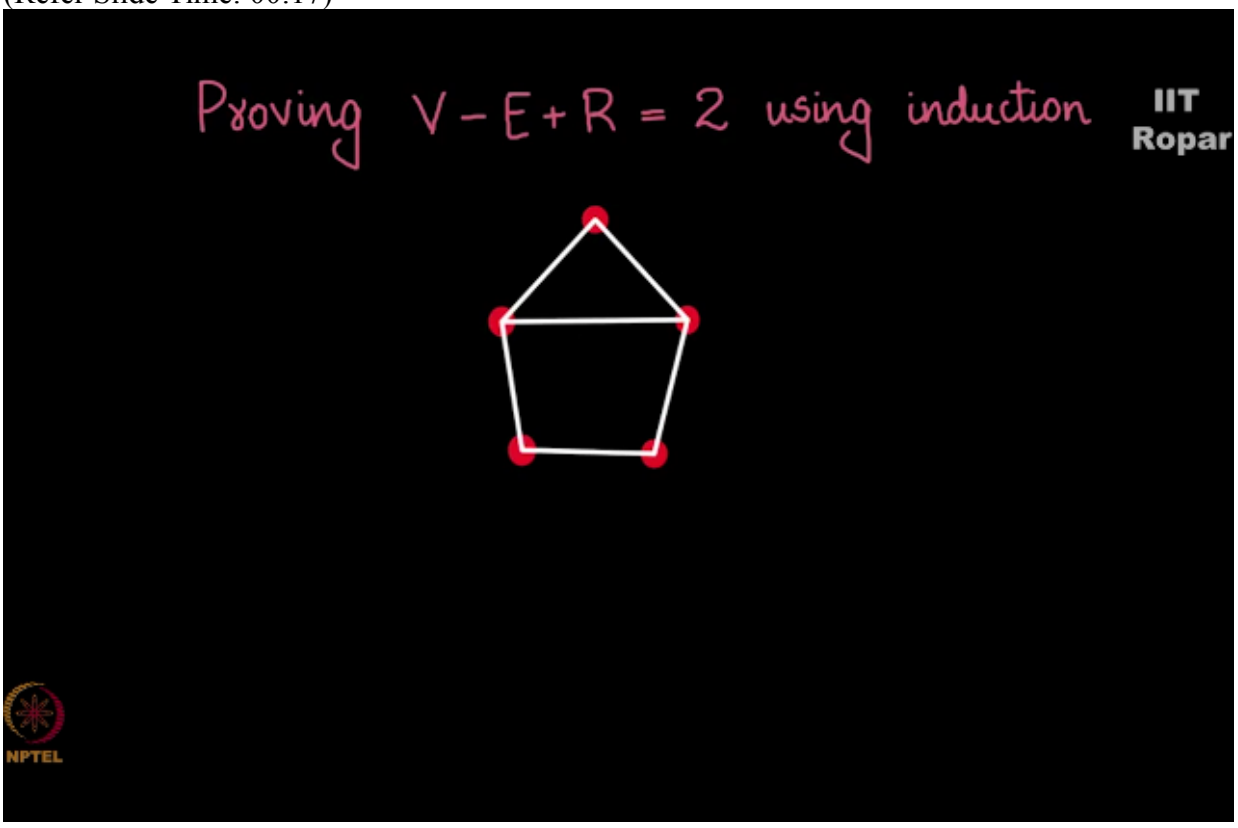

this graph has, how many? 5 vertices, and 1, 2, 3, 3 regions, how many vertices do you have? 1, 2, 3, 4, 5, let me say V = 5, how many edges? 1, 2, 3, 4, 5, 6, edges and the regions are 3 in number as you can see this is actually equal to 2,
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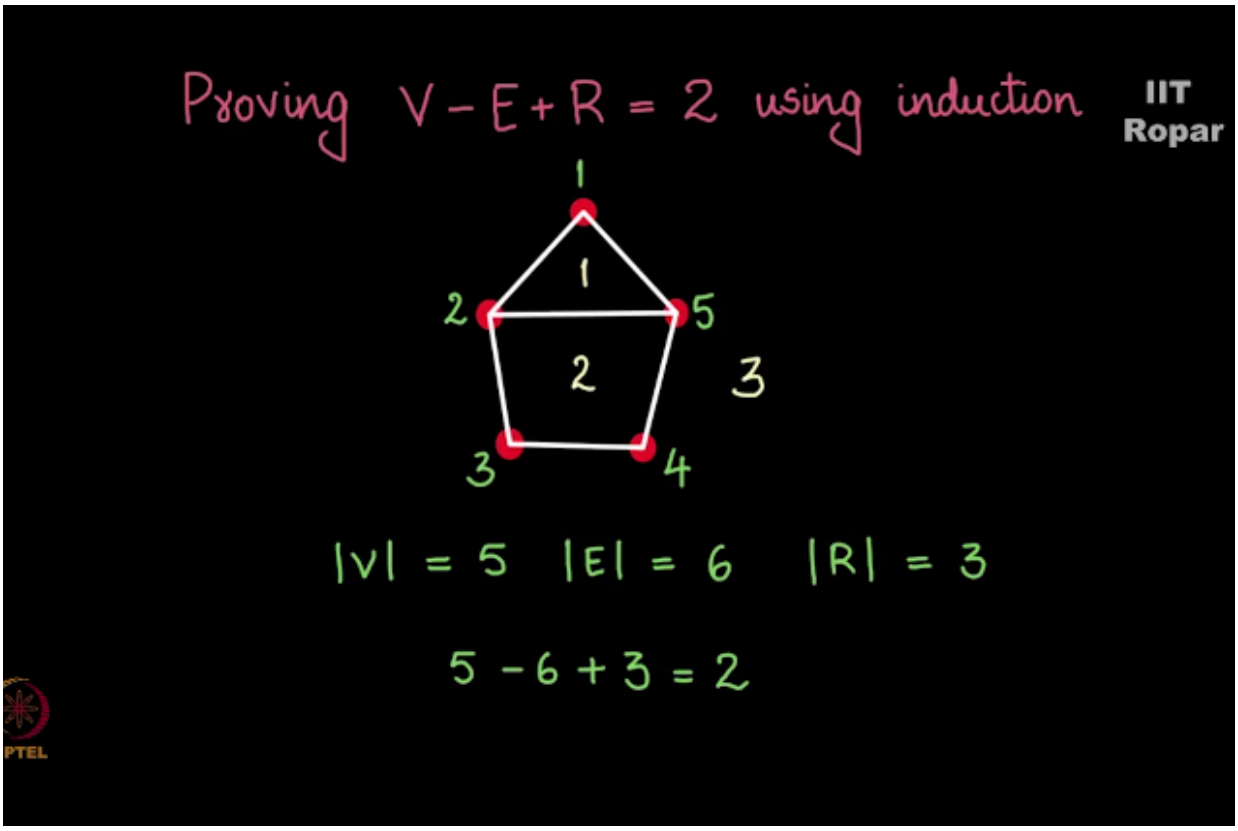

now what is the magic here, why is this formula always true? You see when I put another edge like this,

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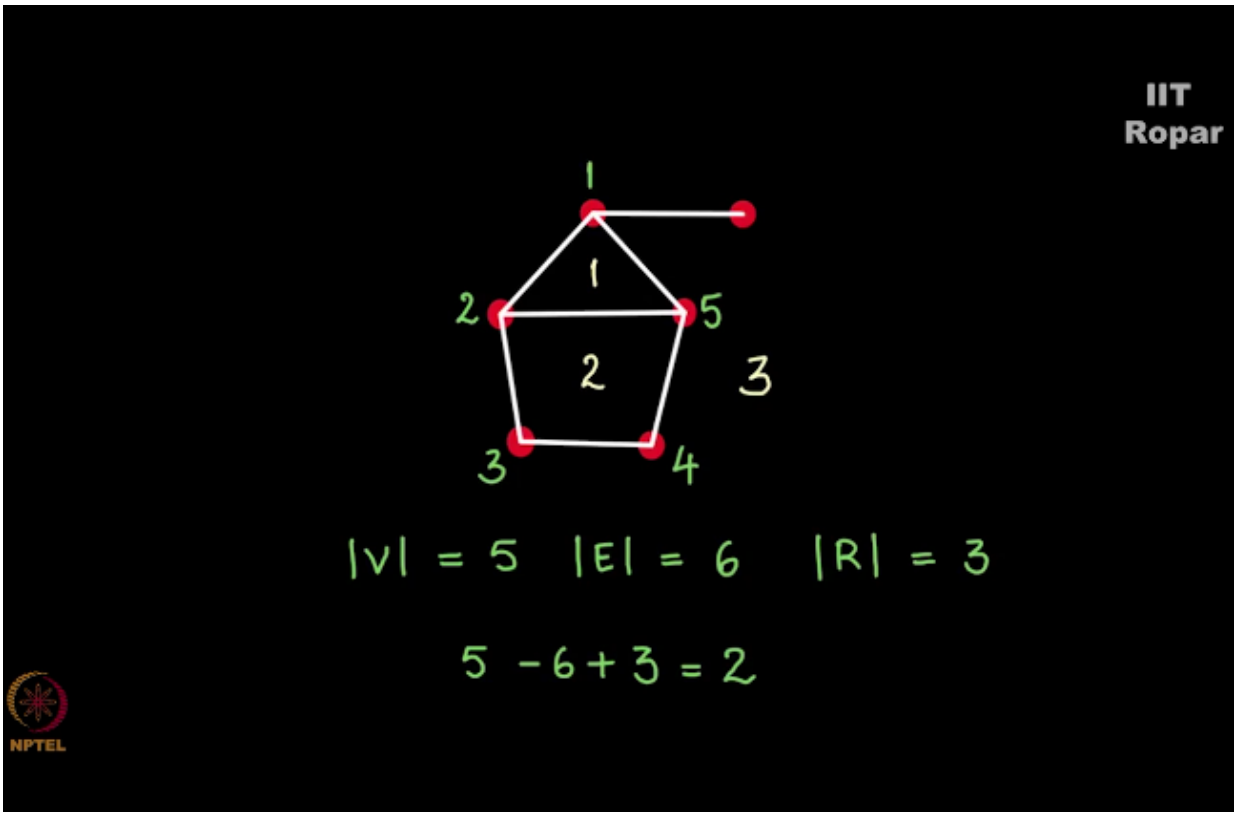

and introduce a vertex, the number of regions don't increase here, but vertex increased by one, edge increased by one, this is like saying when V - E + R is what you are looking at, when you increase vertex by 1, and edges by 1, it results in +1 and -1 to your formula, think about it.

So if I add 1000 rupees to your bank account and then subtract 1000 rupees immediately there is no change to your bank account, so V - E + R was equal to 2, but then addition of a V, and addition of an E results in a +1 and a -1 and hence there is no change in the number 2 here, so whenever you introduce an edge to an existing graph there is no change to the formula.

Now what about you introduce an edge without introducing a vertex, then let's say you do this you will have a fourth region here,
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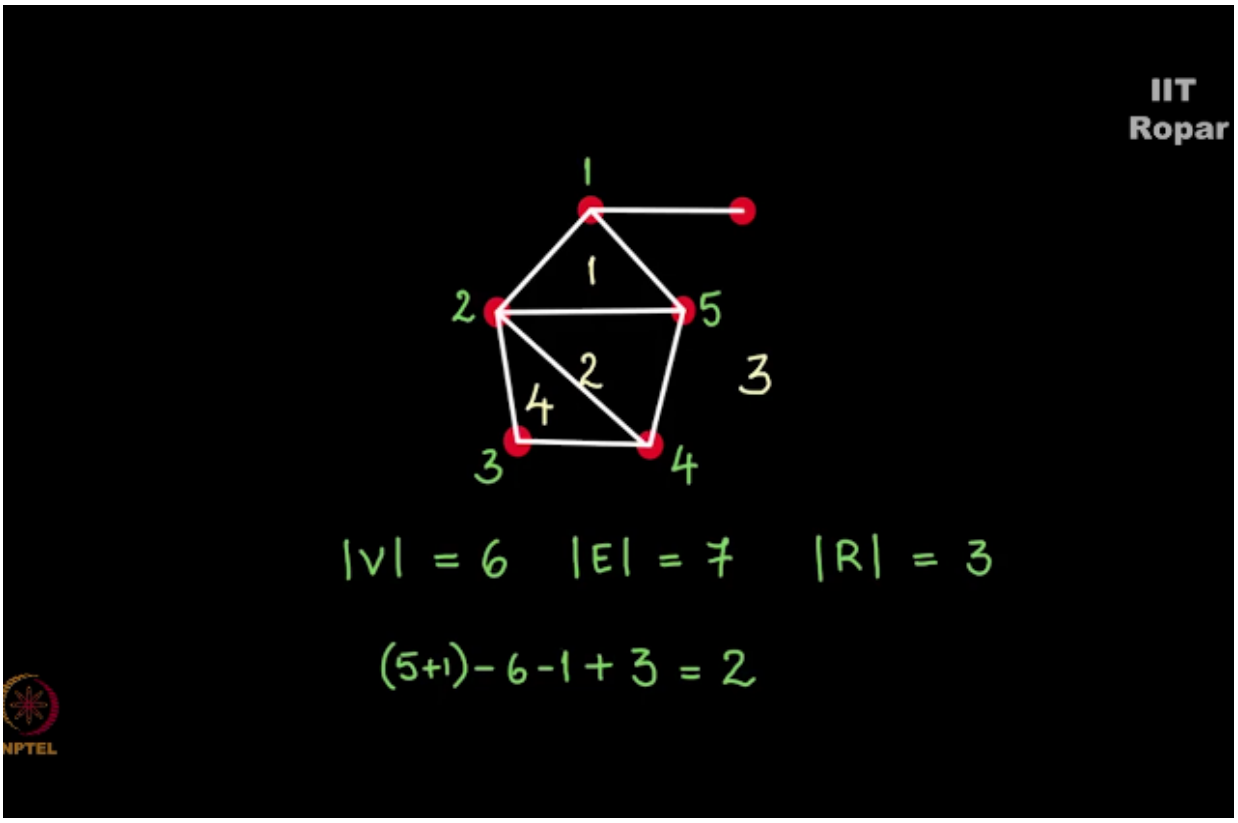

but then you just added one more edge, so edge went up by one unit the region went up by 1 unit, then again then again you have a -1 and a +1 being contributed and hence the value doesn't change whatever it was before remains to be the same, so whenever a graph grows and you have a big planar graph, do you see that it starts from let's say a single vertex and then you introduce an edge and a vertex and then an edge and a vertex, and maybe only an edge, maybe only an edge, I don't know then a vertex and so on you always introduce a vertex, or a vertex and an edge, but we observe that starting from the single node where vertices are only one in number, edges are none and regions happen to be 1 in number, V - E + R turns out to be 2, when there's only 1 vertex,

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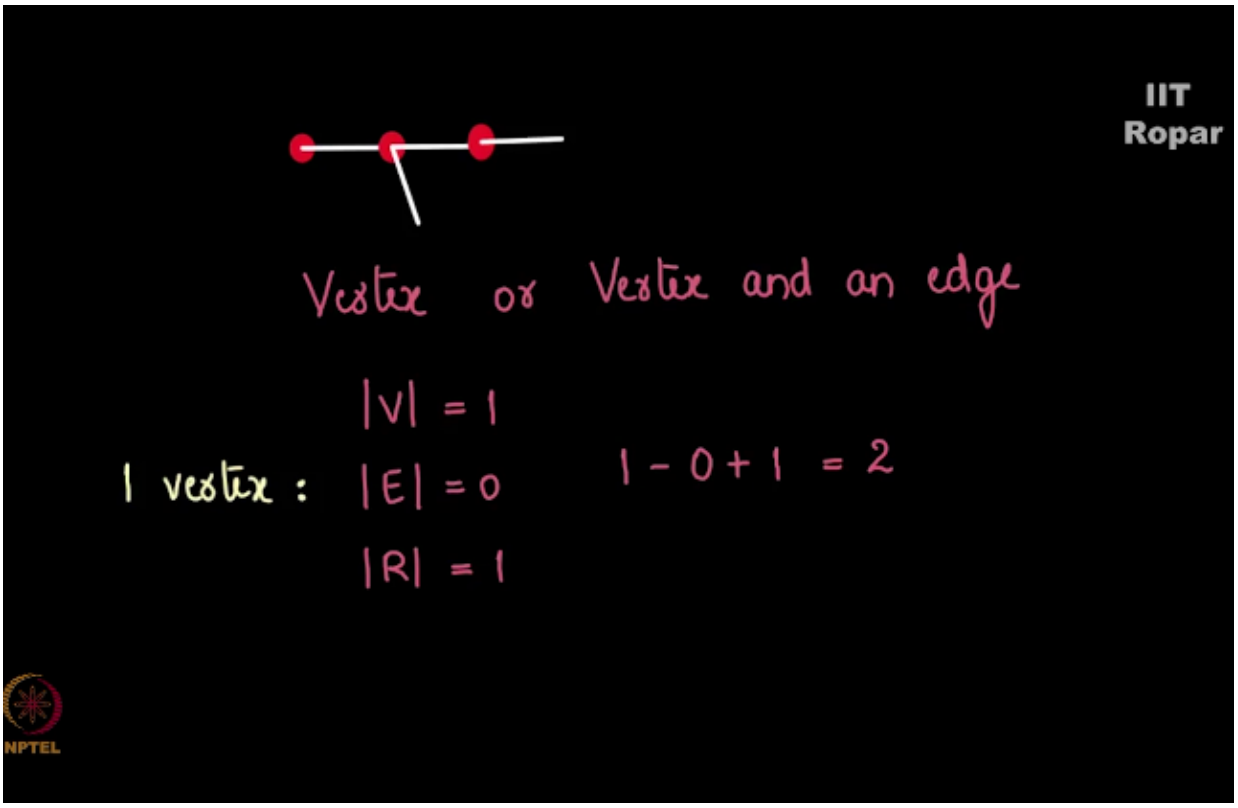

and when you introduce one more vertex it just go up by 1, vertex goes up by 1, the formula remains the same.

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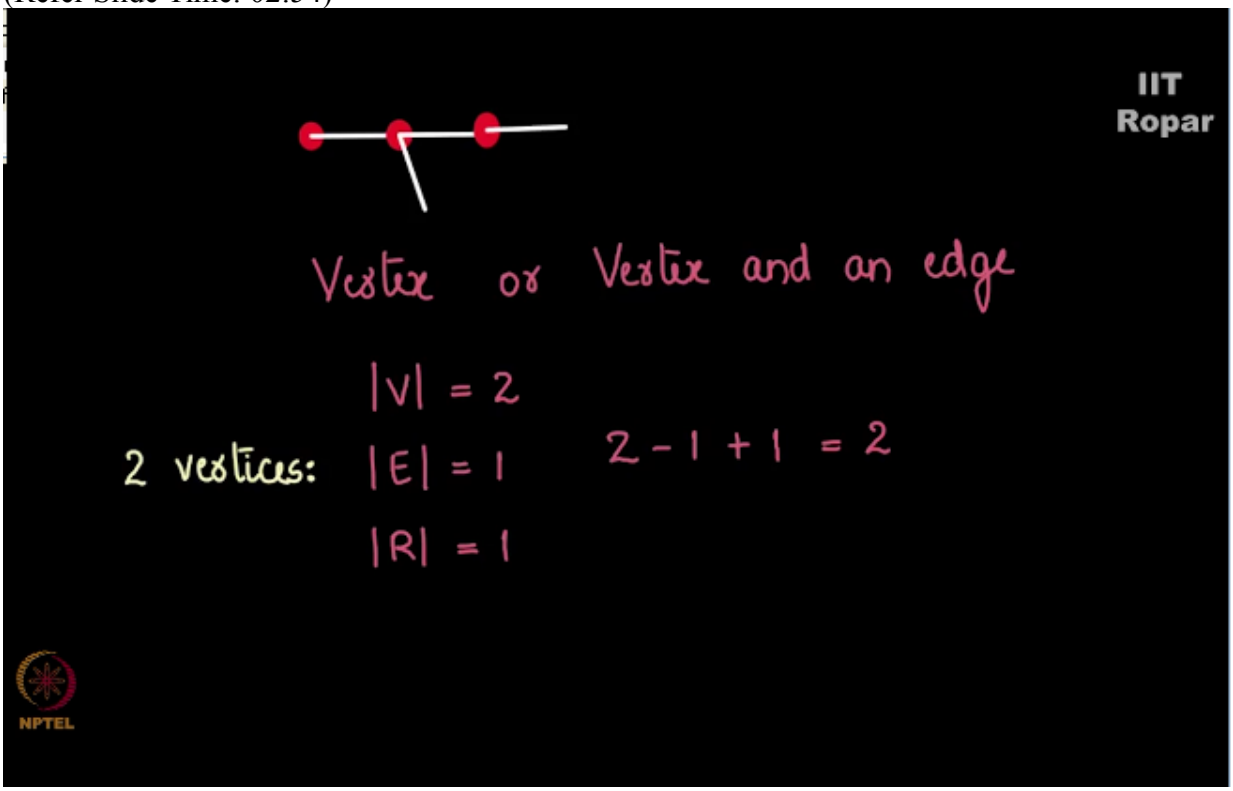

And at every stage the formula remains the same which means V – E + R is always equal to 2,

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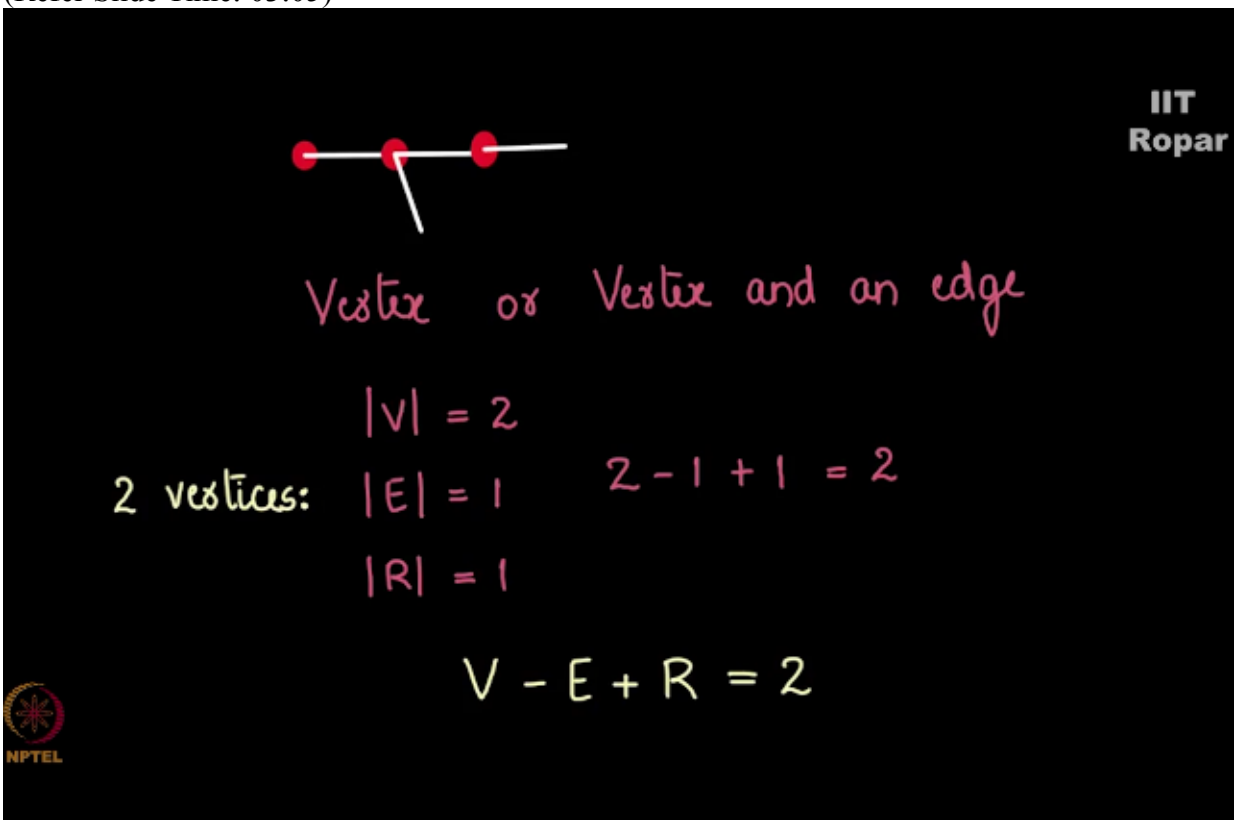

and if you noted something I said I will use induction, doesn't look like I used induction anywhere I purposefully did this without talking about induction to give you the intuition that most of the times we need not look at the K = 1, let it be true for K = something, and then something + 1, and hence it's true for everything is a very mechanical way of showing induction, I tried making you people feel for induction the natural way, right, we are actually inducting on the number of edges here,

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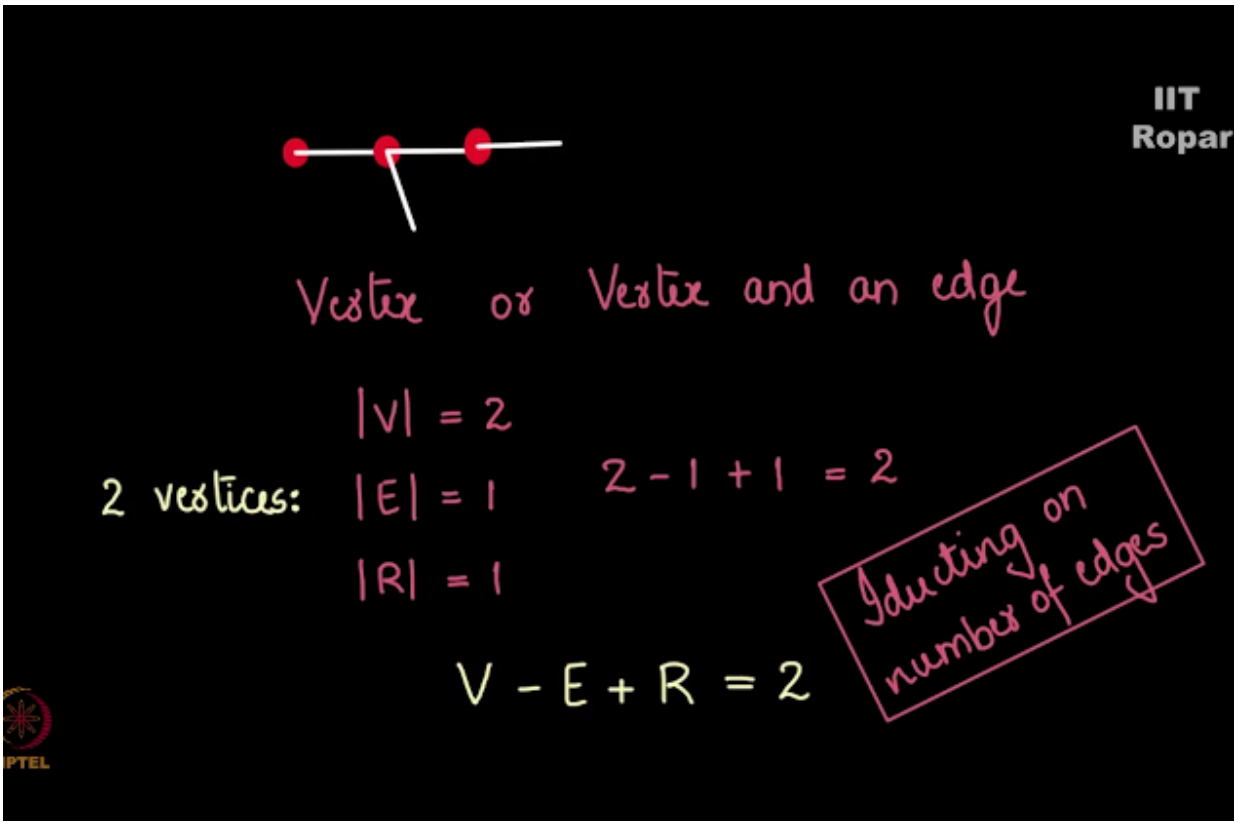

right, we were able to go ahead step by step introduce a new edge and still we observed that there is no change in the formula, and hence V - E + R = 2 and you now know induction more than the mechanical way of doing it, this is my favorite example of how induction can powerfully come to your rescue and induction is actually very intuitive, correct, but I leave it to you as a homework to properly do the mechanical version of induction in solving this formula.