NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

A result on Path

By
Prof. S.R.S Iyengar
Department of Computer Science
IIT Ropar

I'm going to prove a major theorem, a major result before which I am trying to cover all the prerequisite results, the first result is what you saw just now that the degree when it is bounded by n/2, lower bounded by n/2 which means degree of every node is at least n/2, then the graph is connected

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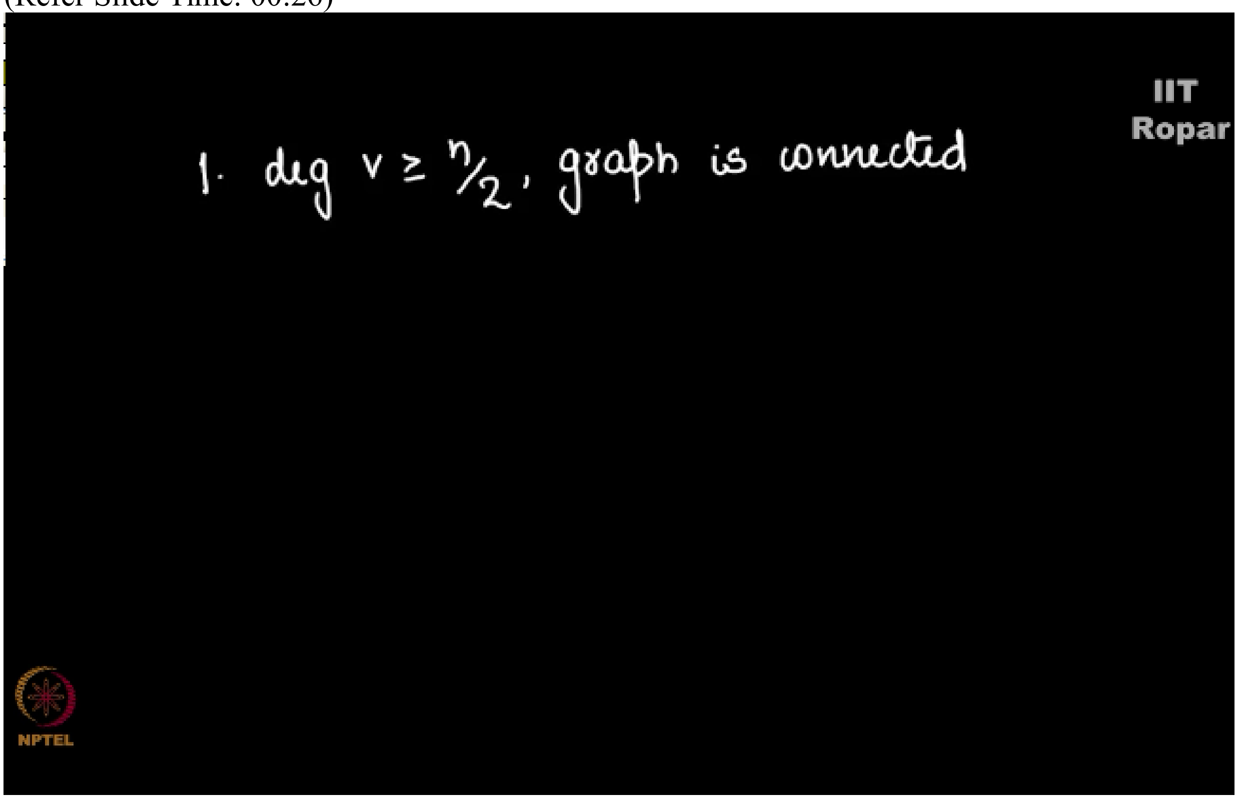

a second point is not a result, but a small observation. Look at this, here is a path of length 3, by length we mean the number of edges in the path as you can note the number of edges is, in a path is always 1 less than the number of vertices, so here is a path with 4 vertices its length is 3, so whenever you take a path like this,

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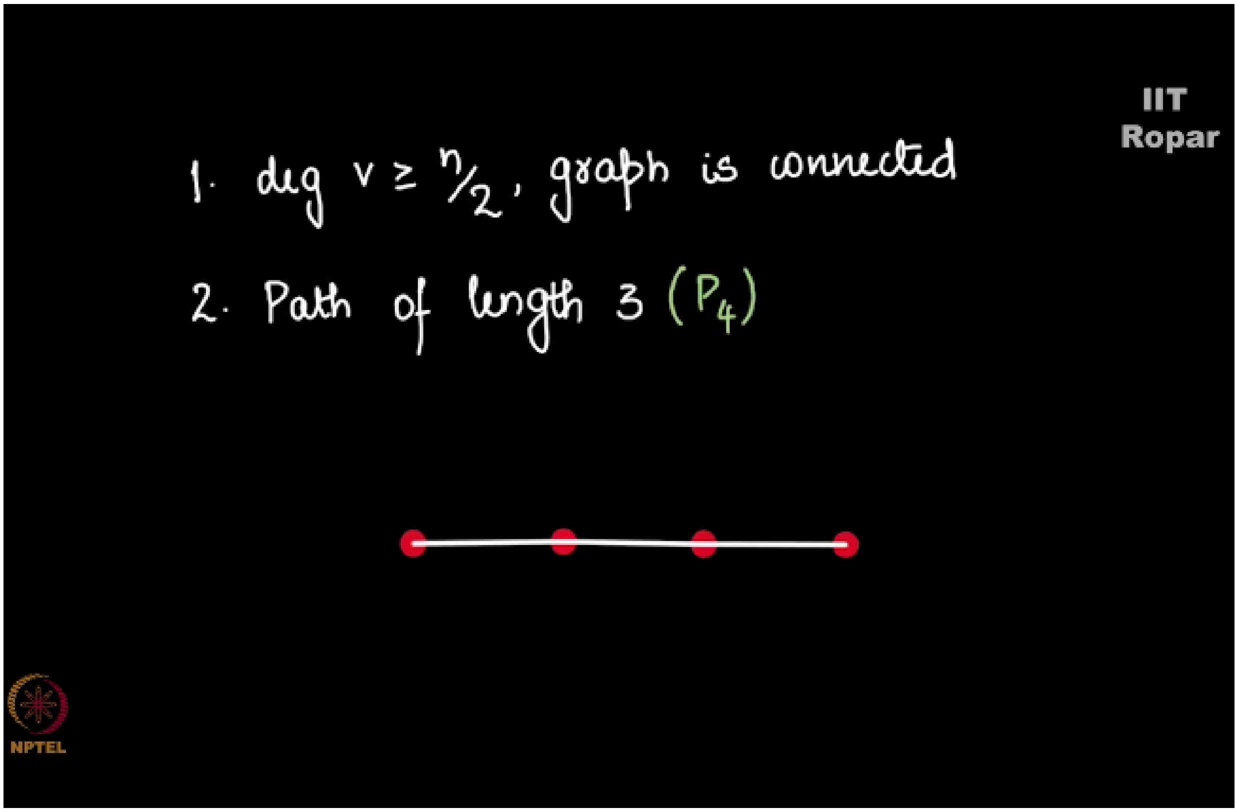

right, if you have an edge let's say from U to V, from V you have an edge to X, and from U you have an edge to Y, do you see what happens,

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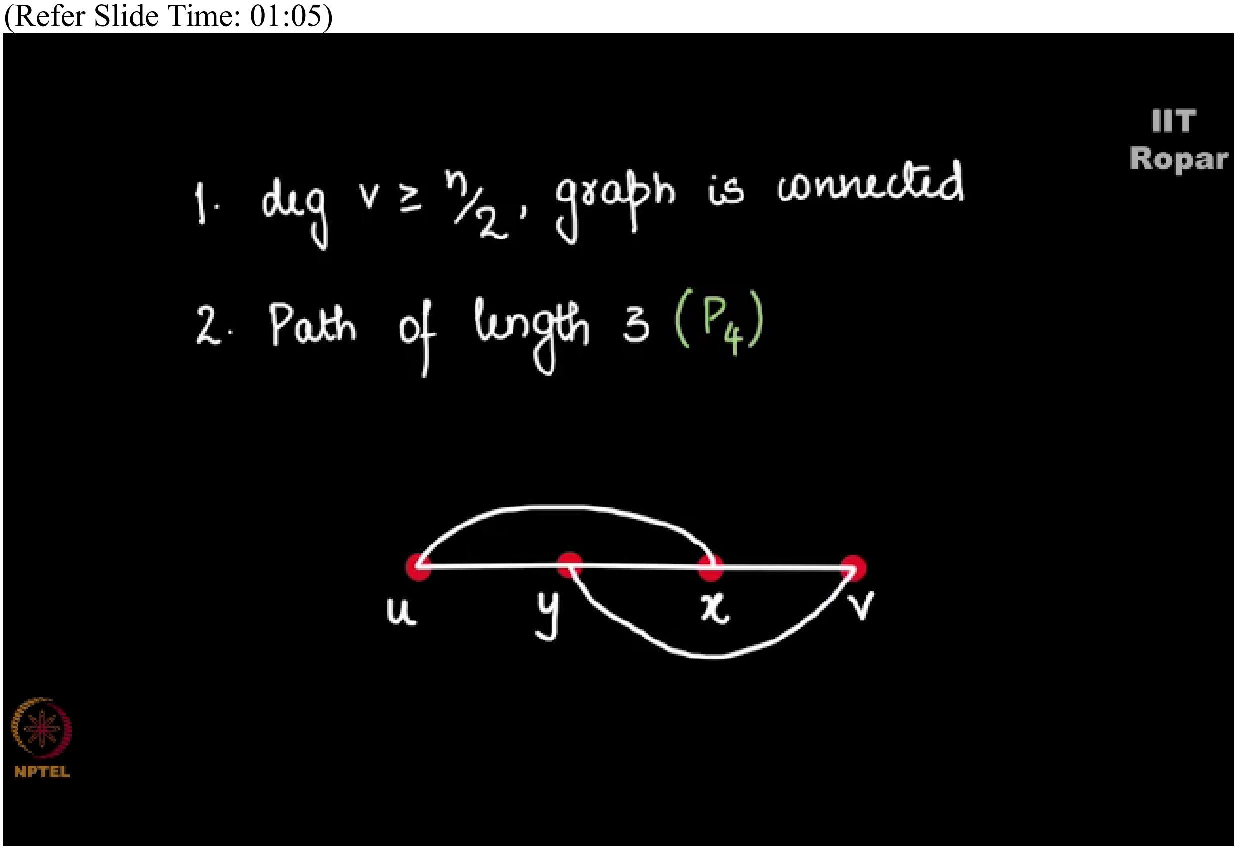

if you remove this edge you will get a cycle U, Y, V, X and U,

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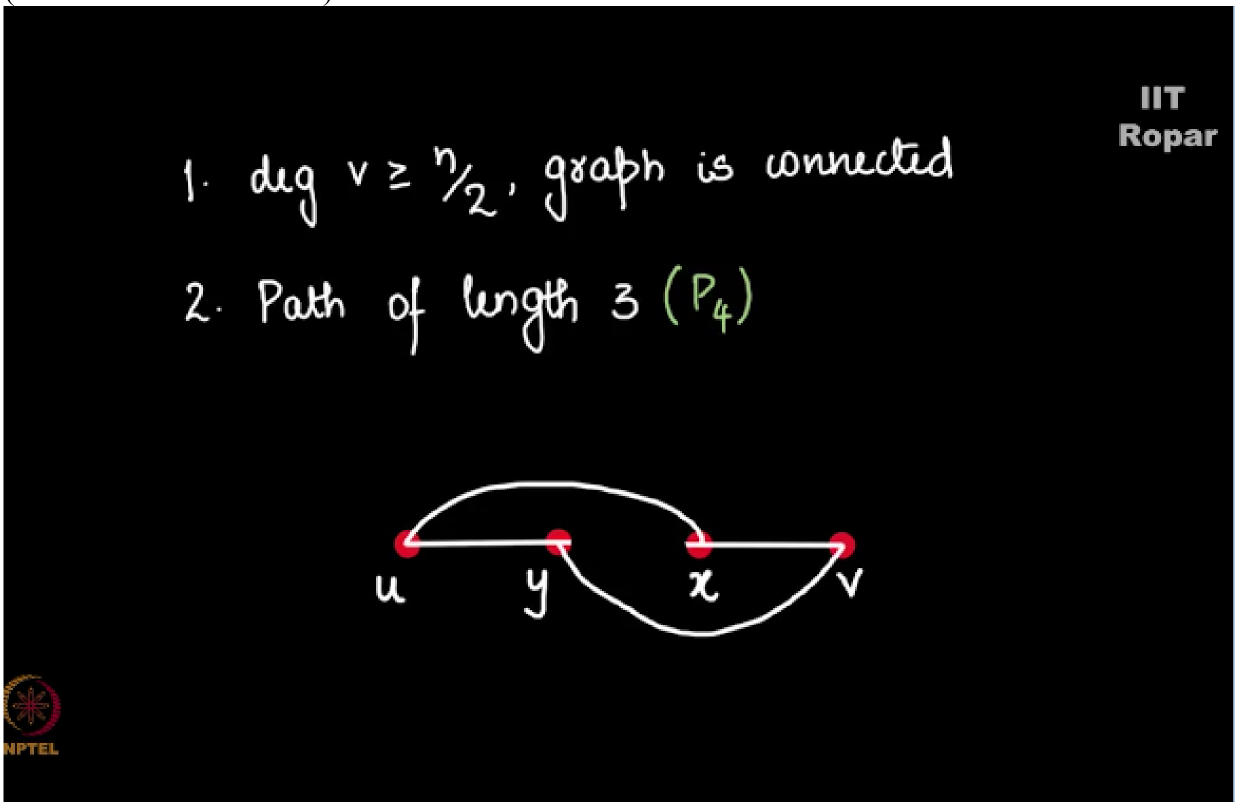

do you see what I say, U,Y, V, X and U,

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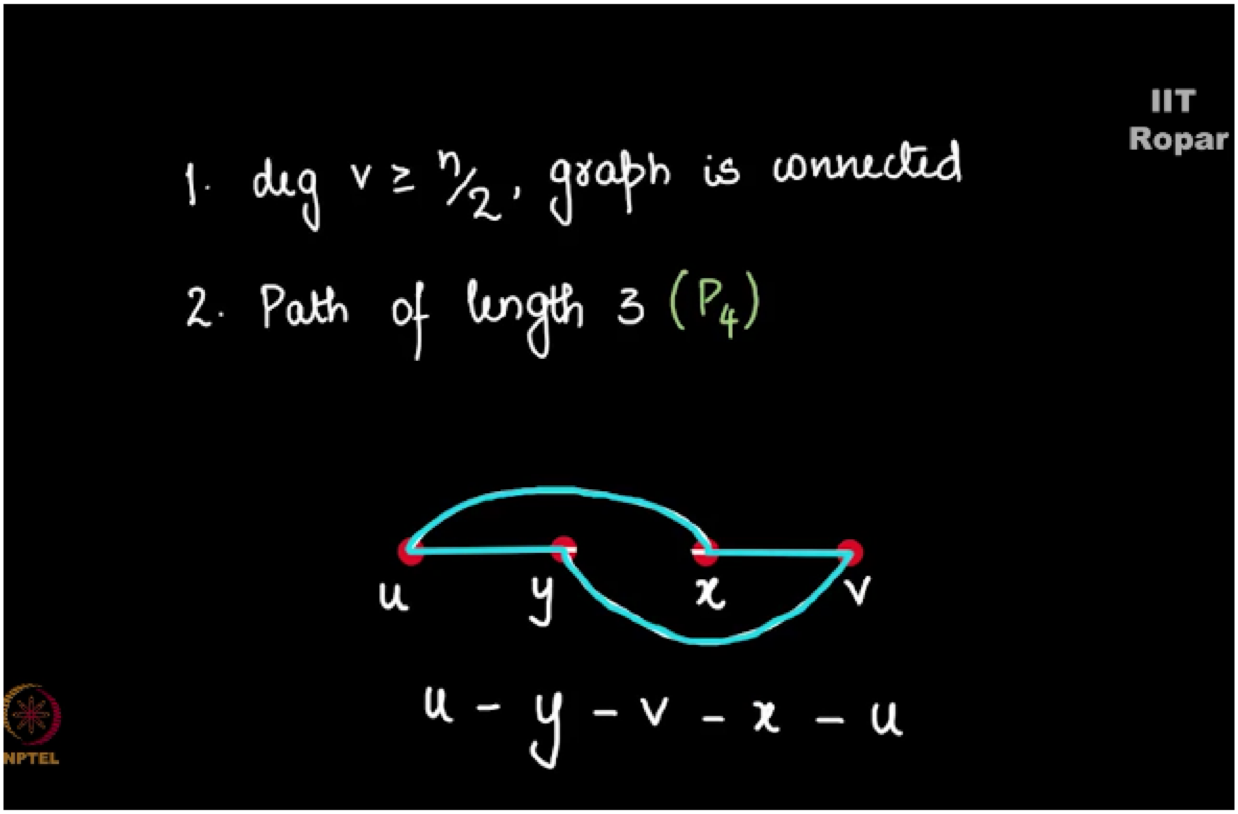

so given any path, path of let's say length 100 if we take U1, U2, U3 so on up to U100,
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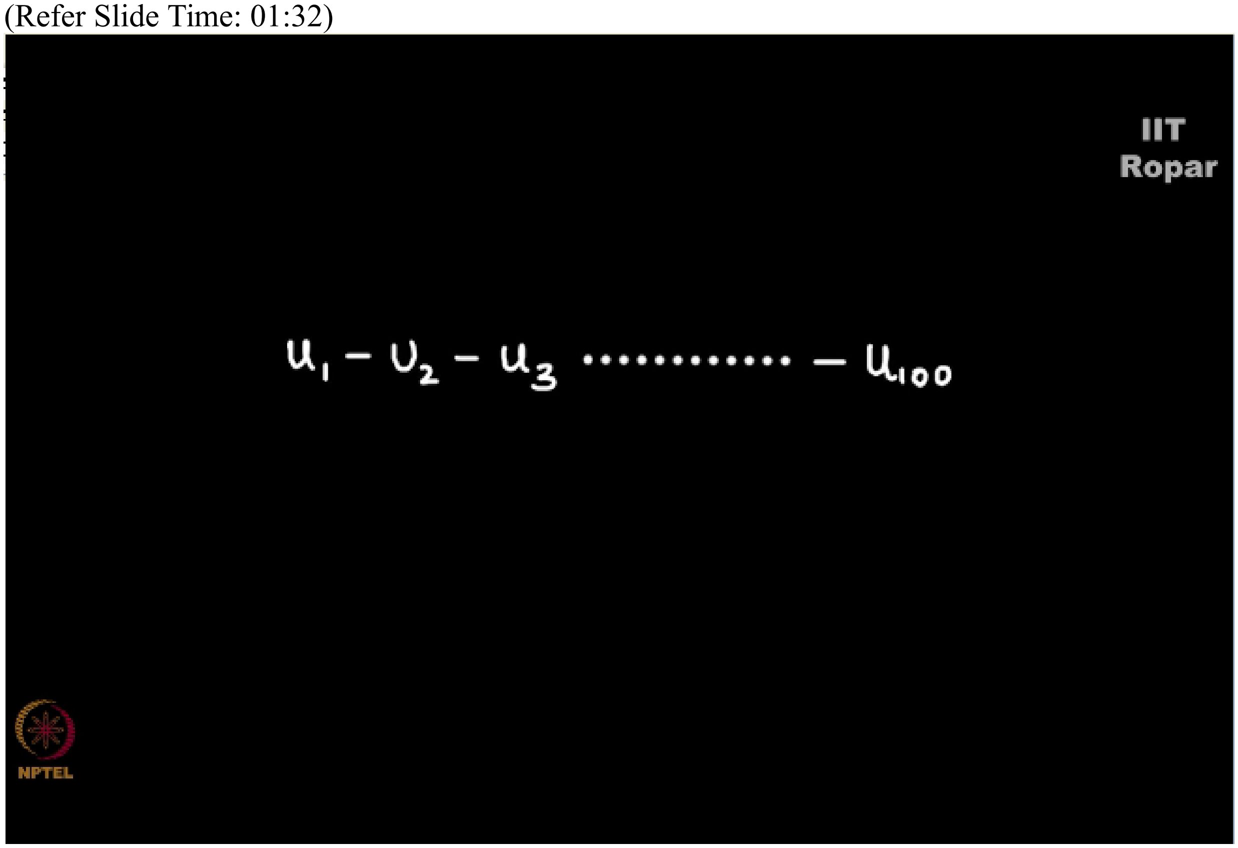

this is a path of length 99 with 100 vertices you will observe that whenever this U100 is adjacent to some vertex, and that vertex, next vertex, by next vertex I mean the successor is adjacent to U1

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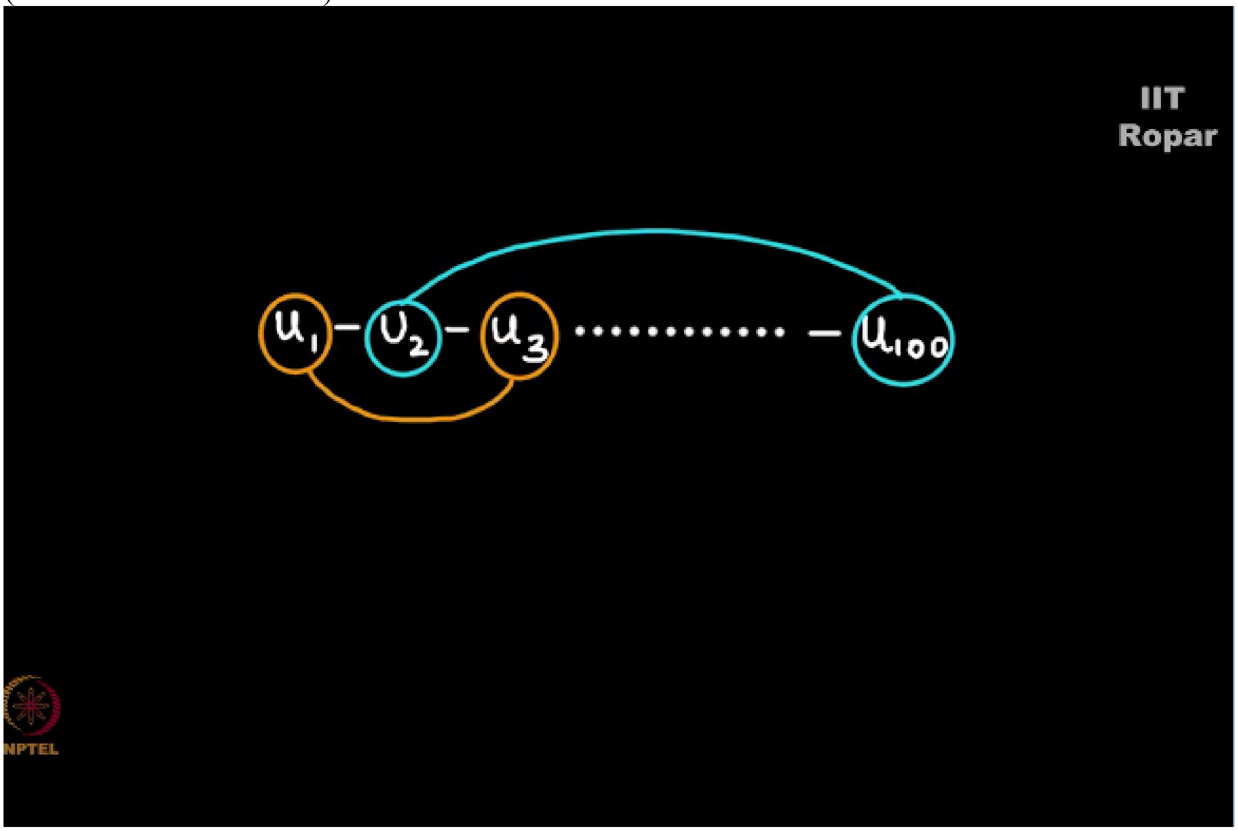

then you can connect these two things and remove this particular edge

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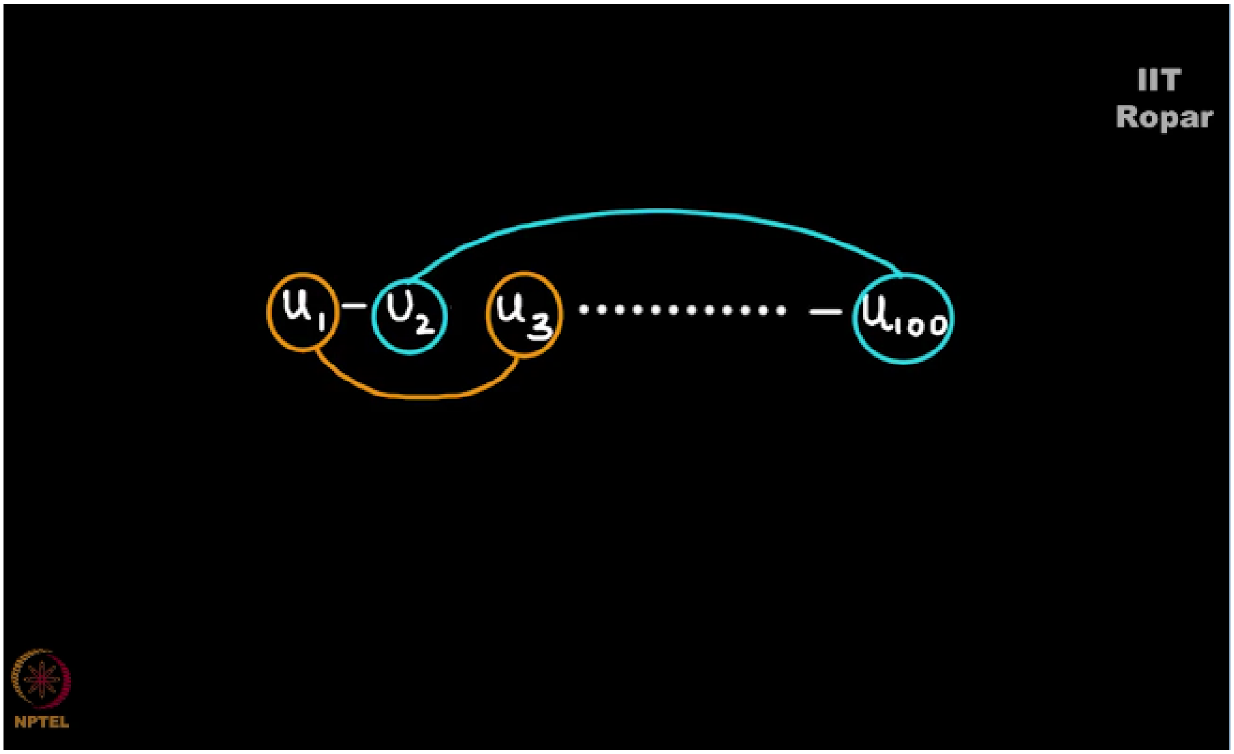

and what you will have is a cycle on these 100 nodes only, look I did not delete any node here, I only deleted the edge here, one edge.

Let me paraphrase my observation whenever in a path the extreme vertices have this property that the last vertex has an edge to some vertex in this path,

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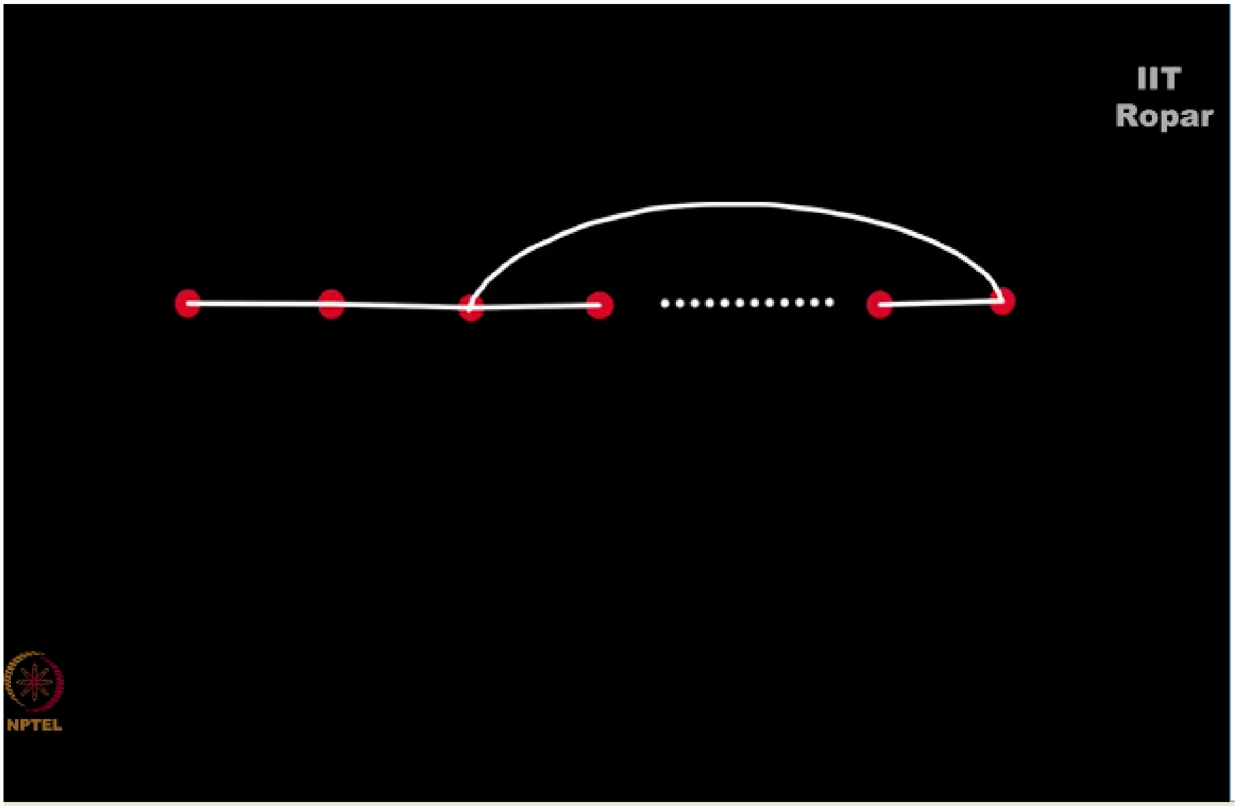

and the beginning vertex has an edge to the immediate successor of this vertex
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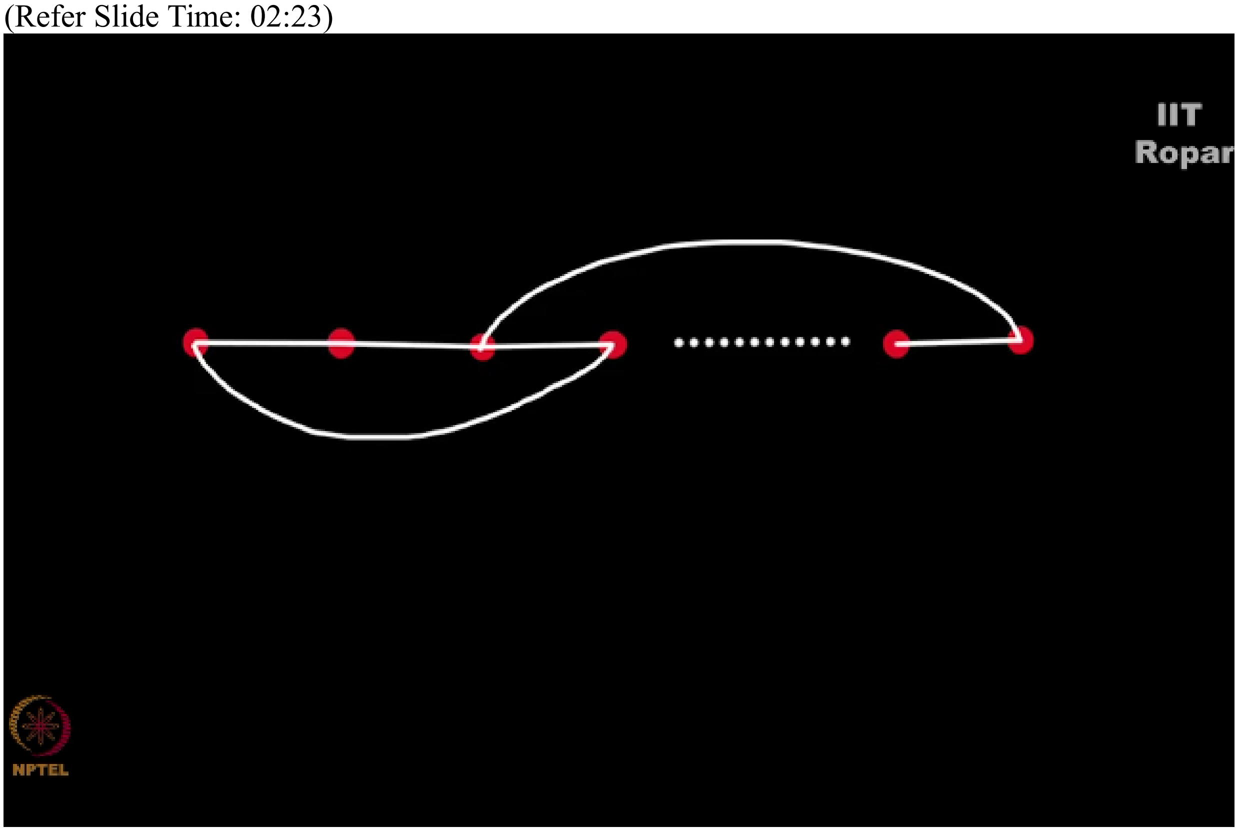

as you can see in this illustration, then you can delete this particular edge and make this a cycle and the cycle will have all the vertices of the path, although in some other order, not necessarily U1, U2, up to U100.

Look at this graph G, it is a graph with several vertices, but do you see there's a cycle here, X1 X2, X3, X4, X5 and then again X1,
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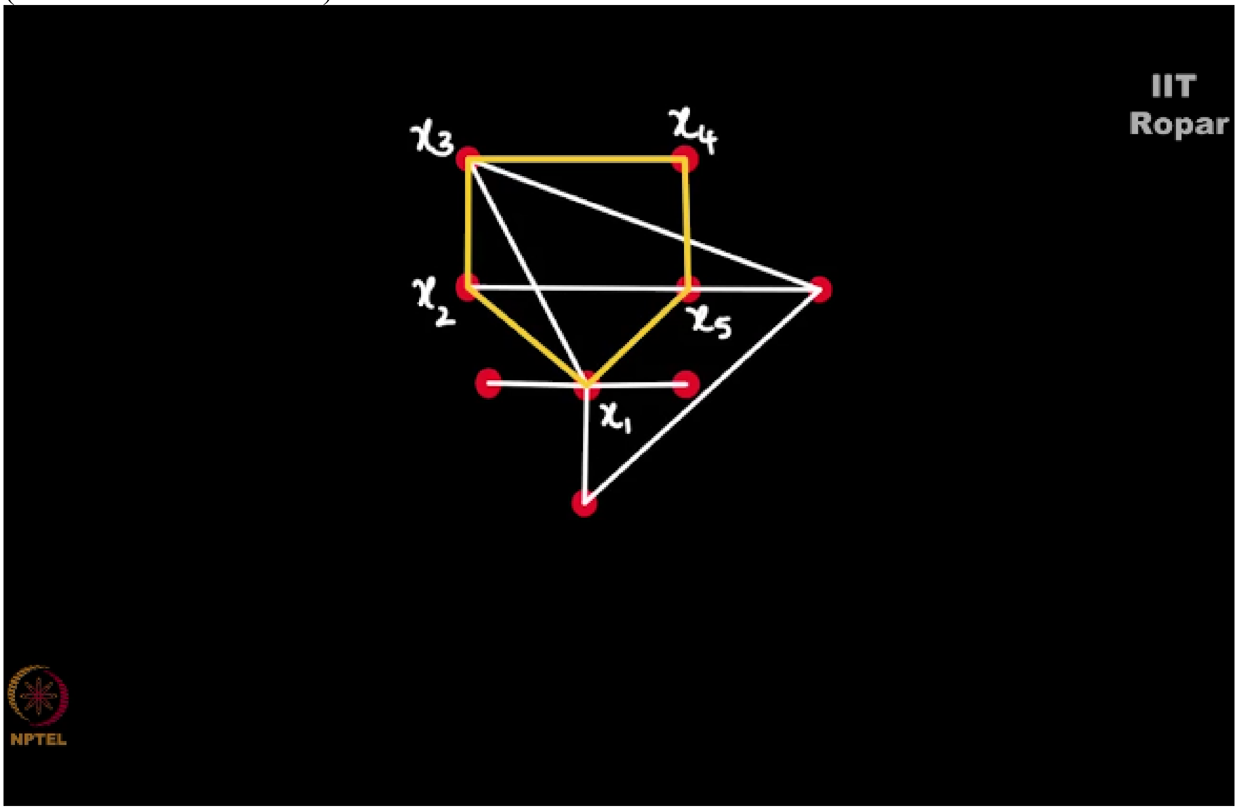

correct, there are 5 vertices and hence 5 edges in the cycle, but the graph is bigger than just 5 vertices as you can see.

Now what do I observe? I take another vertex X6 slightly far away from the cycle,
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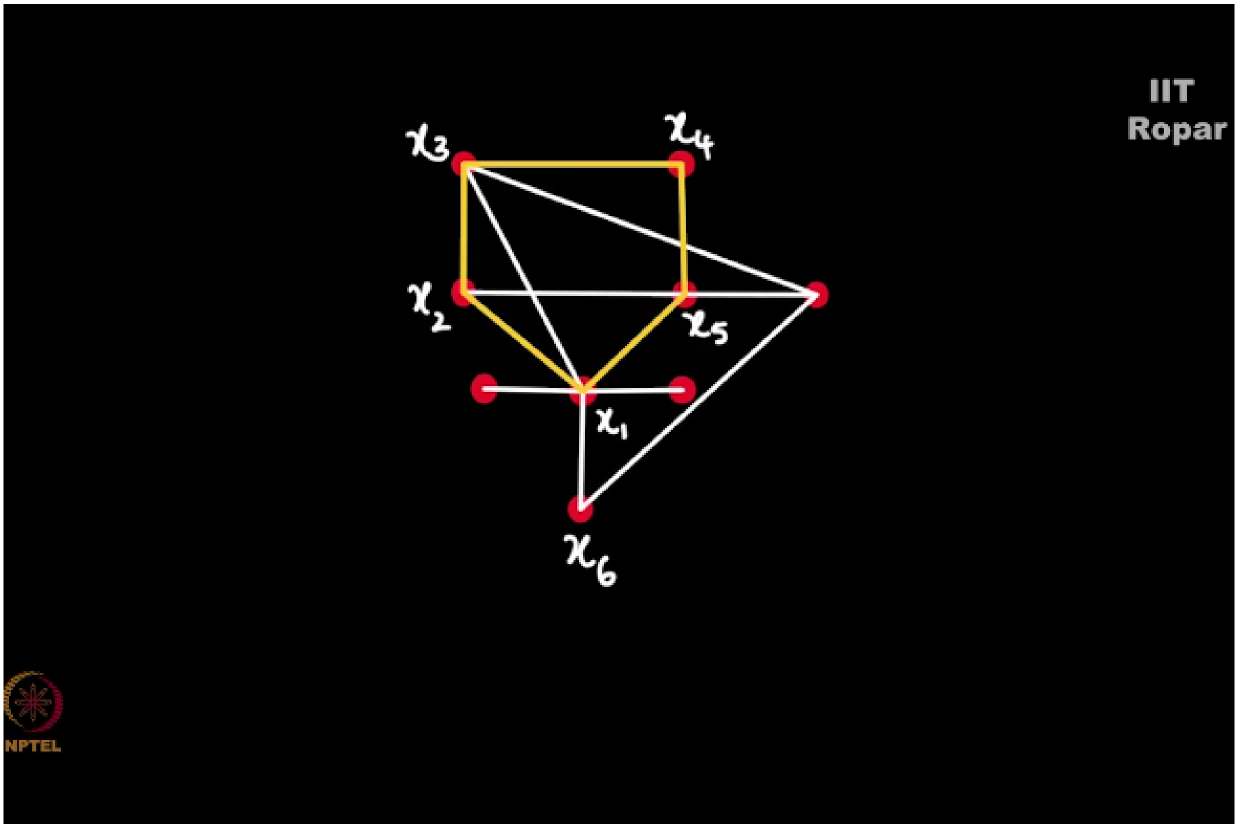

and you see what I can do, X6 is adjacent to X7, and X7 is adjacent to X3, now what I do is I remove this edge X2, X3,

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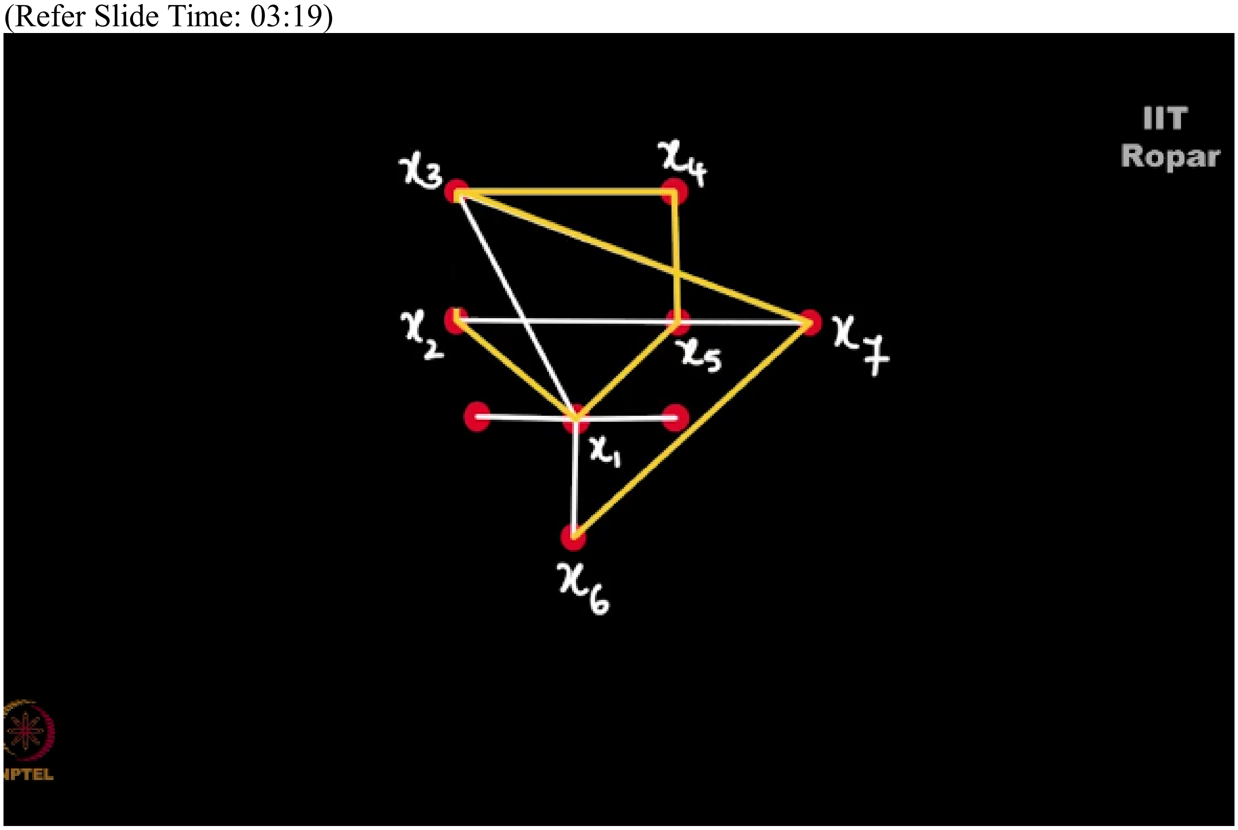

now I have got a path bigger than the given cycle, right, there are more vertices than the vertices in the cycle, now let me paraphrase this little result that we have discovered, given a graph G if there is a cycle of length K, by that I mean a cycle with K vertices then it implies that you can find a path of length at least K+1,

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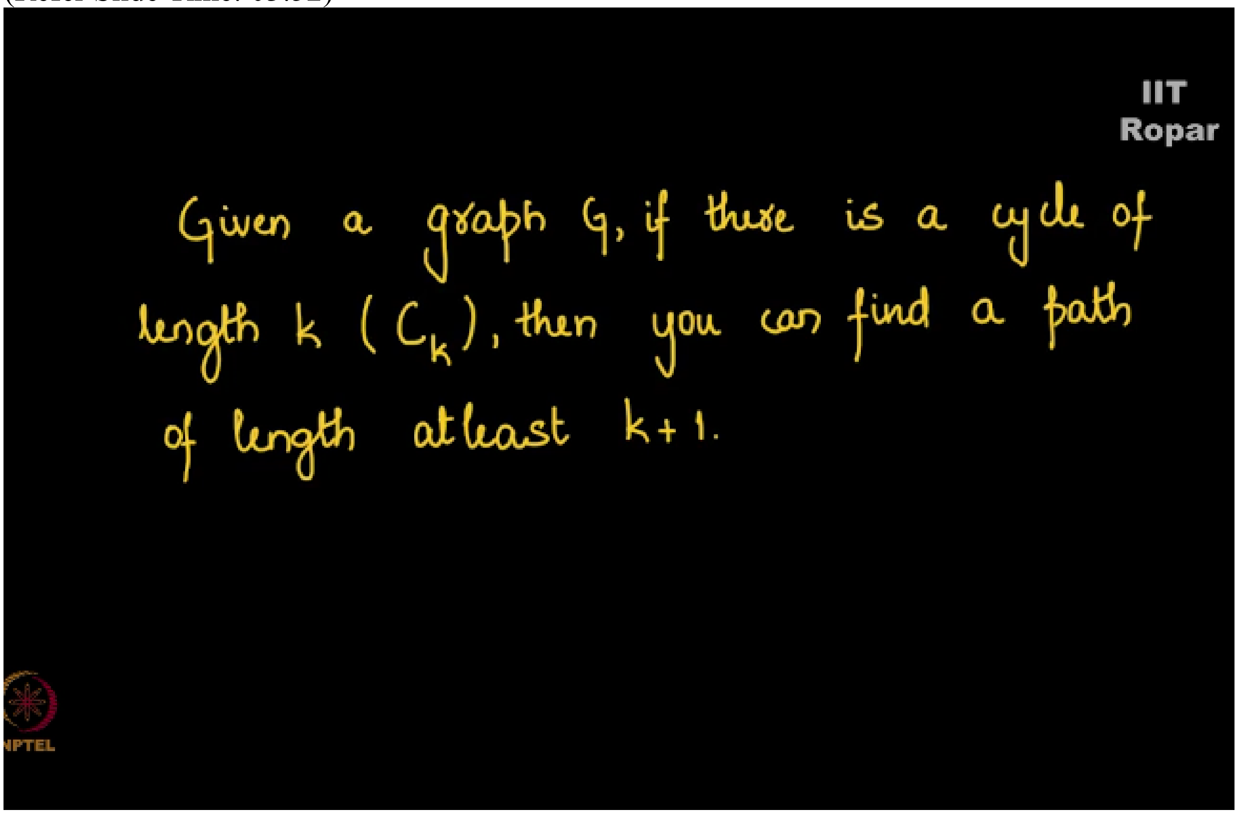

by that I mean you will go ahead and find out one more vertex and make this cycle become a path with one vertex more at least, you see the point, right and it is a pretty straightforward point, but then you might be wondering what if the cycle covers all the vertices then it is not true, so the theorem states given a graph G with N elements if there is a cycle of length K, and K is less than N then you can find a path of length K+1 correct, so this is true only if the graph is connected you all know that, right, that goes without saying.

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