**NPTEL**

**NPTEL ONLINE CERTIFICATION COURSE**

**Discrete Mathematics**
**Graph Theory – 2**

**Examples of Planar graphs**

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Here are a few examples of planar graph, so for the next few minutes we will be trying to draw or check if these graphs are planar,
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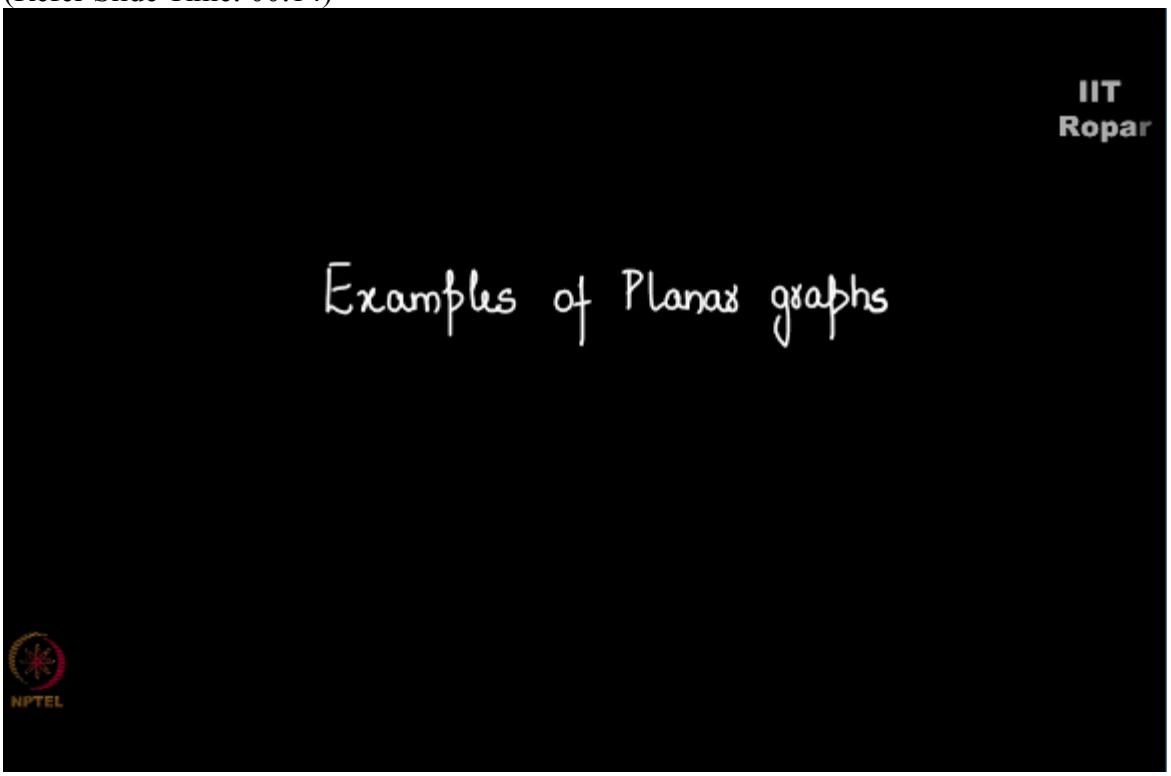


consider this graph it's a star graph on 1, 2, 3, 4, 5 vertices,
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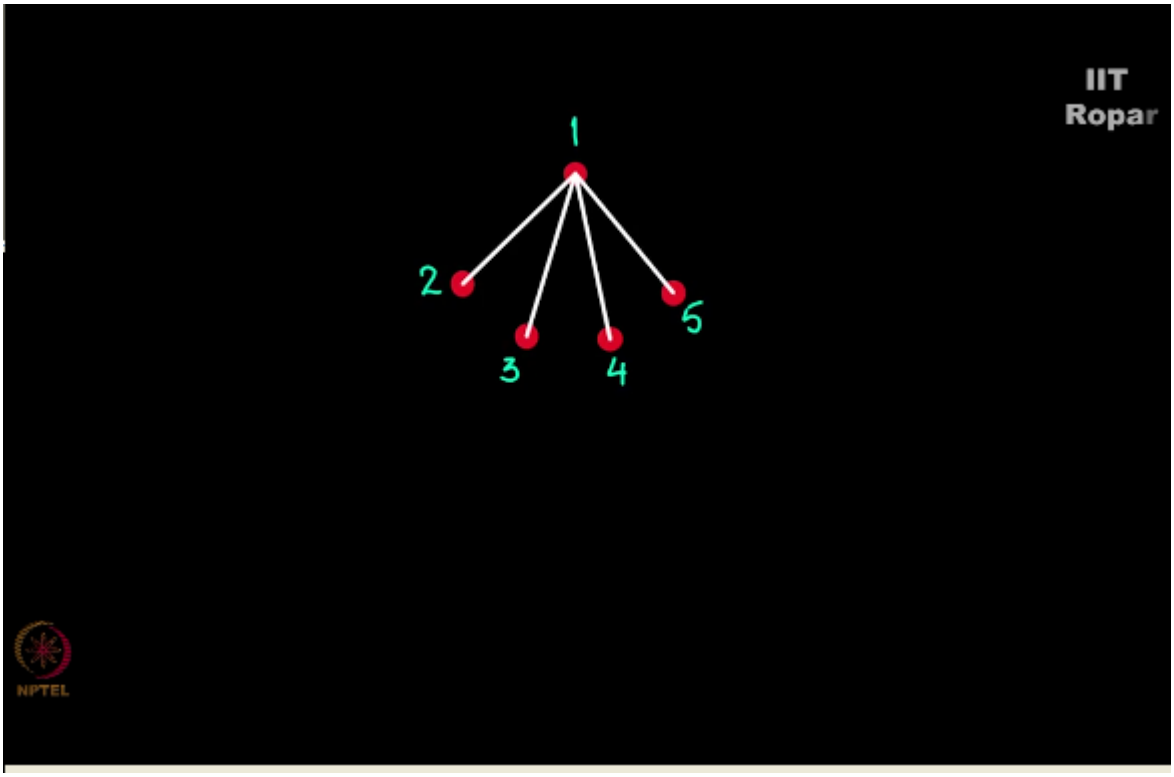


this is called the center vertex, so we have a star graph on 5 vertices, this you see it is planar,
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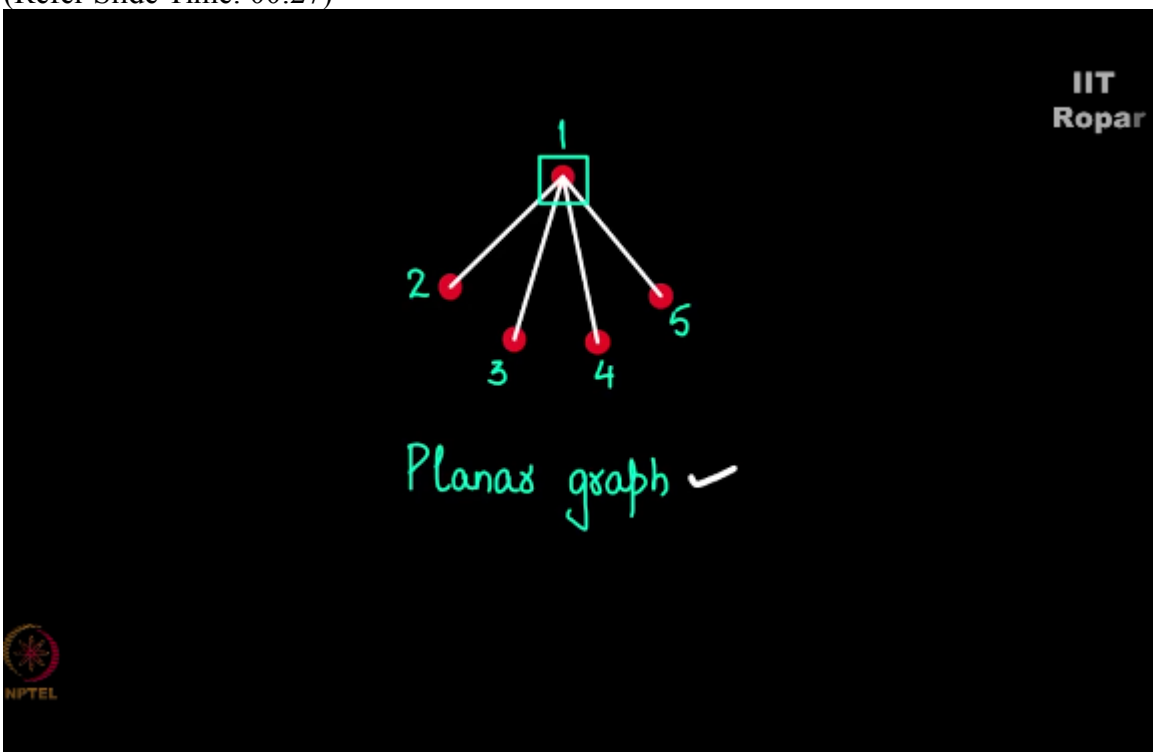


there is no doubt about it because we do not have any intersections here, so in general any tree is a planar graph,
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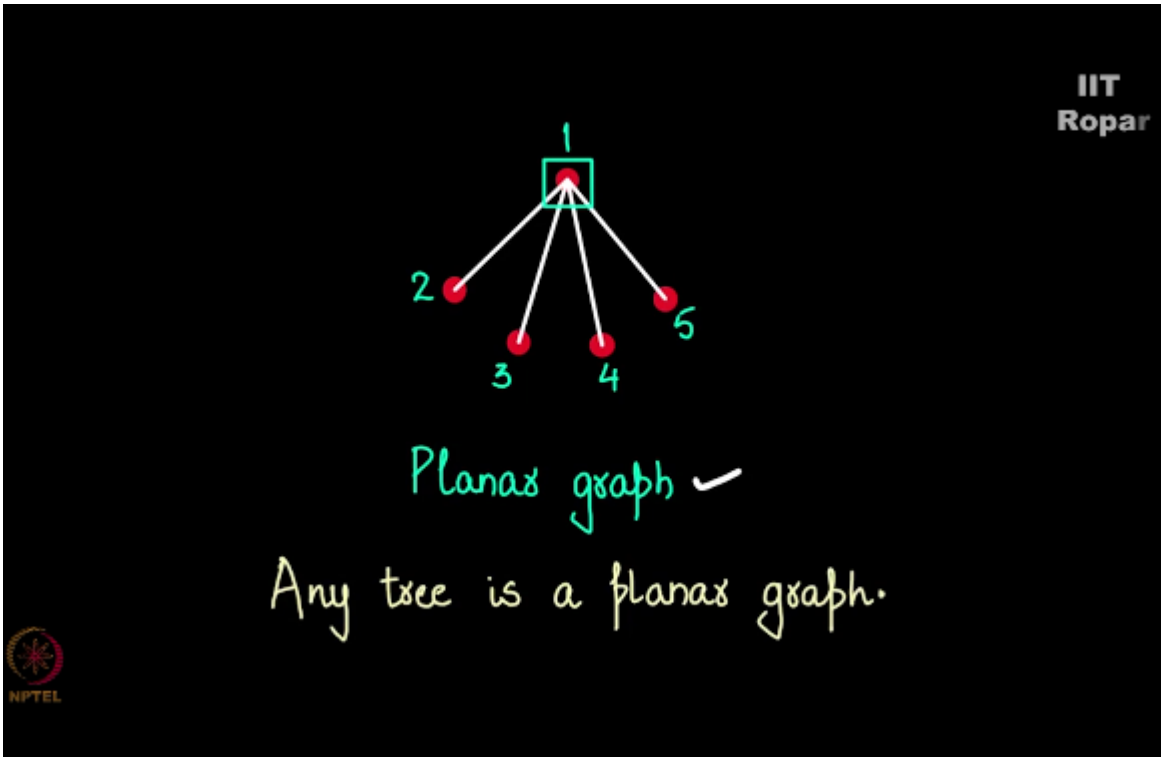


star is a particular type of a tree and any tree is a planar graph.

Consider this graph,
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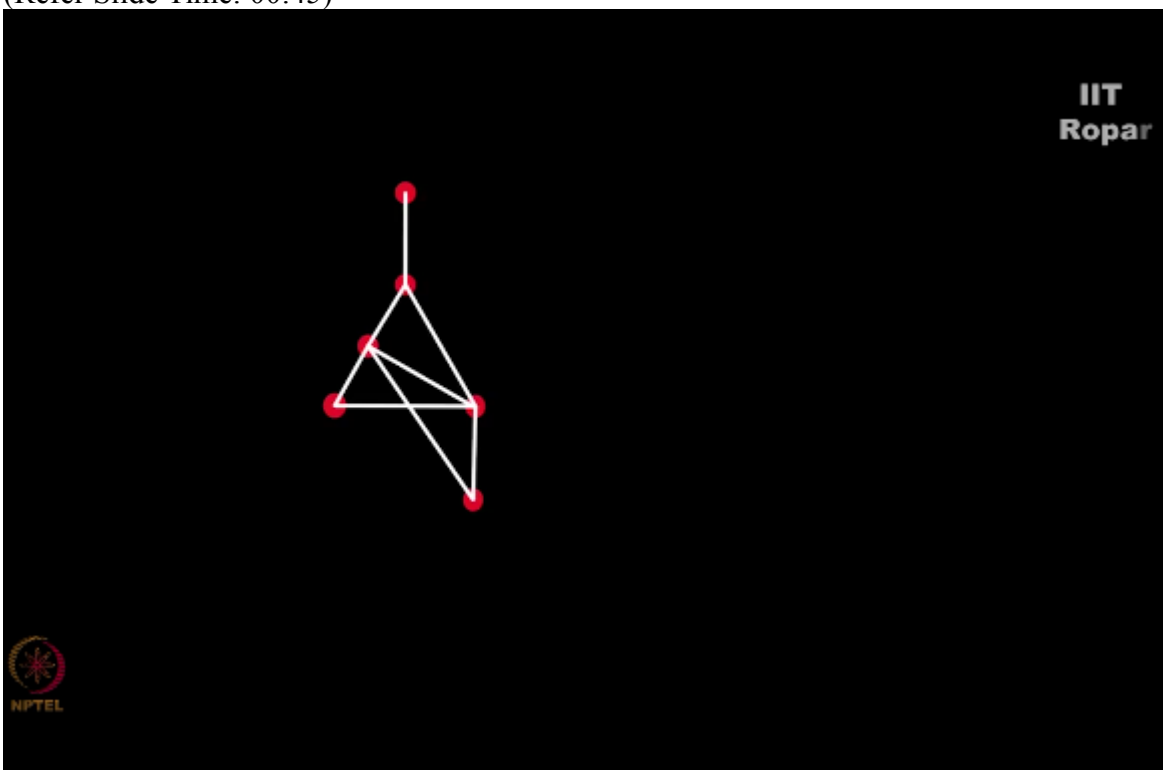


so let me check if it is planar, I'll draw it like this, you see here it is a C3

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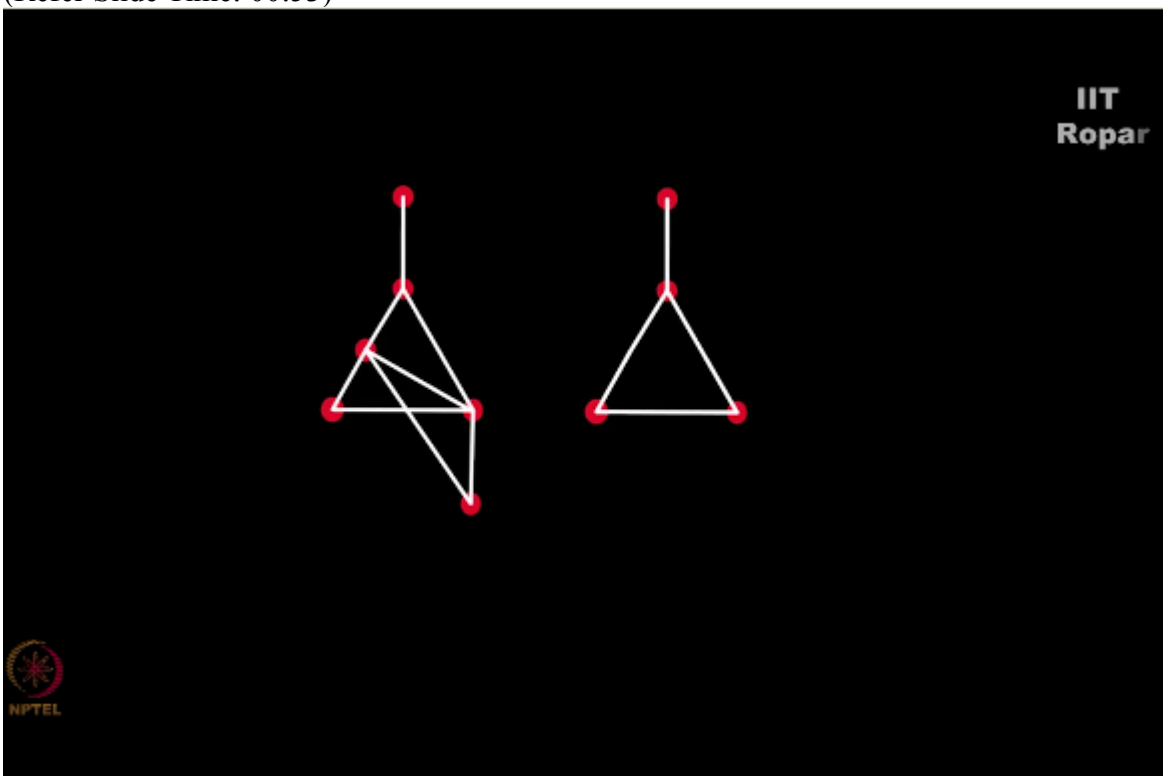


and then now pendant vertex, a pendant vertex, this is called as a pendant vertex,

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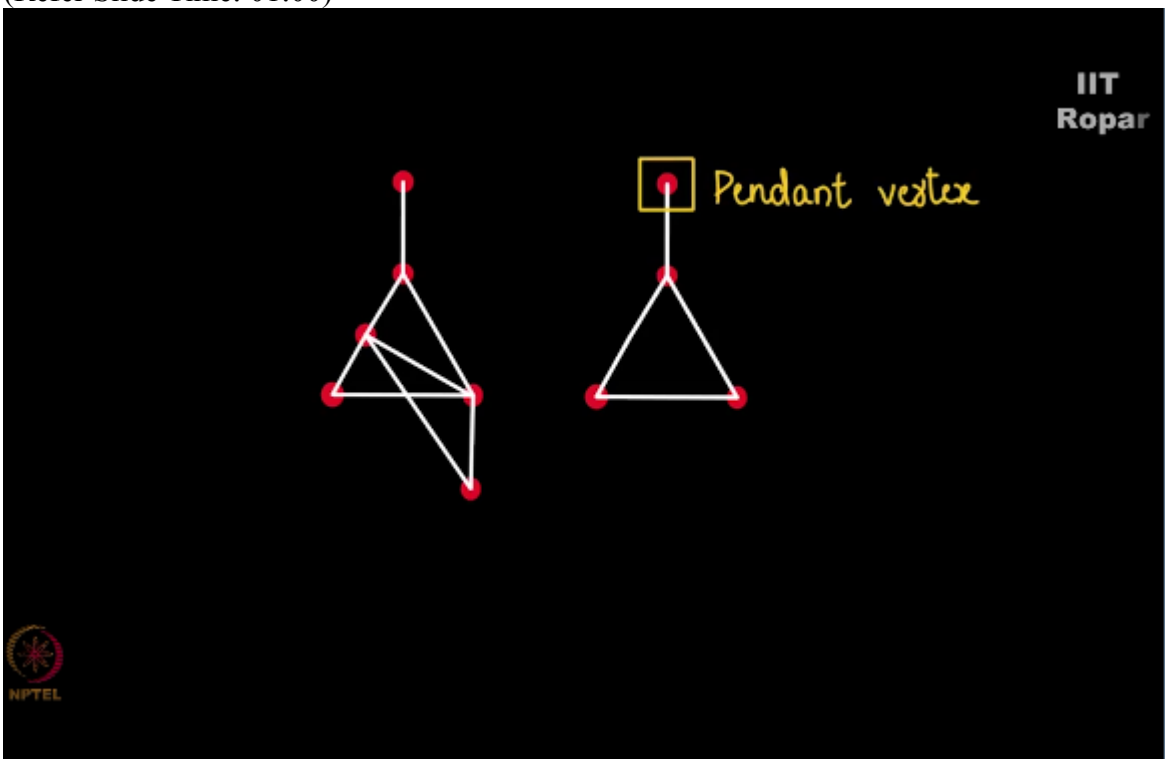


the vertex whose degree is just one like this, it's like a pendant falling out, right, this is called as a pendant vertex.

Now we have another vertex here and there is a edge here like this, now you have one more vertex remaining outside like this,
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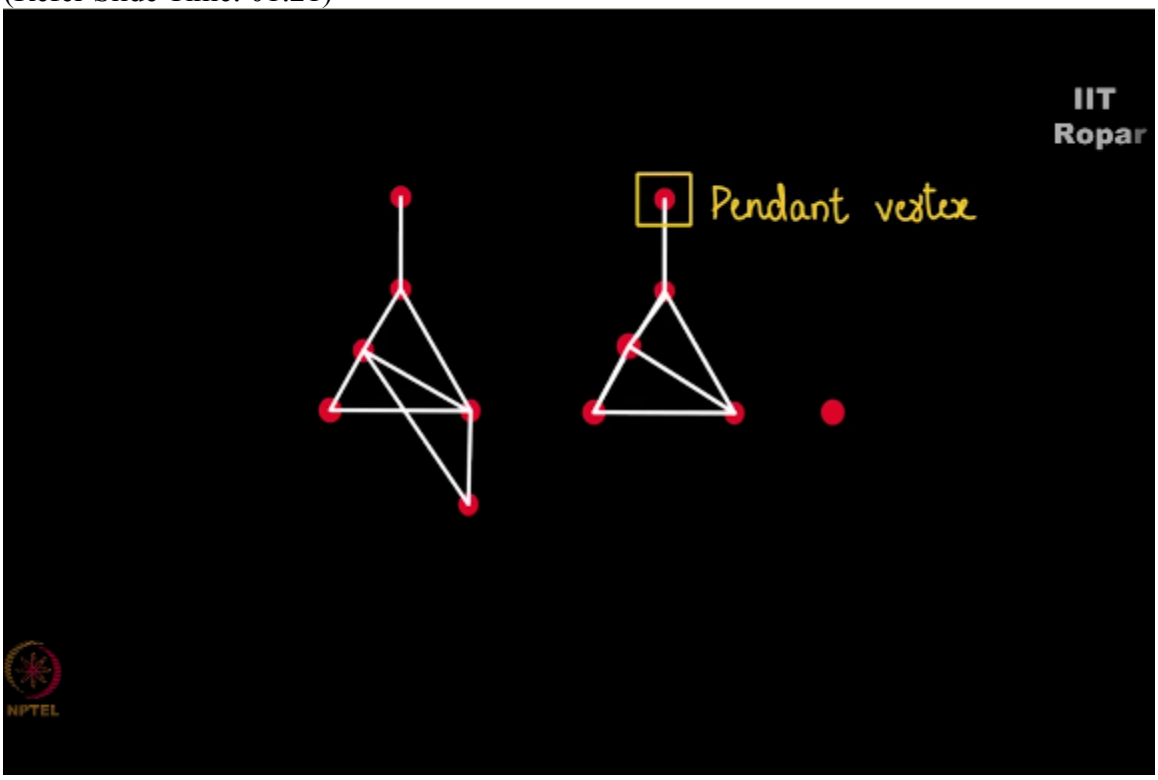


right, and there is an edge from this vertex, so last one edge is remaining to this one, how do I connect it? I can do it like this,
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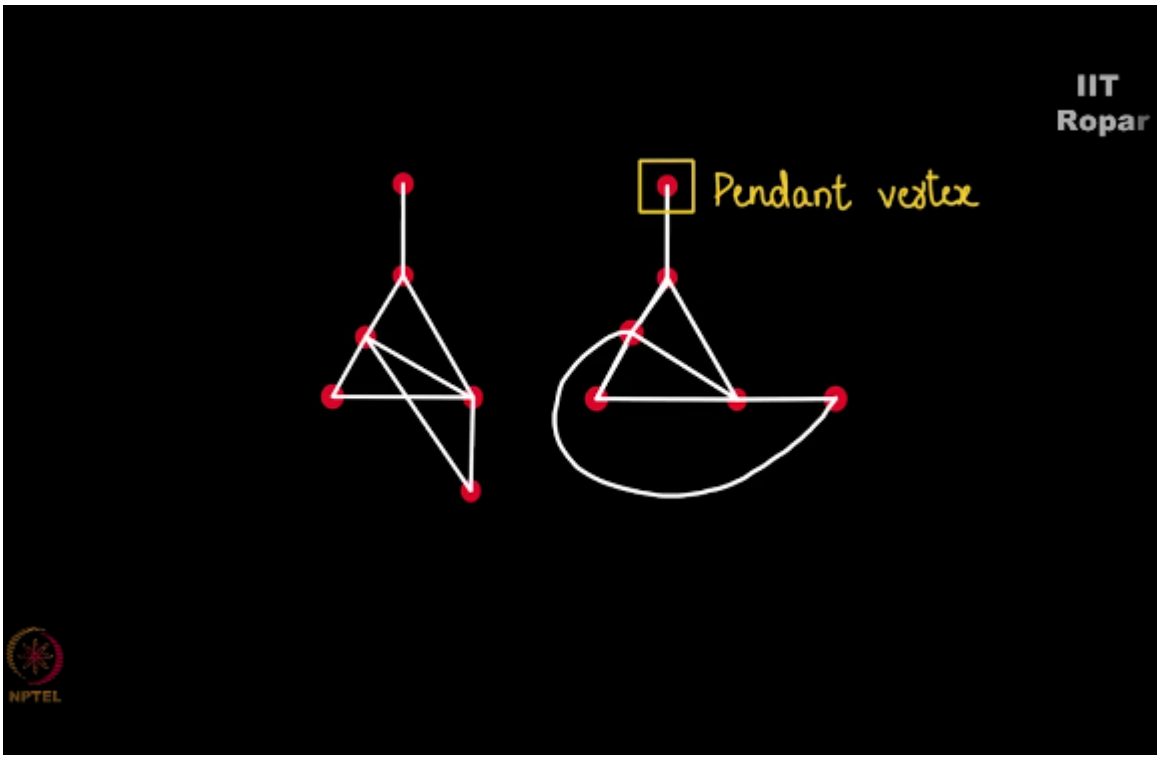


yes, I have obtained it, the graph is planar.

Now consider this graph on 6 vertices let us check,
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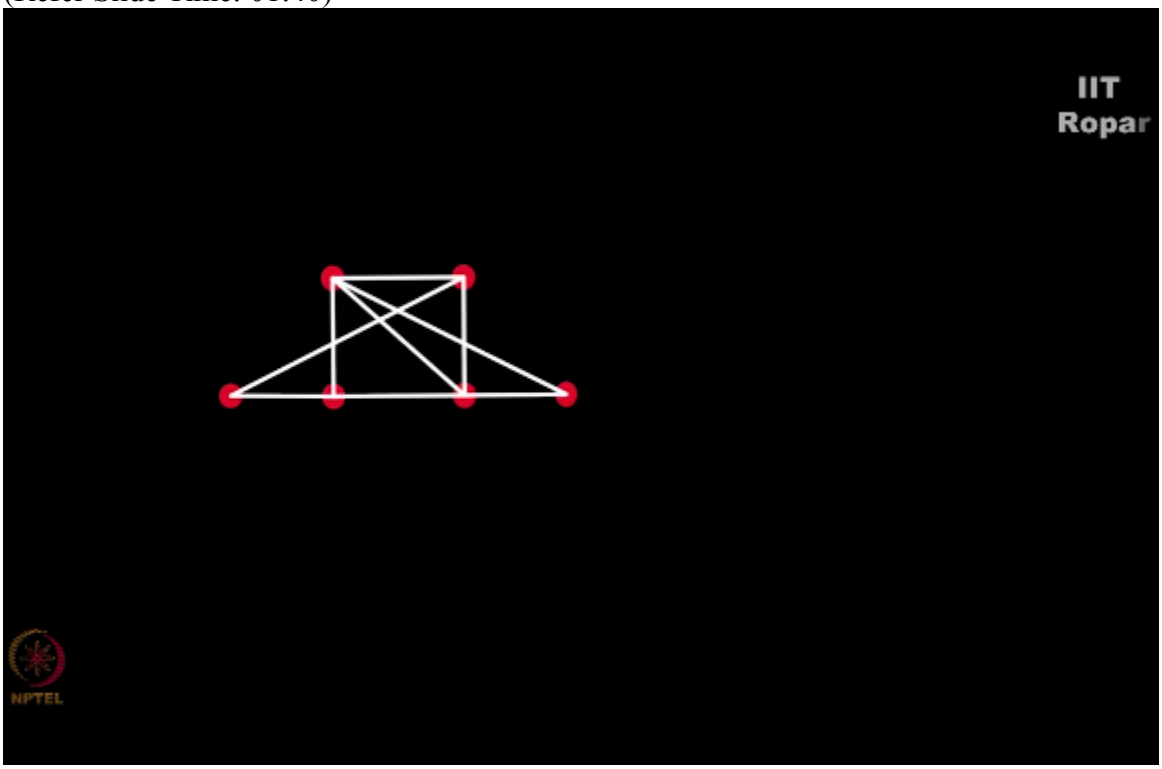


I draw a C4 first like this,
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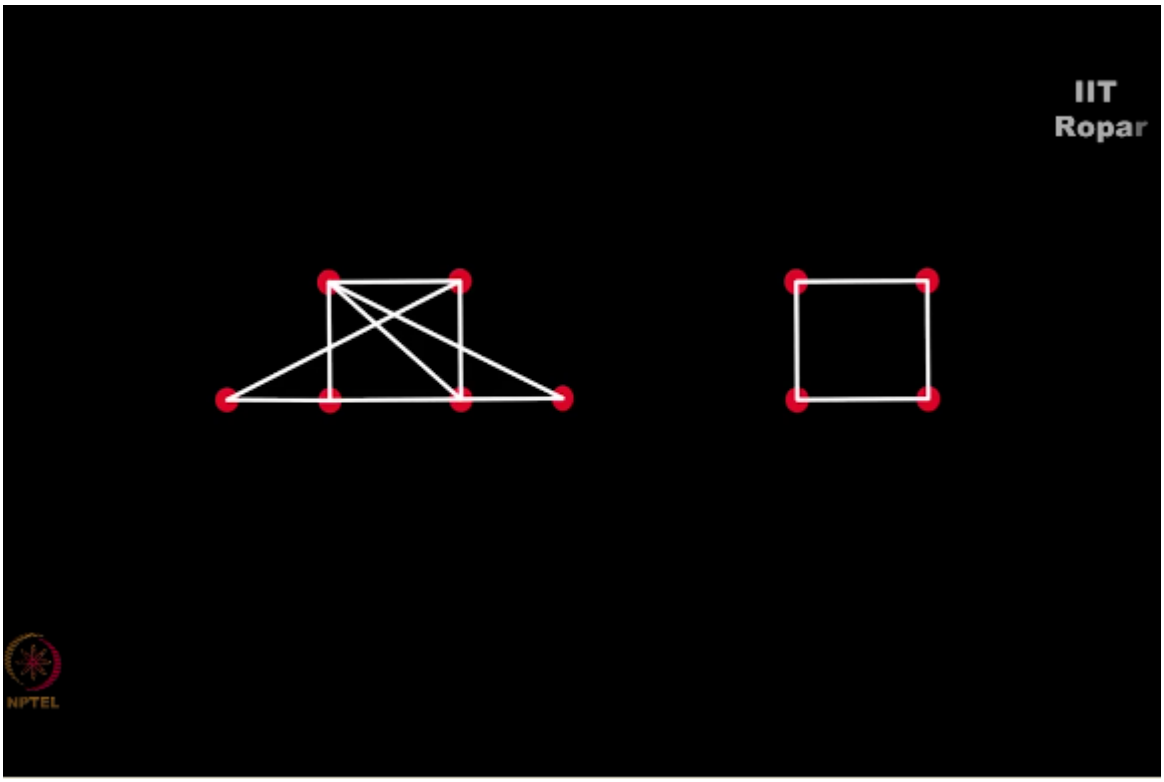


and then a diagonal like this,
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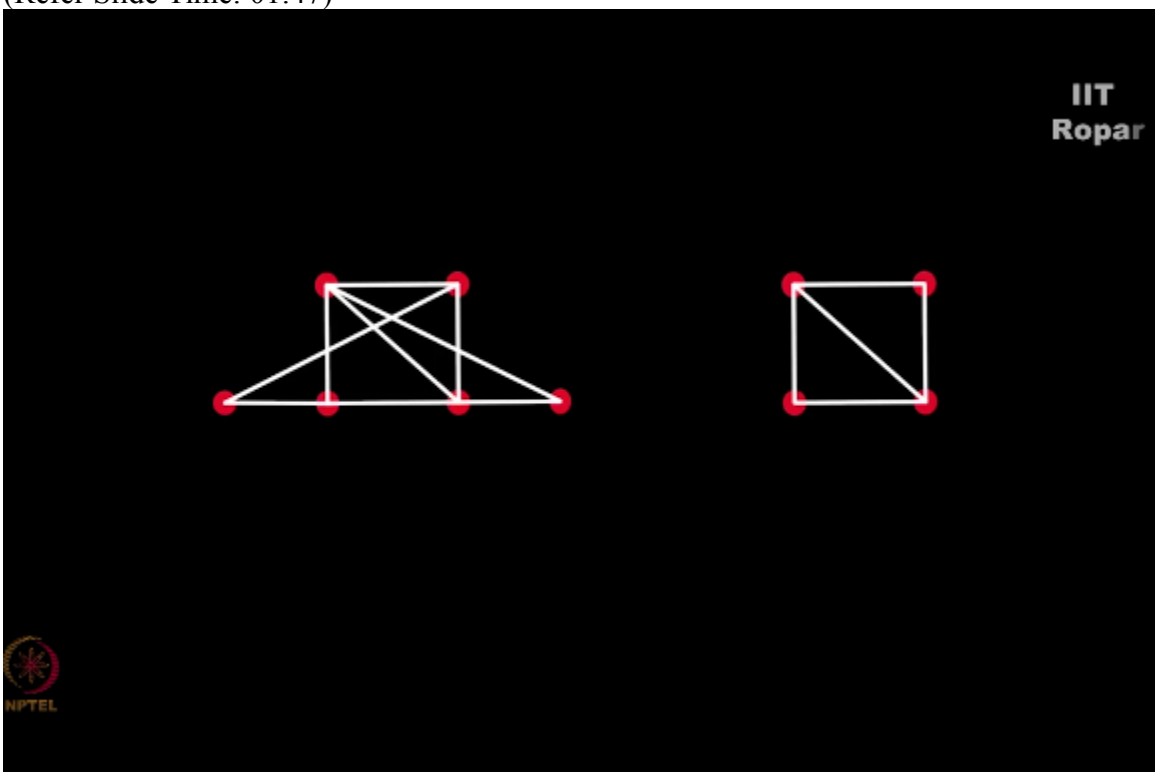


now you see this vertex I'll shift it here this way,
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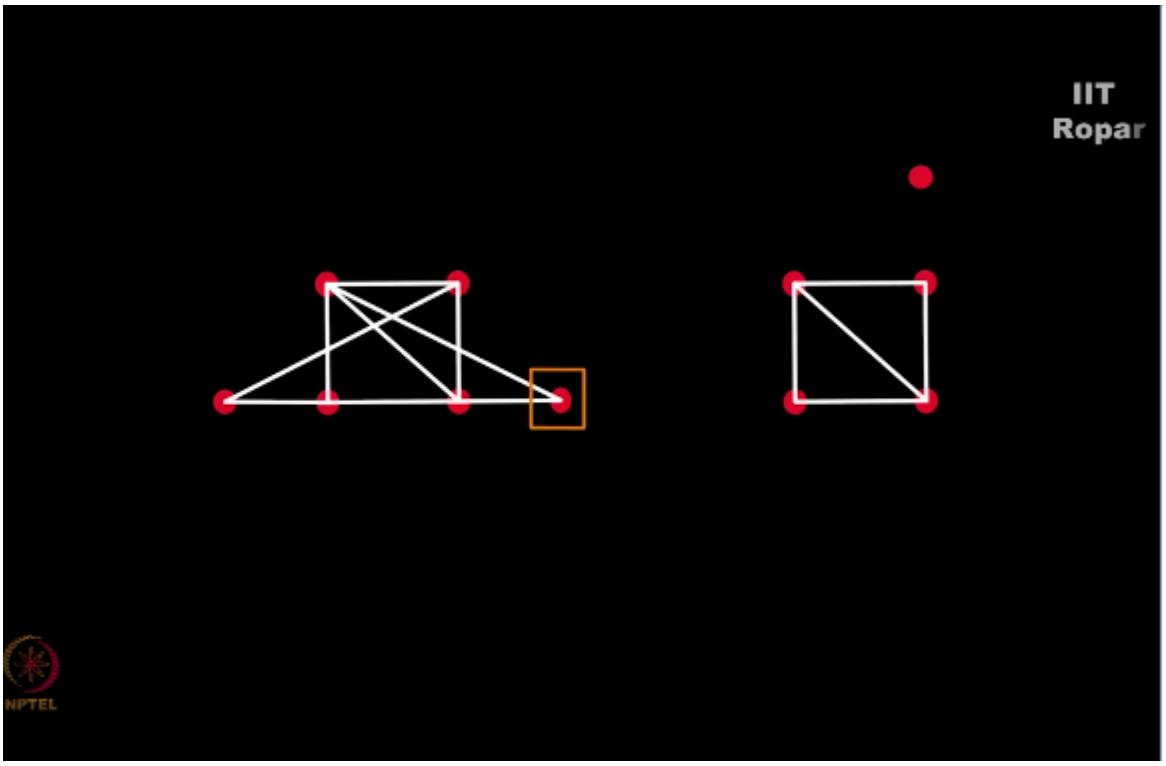


And I'll draw these edges,
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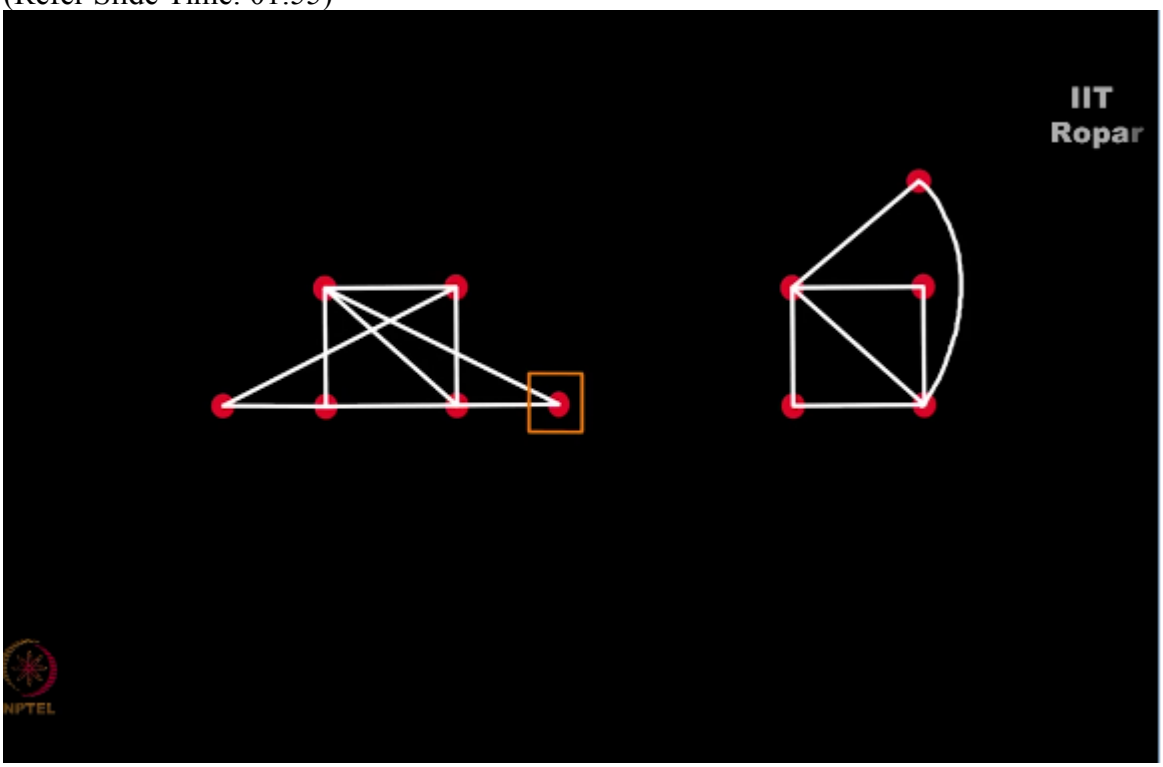


now I have one more vertex to be drawn and 2 more edges, now if I draw it here like this, one edge is done, another edge if I connect it we get 2 intersections,
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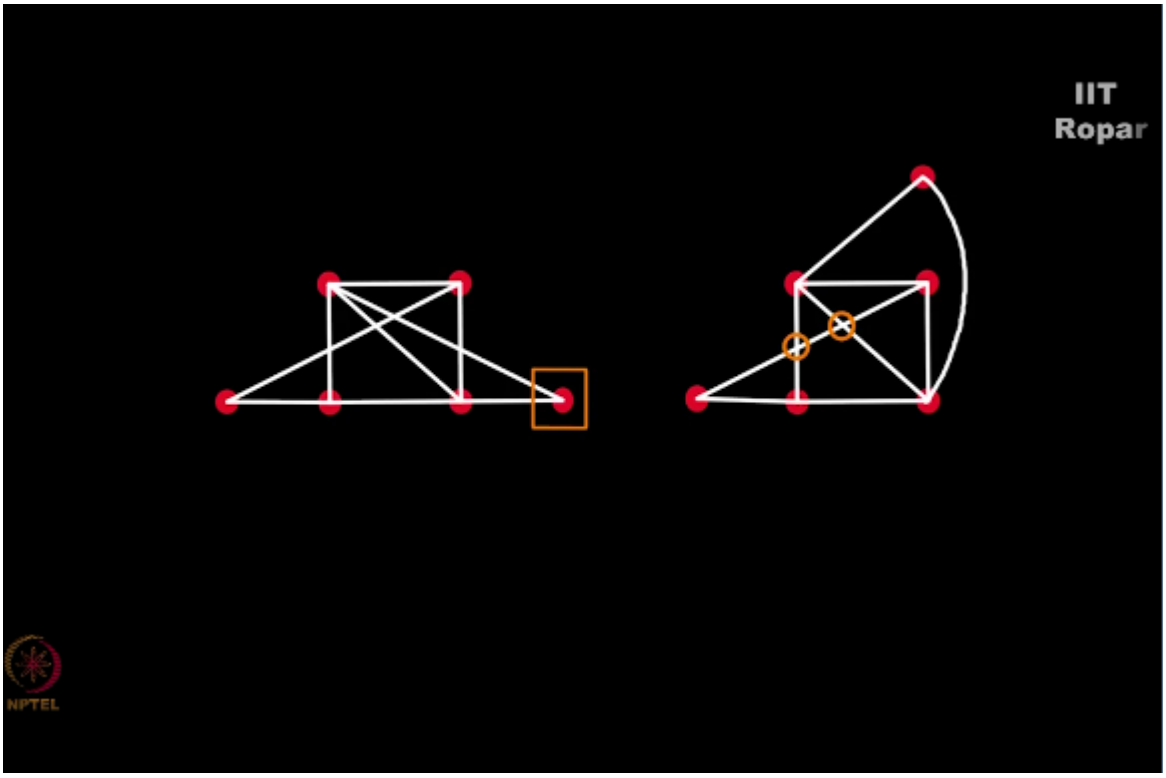


this is not allowed, now if I shift it here, and I connect it, let me say this way,

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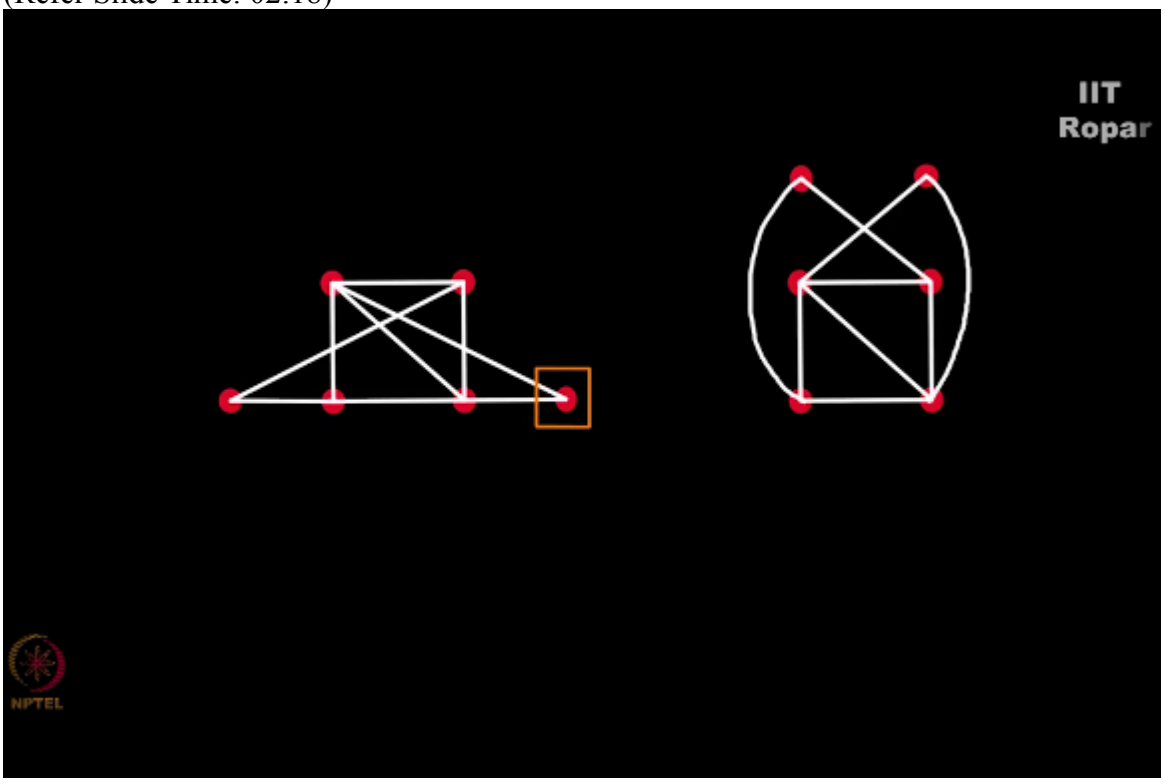


I get a intersection here.

Now let us try by taking some other way and checking if it is planar or not, let me put the vertex inside here,
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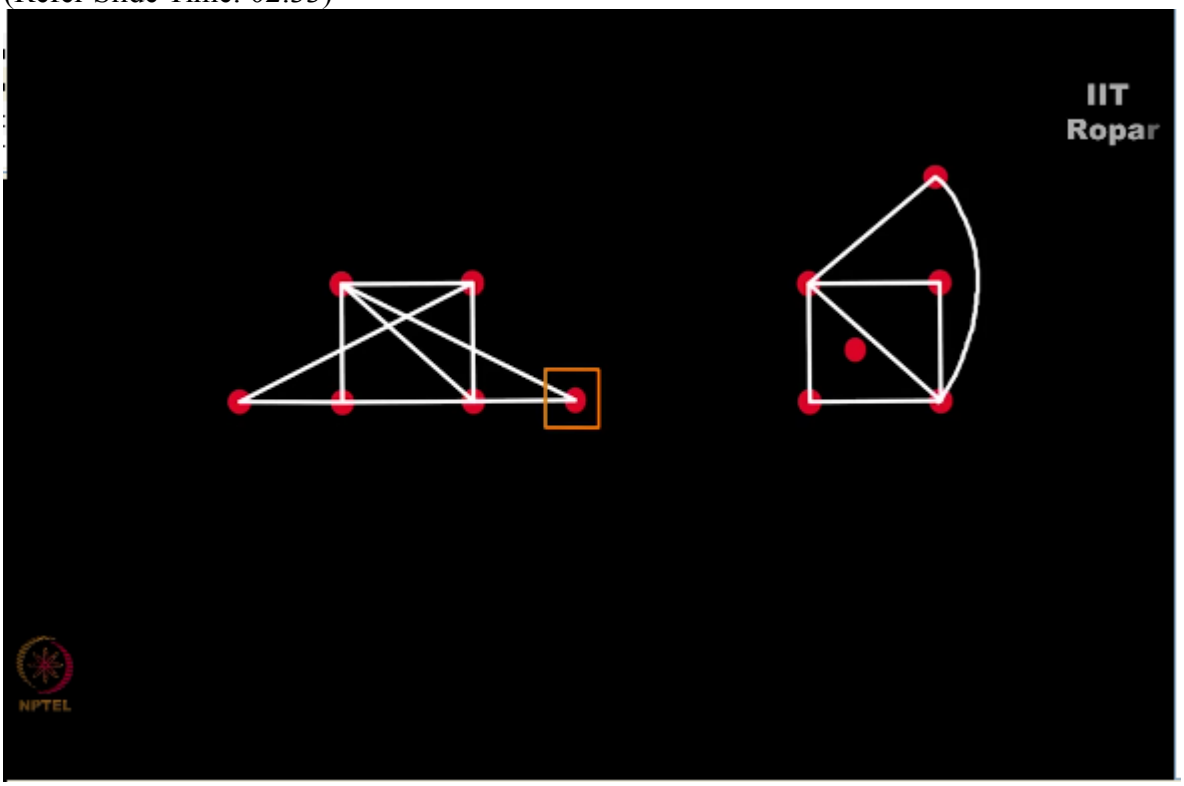


if I draw this edge and this edge you see
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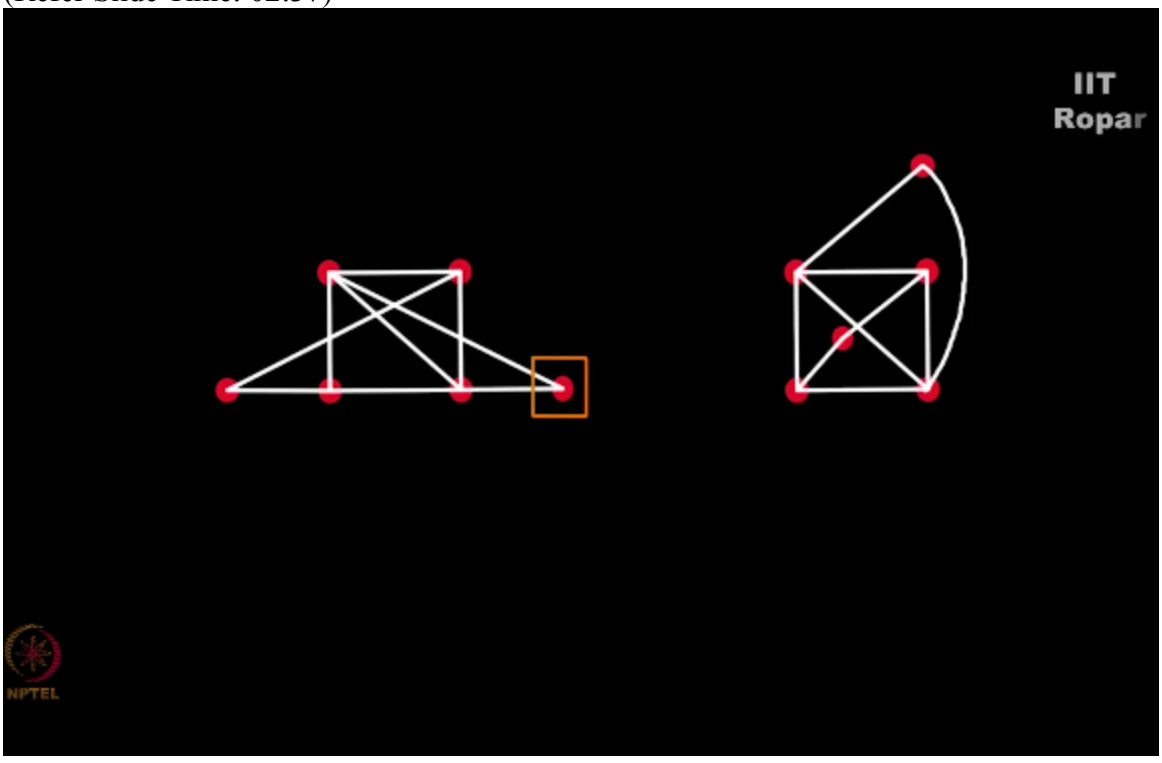

I'm again obtaining an intersection, right, looks like this graph is non-planar,
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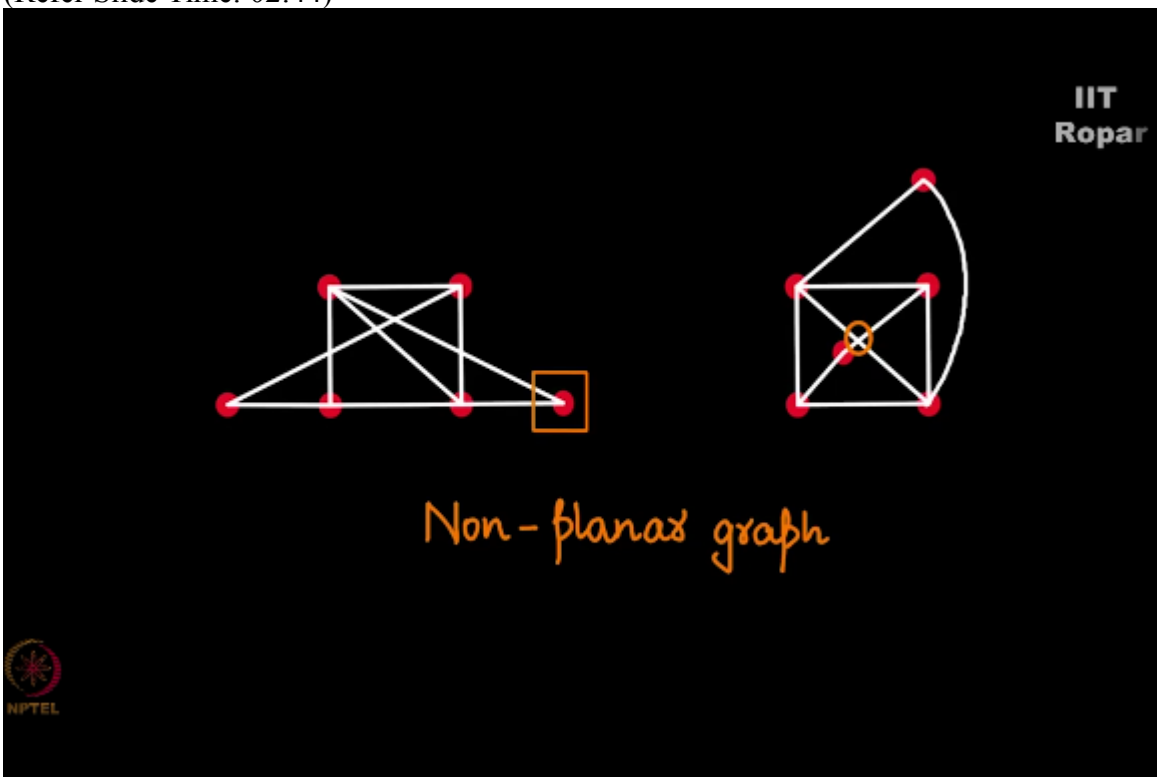


now let me take this graph,
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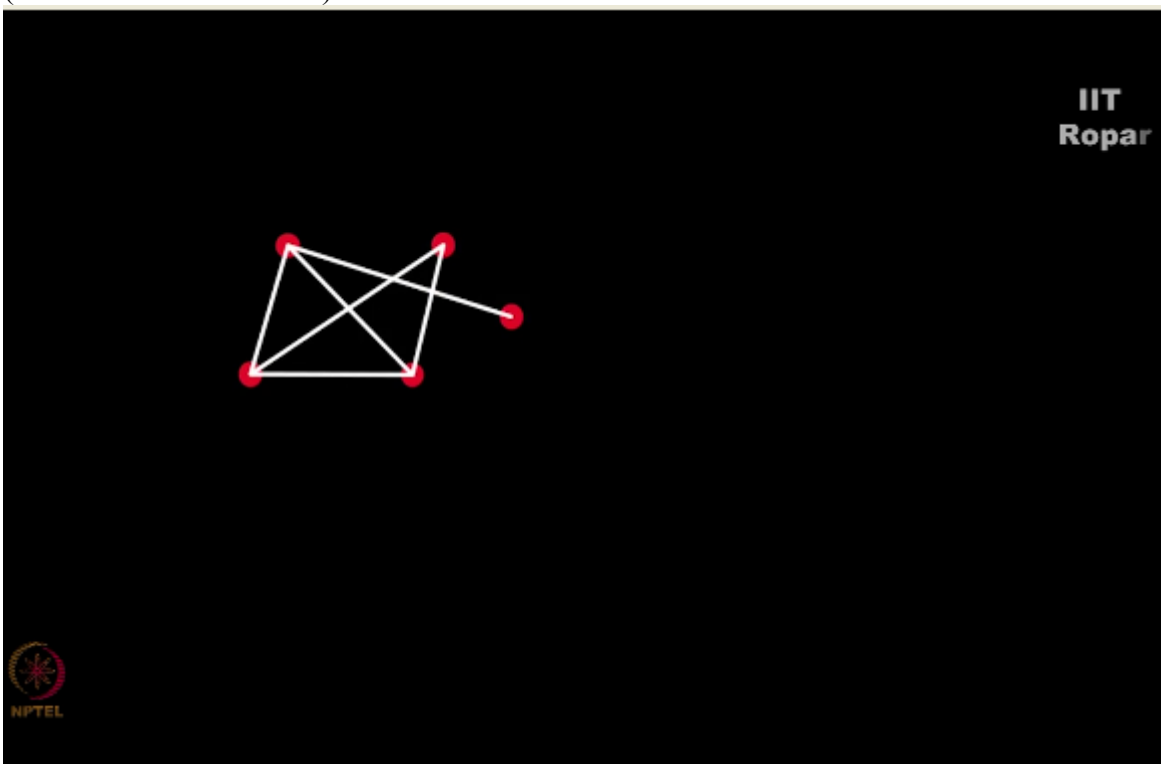


I can see some 2 triangles here, this is one triangle,

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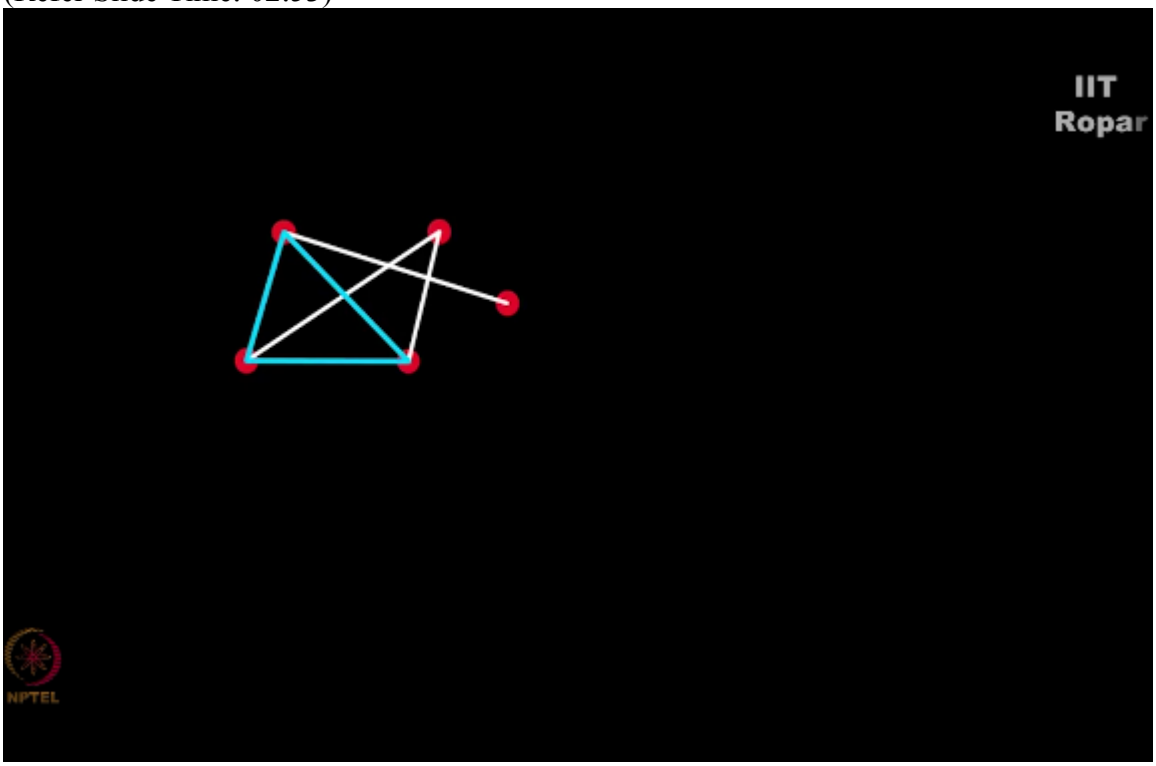


and this is another triangle,
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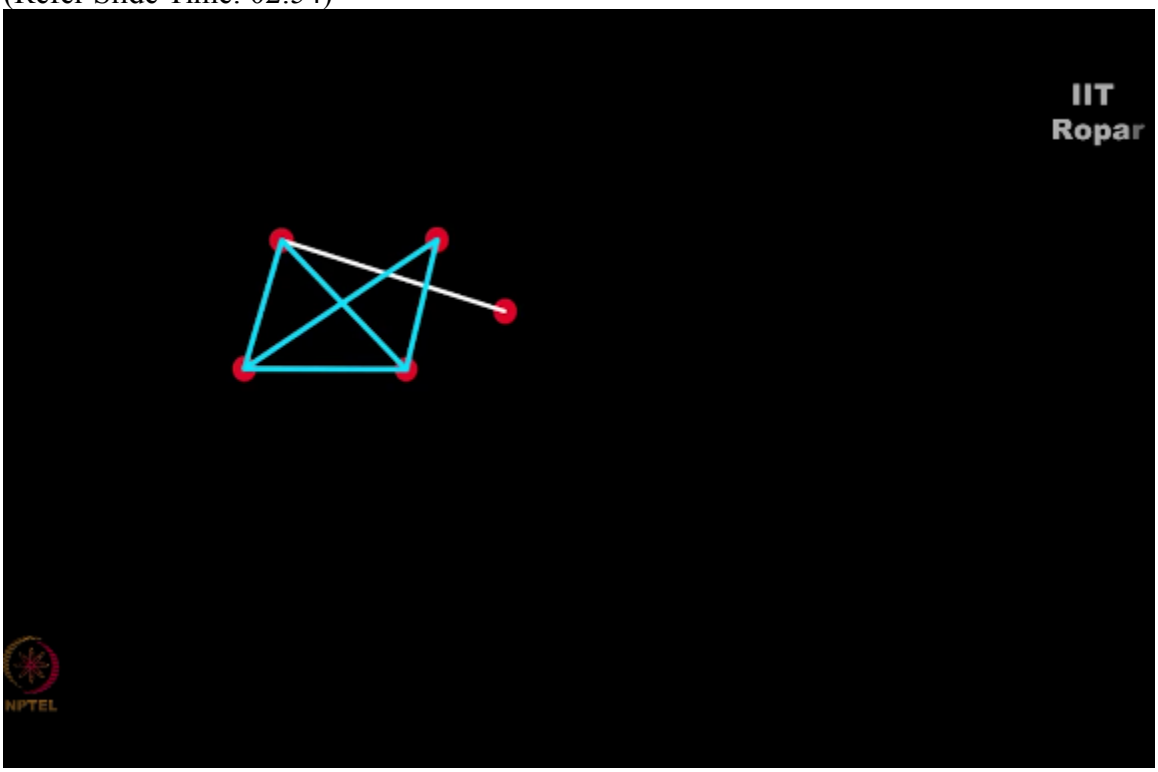


what I do is I neatly shift this triangle downwards and draw it like this,
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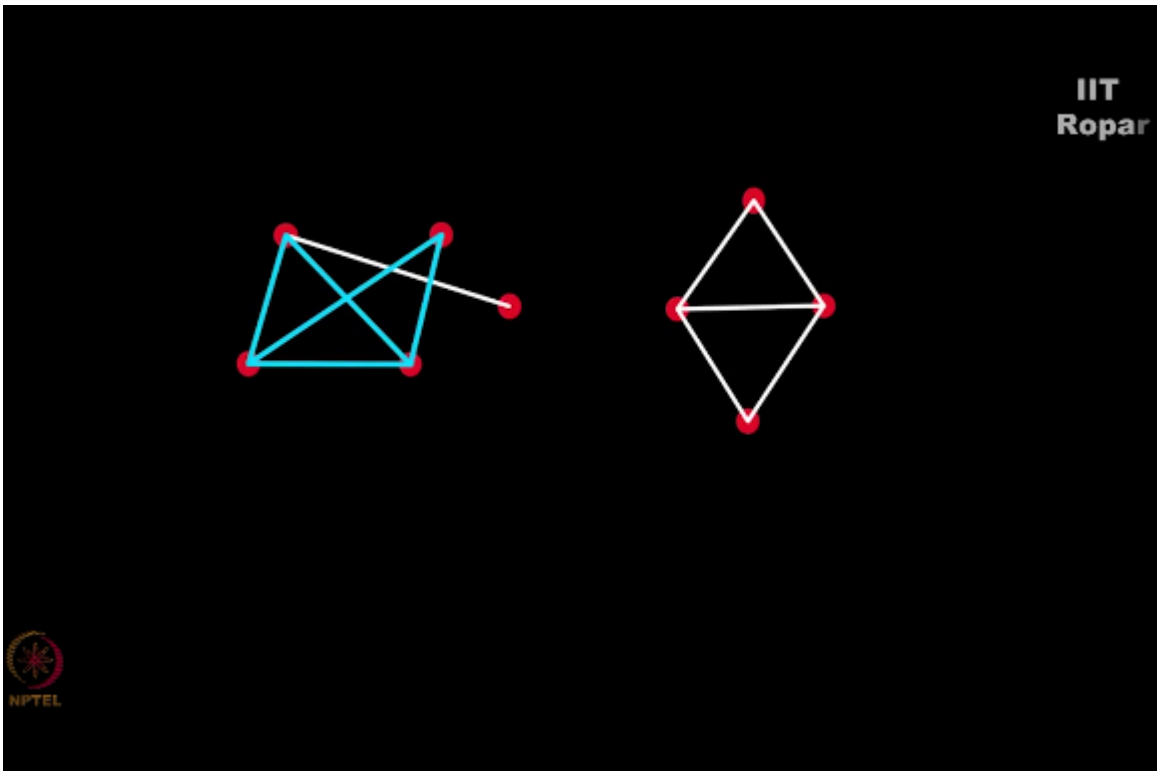


do you see this triangle was just above, I have pulled it down, and this edge and the vertex, this pendant vertex very easily we see that this graph is planar.

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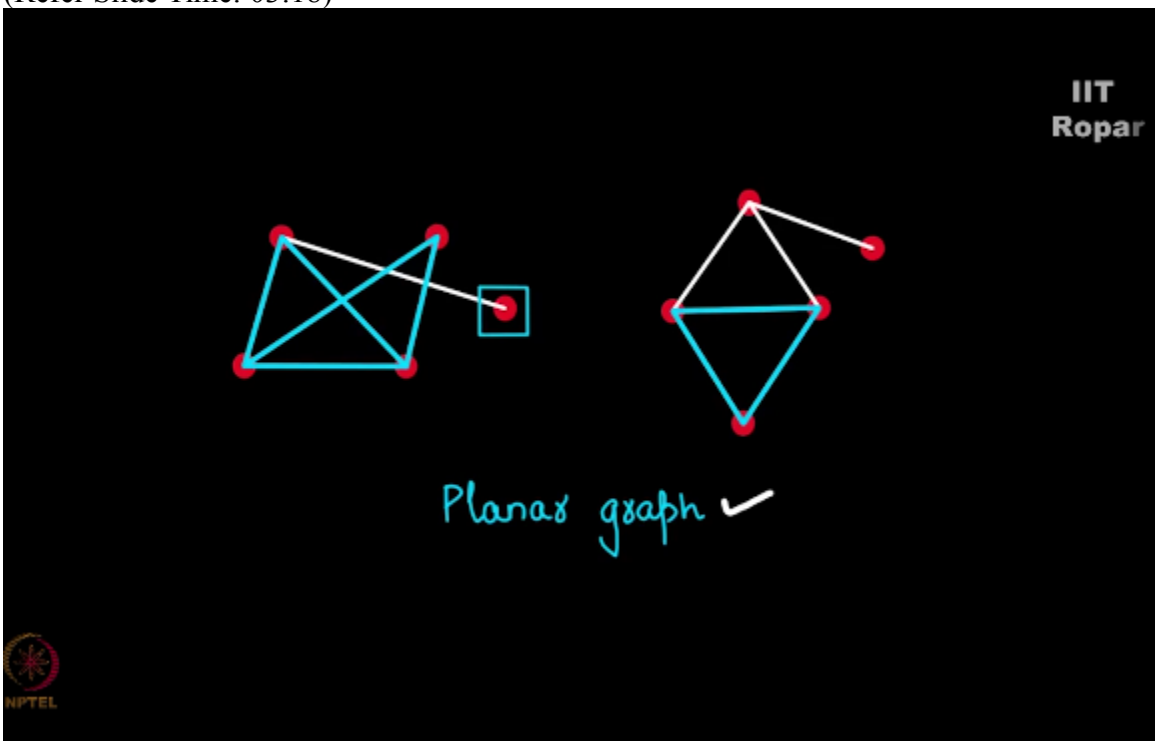


Now consider this cycle C6, cycle on 6 vertices,

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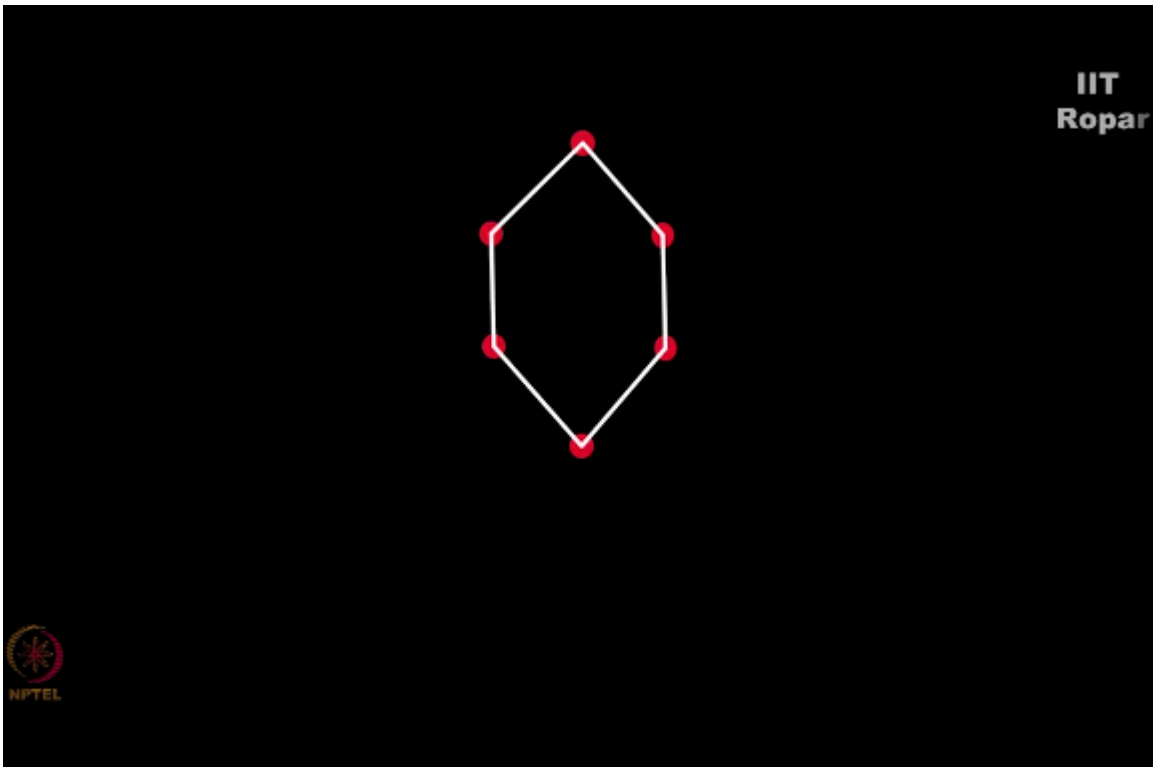


you see it is planar for that matter any cycle is planar, now if I introduce two edges, will this become planar or non-planar? No, I can keep one edge as it is like this,

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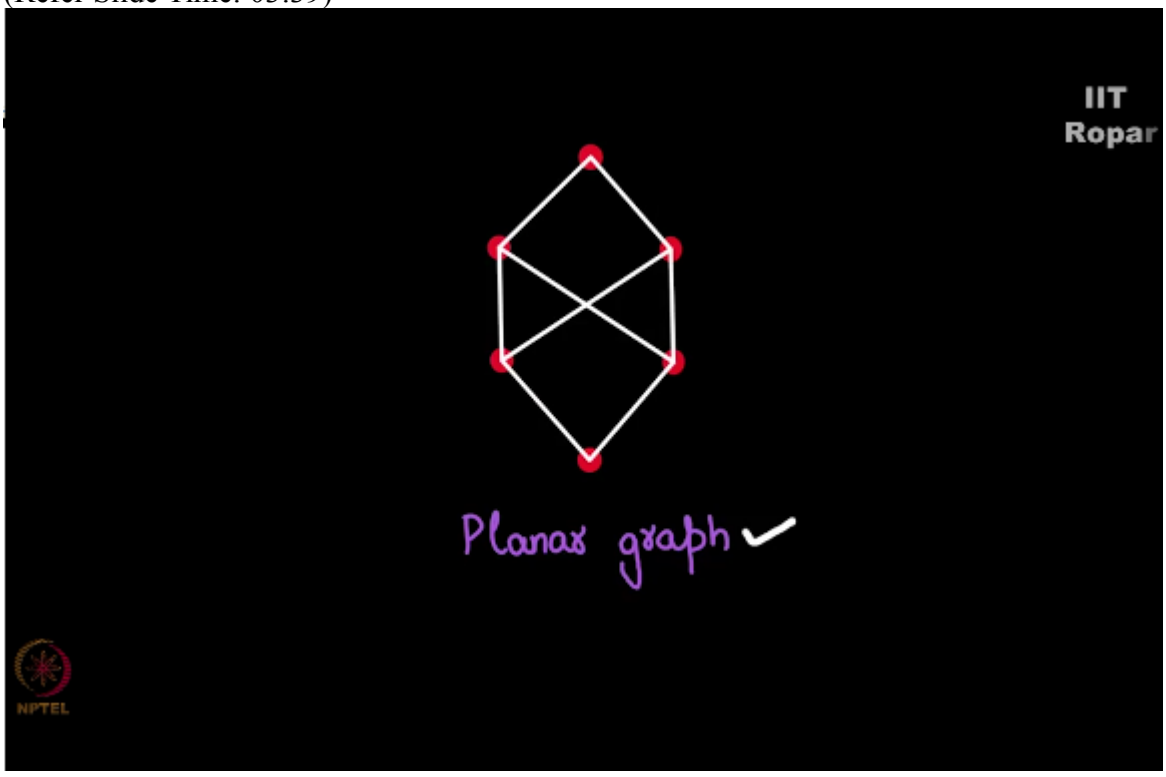


and shift this edge this way,

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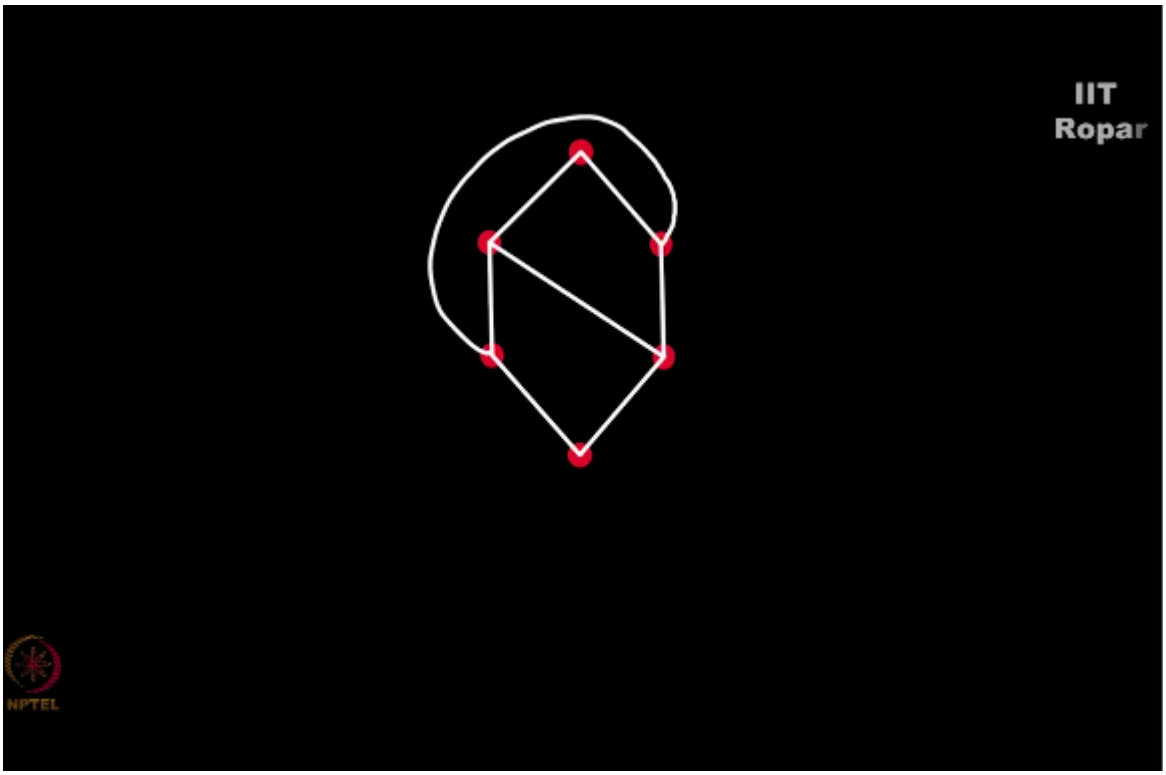


so we see that this graph is planar.