NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory – 2

Examples of Hamiltonian graphs

By
Prof. S.R.S Iyengar
Department of Computer Science
IIT Ropar

Let us see some examples of Hamiltonian graphs,
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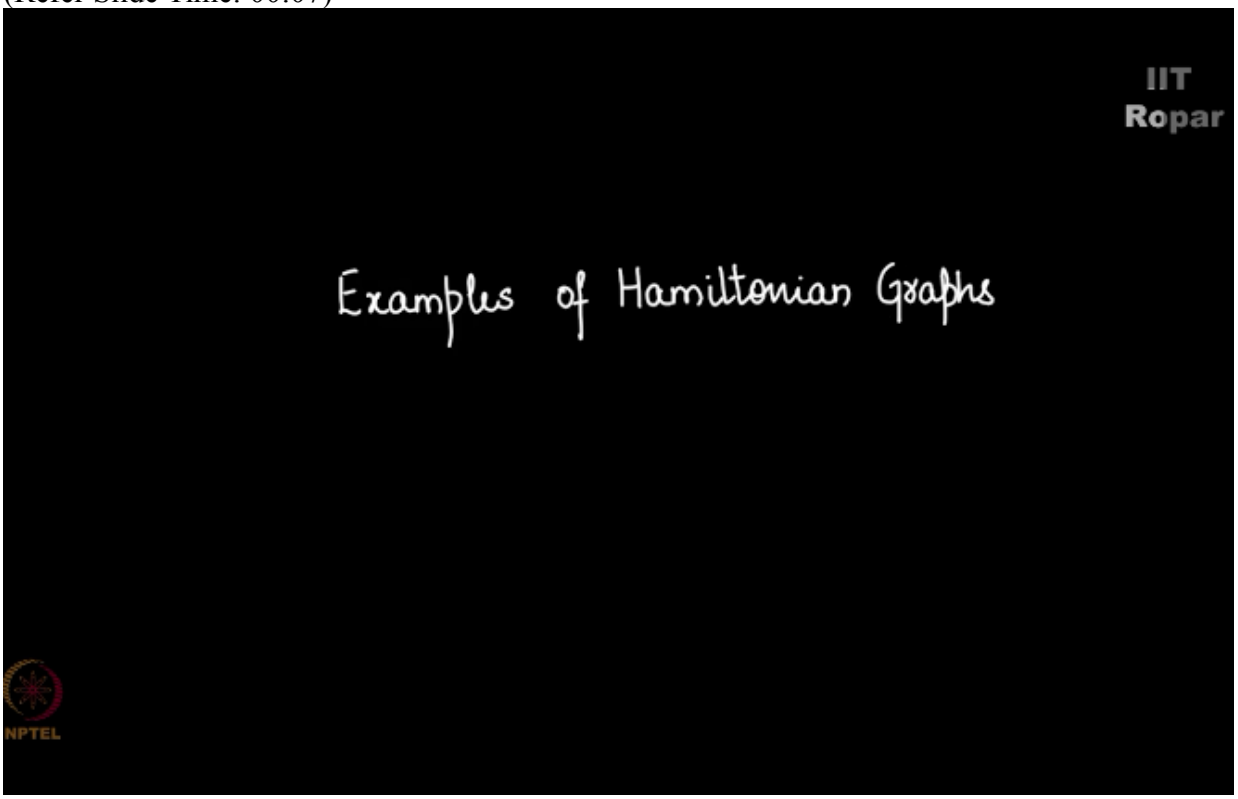

a few graphs will be displayed and we will check if we can find out a Hamiltonian cycle in this graph.

Consider this one as the first example,
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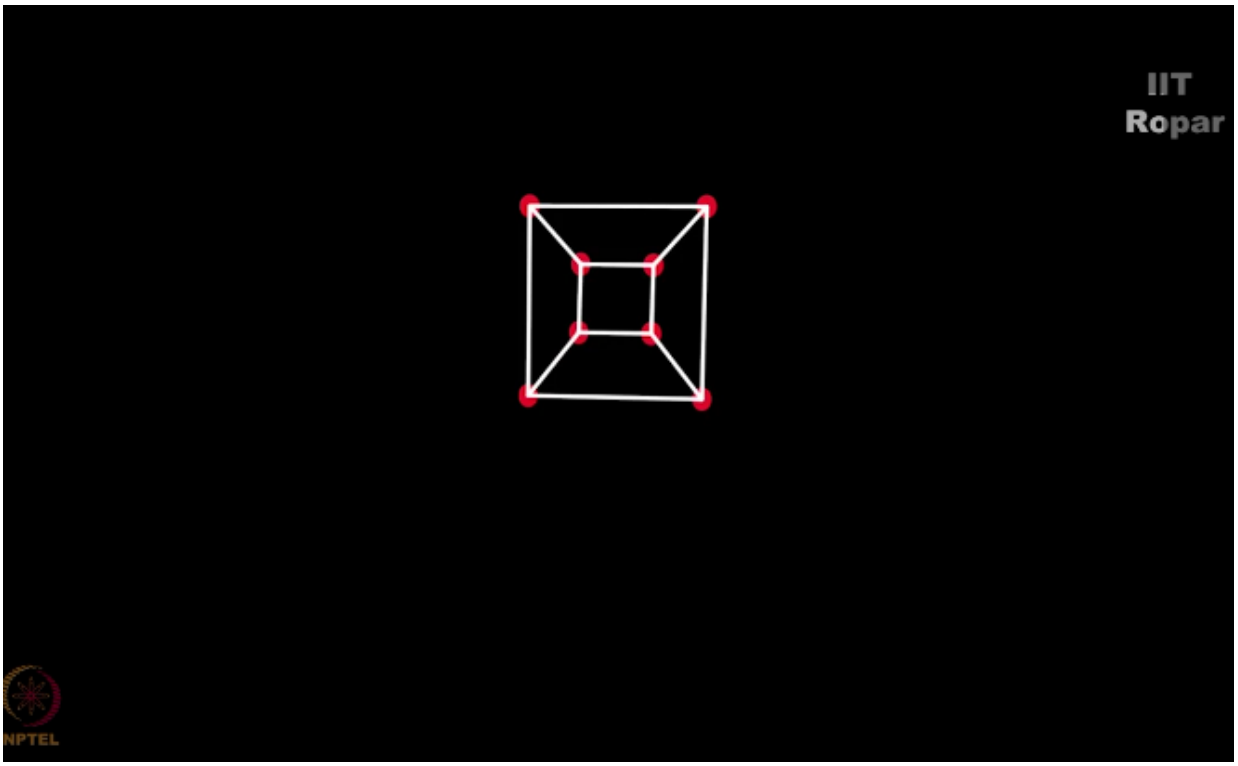

I'll label the vertices as A, B, C, D, U, V, X, and Y,

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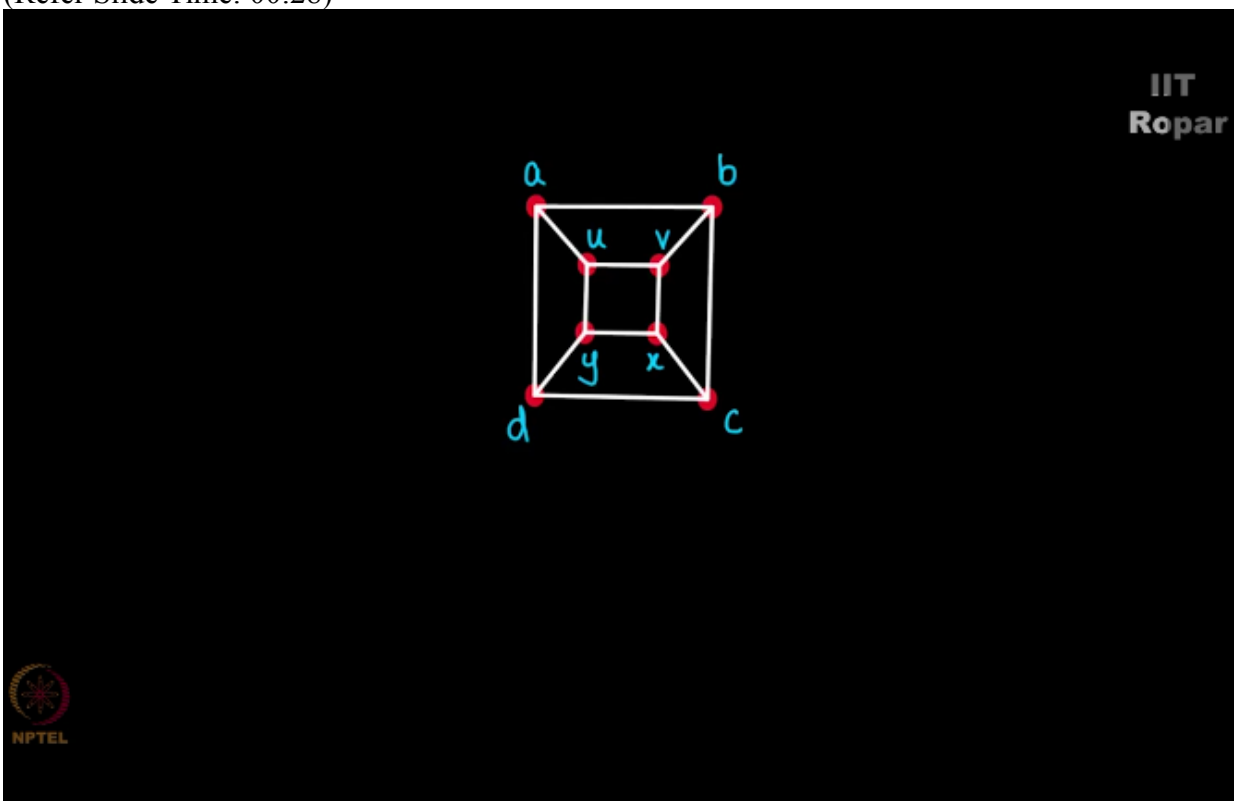

now let me take the path starting from A go through all the vertices and I'll come back to A, let me see if I can do this, I'll go like this A to B

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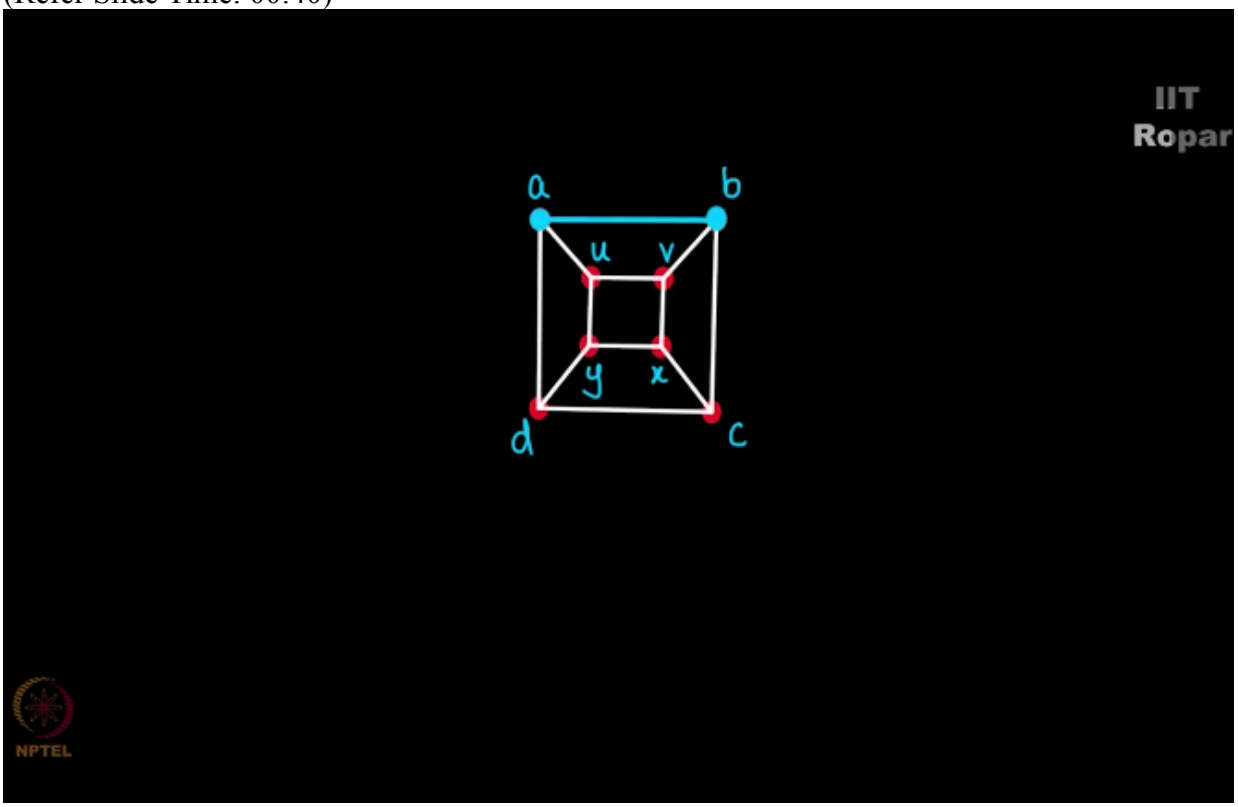

B to C, C to X, X to V, V to U, U to Y, Y to D, and then D to A,

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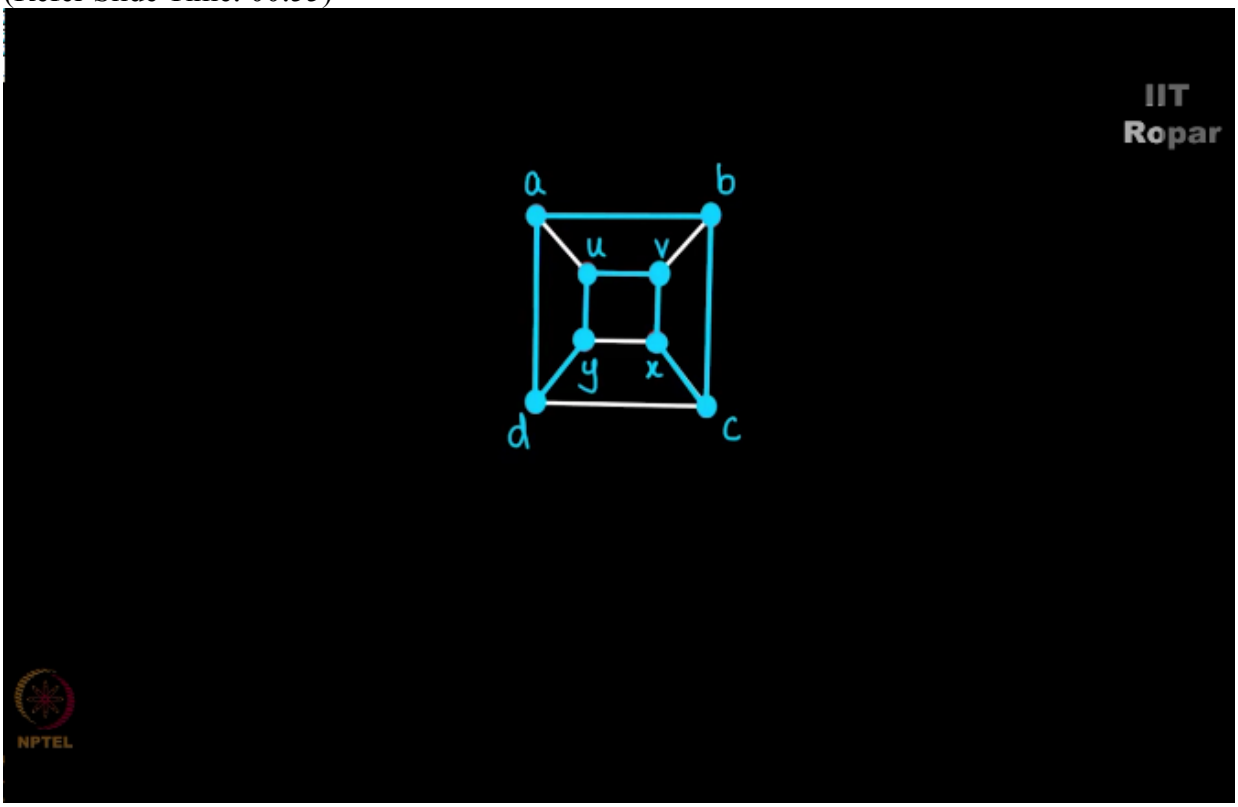

do you see I have covered all the vertices here? Though I haven't covered this edge X, Y, and this edge A, U
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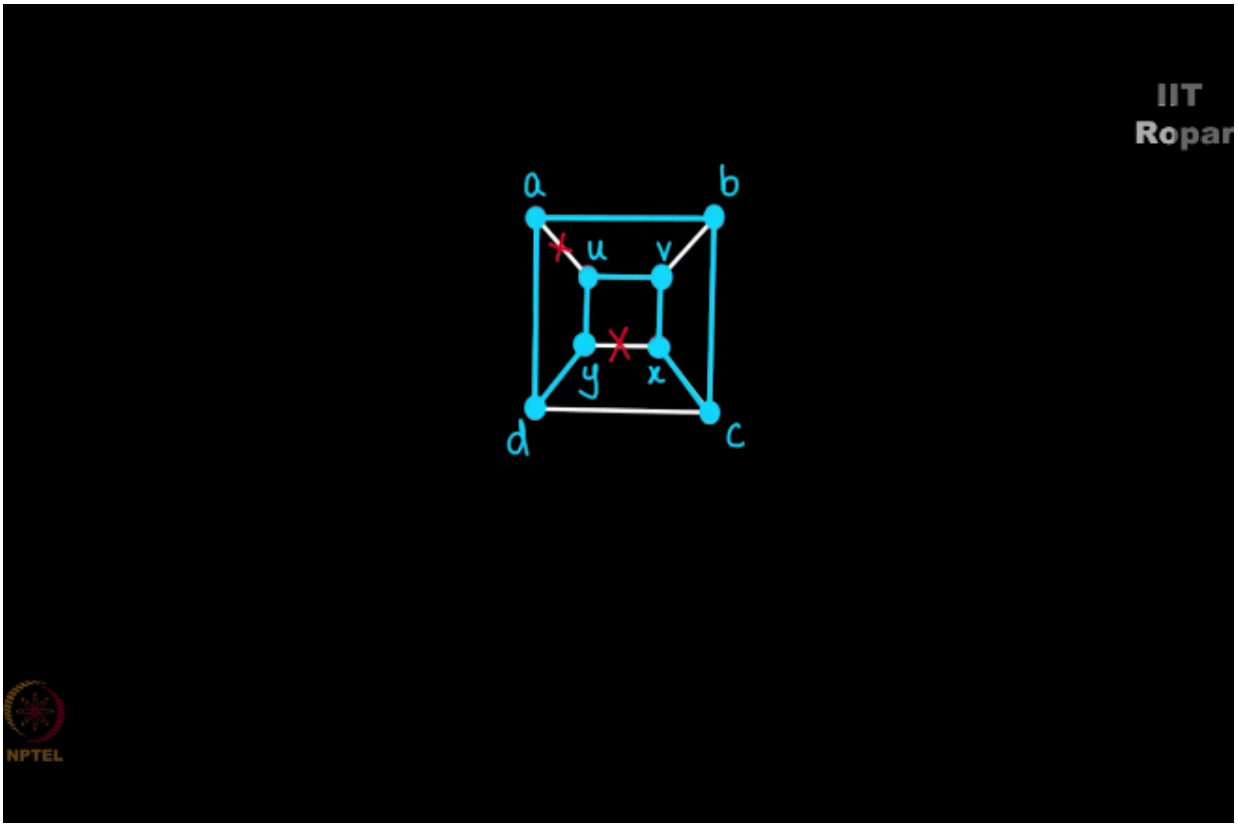

it does not matter, I have covered all the vertices, and hence this graph is Hamiltonian.
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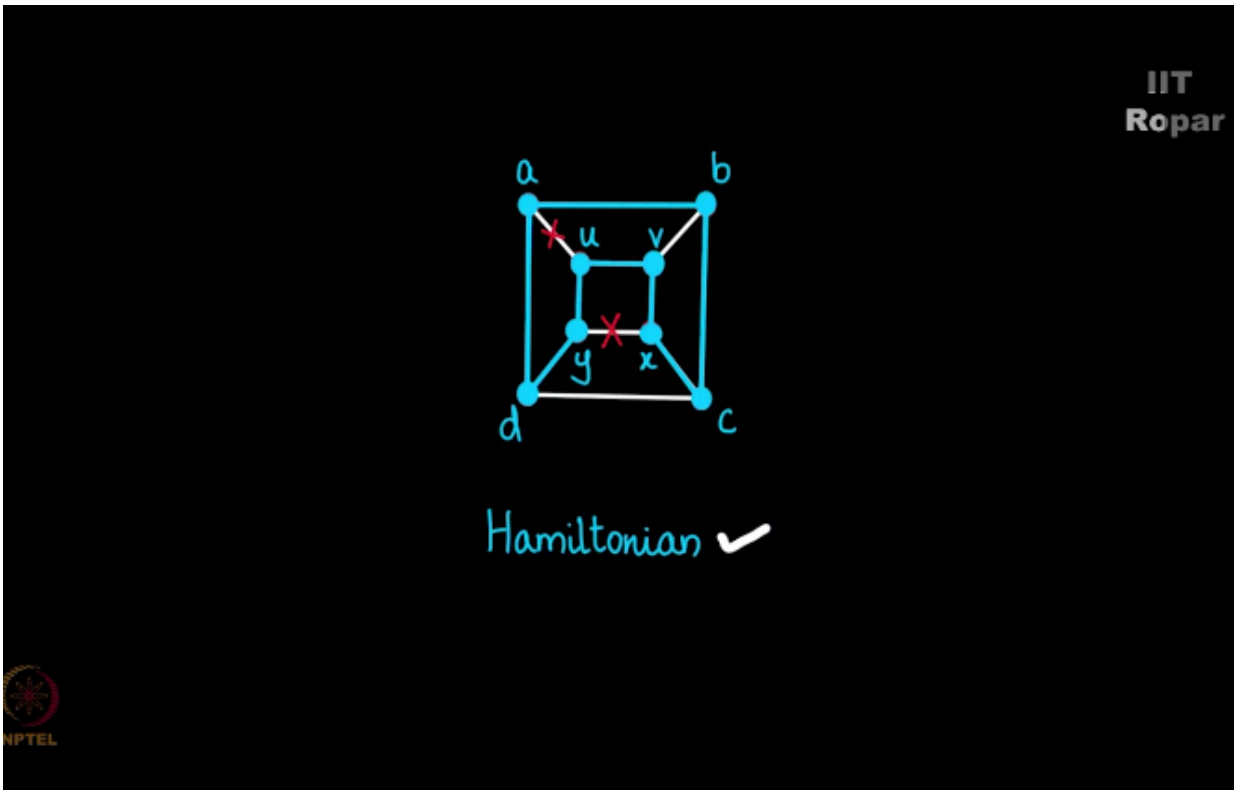

Consider this graph, you must be knowing that this is a case X, complete graph on 6 vertices
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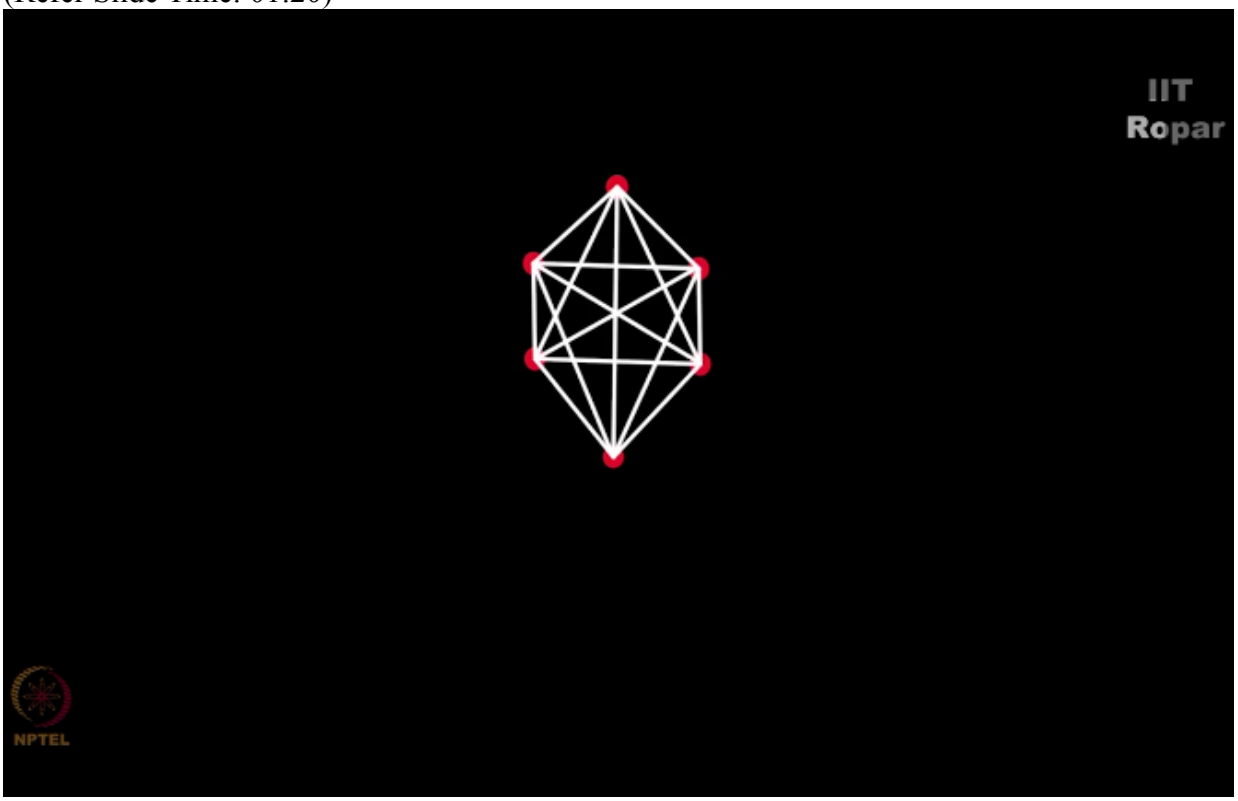

let me label the vertices as 1, 2, 3, 4, 5, and 6
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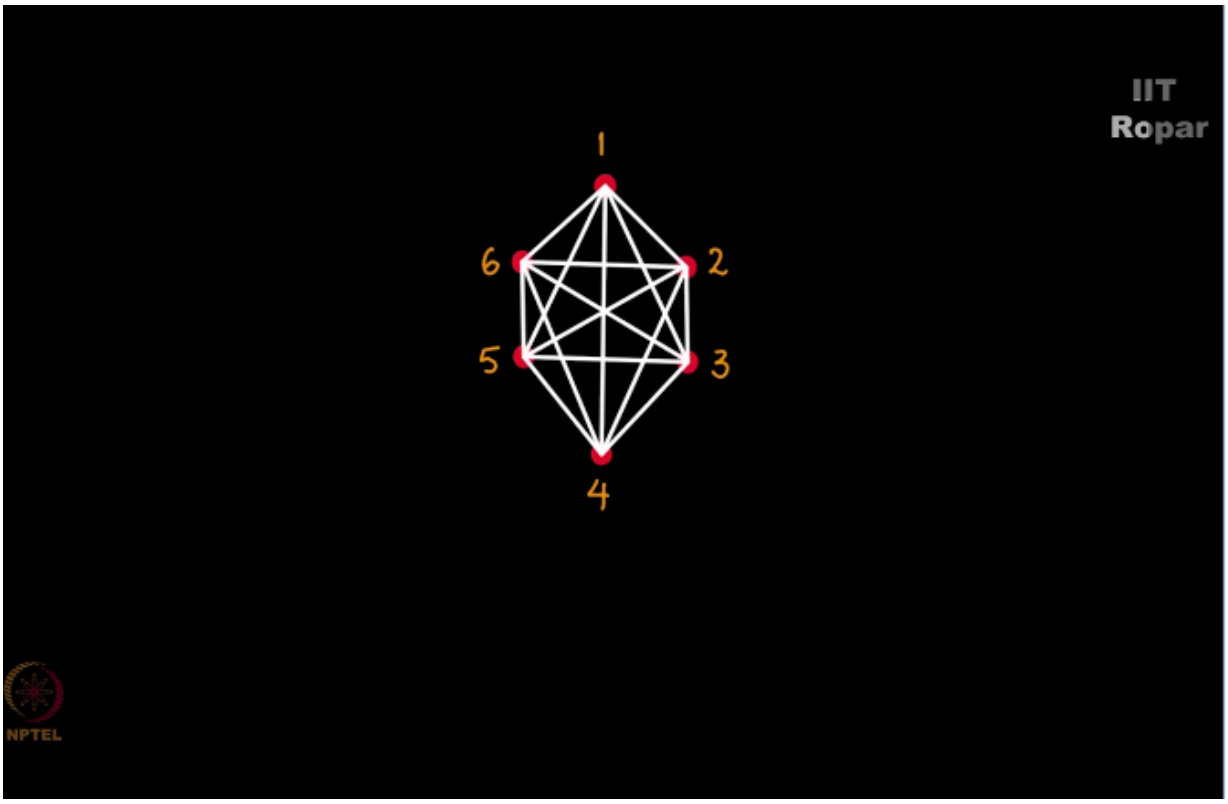

I'll start from 1 go to 2, 2 to 3, 3 to 4, 4 to 5, 5 to 6

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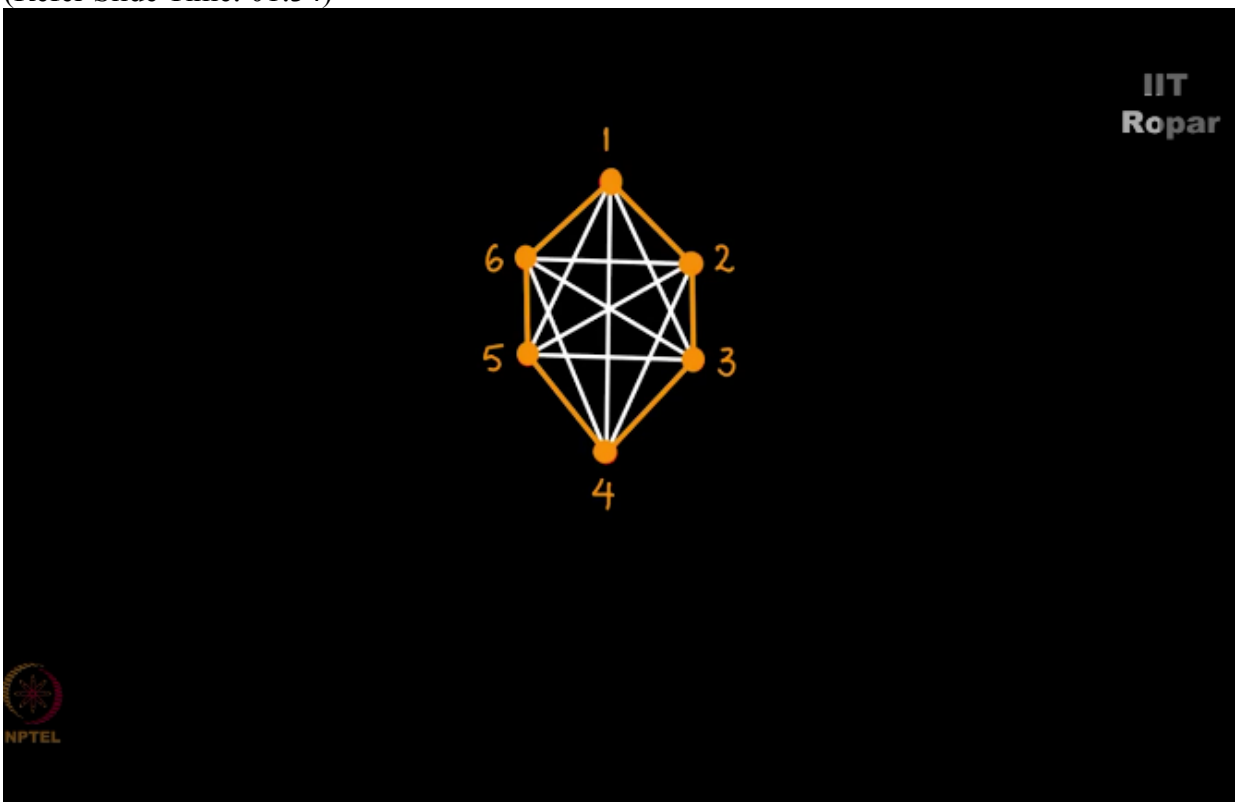

I haven't covered a few edges in middle but have covered all the vertices and the graph is Hamiltonian.
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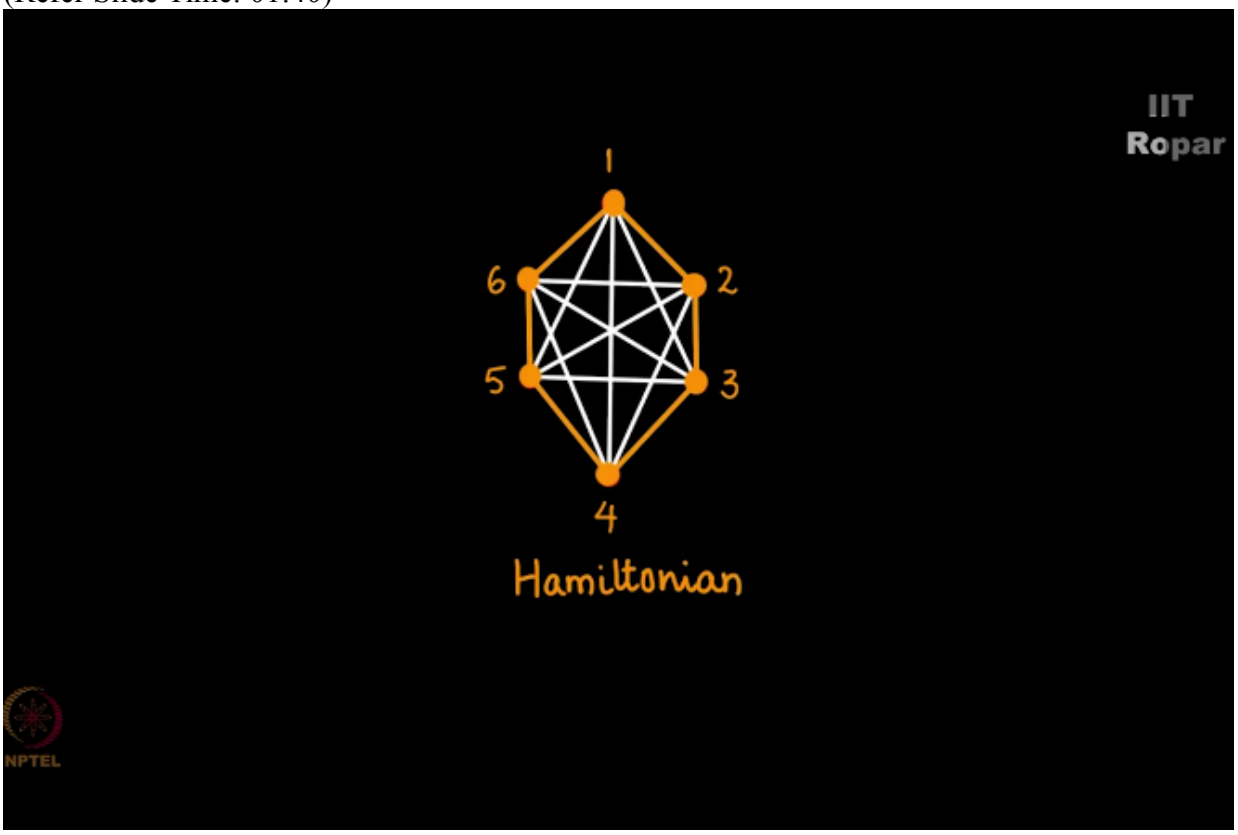

Now consider this example, you see the graph looks quite complicated, let us see if it is Hamiltonian or not?
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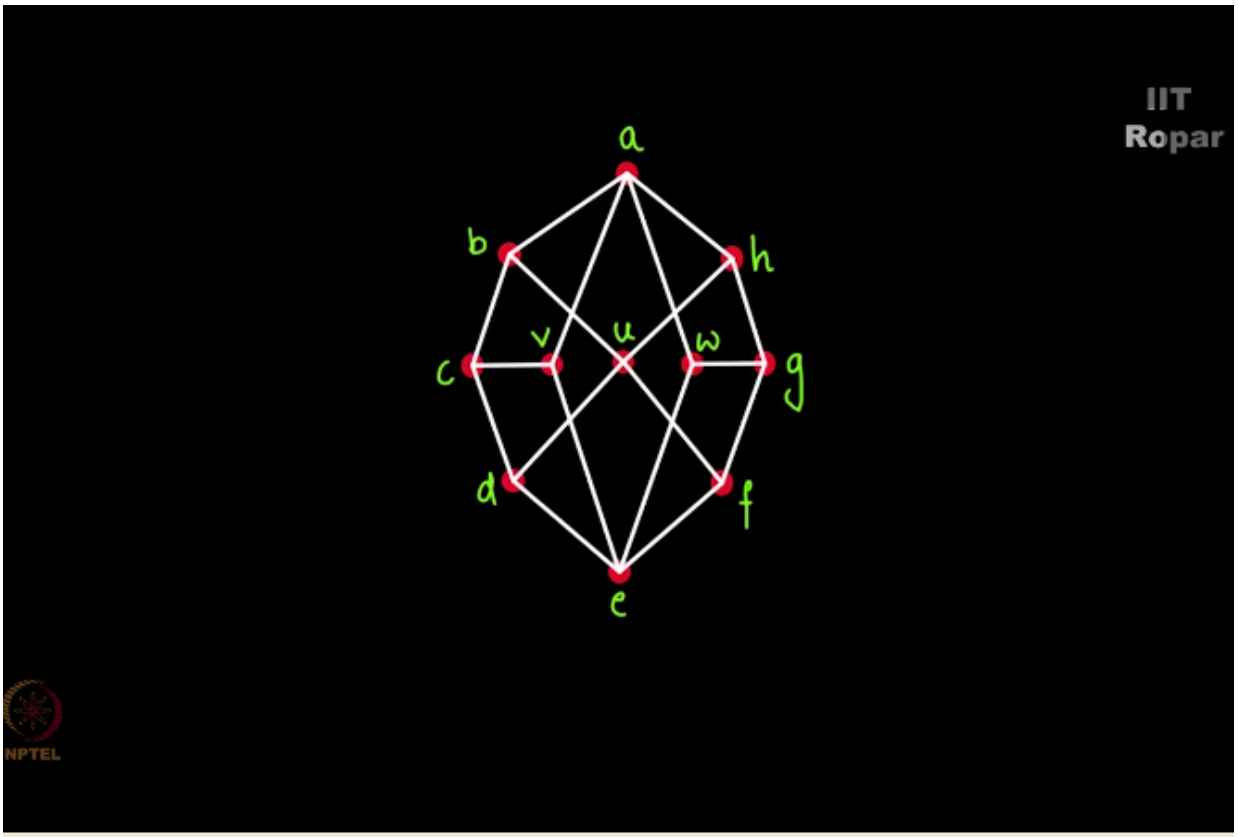
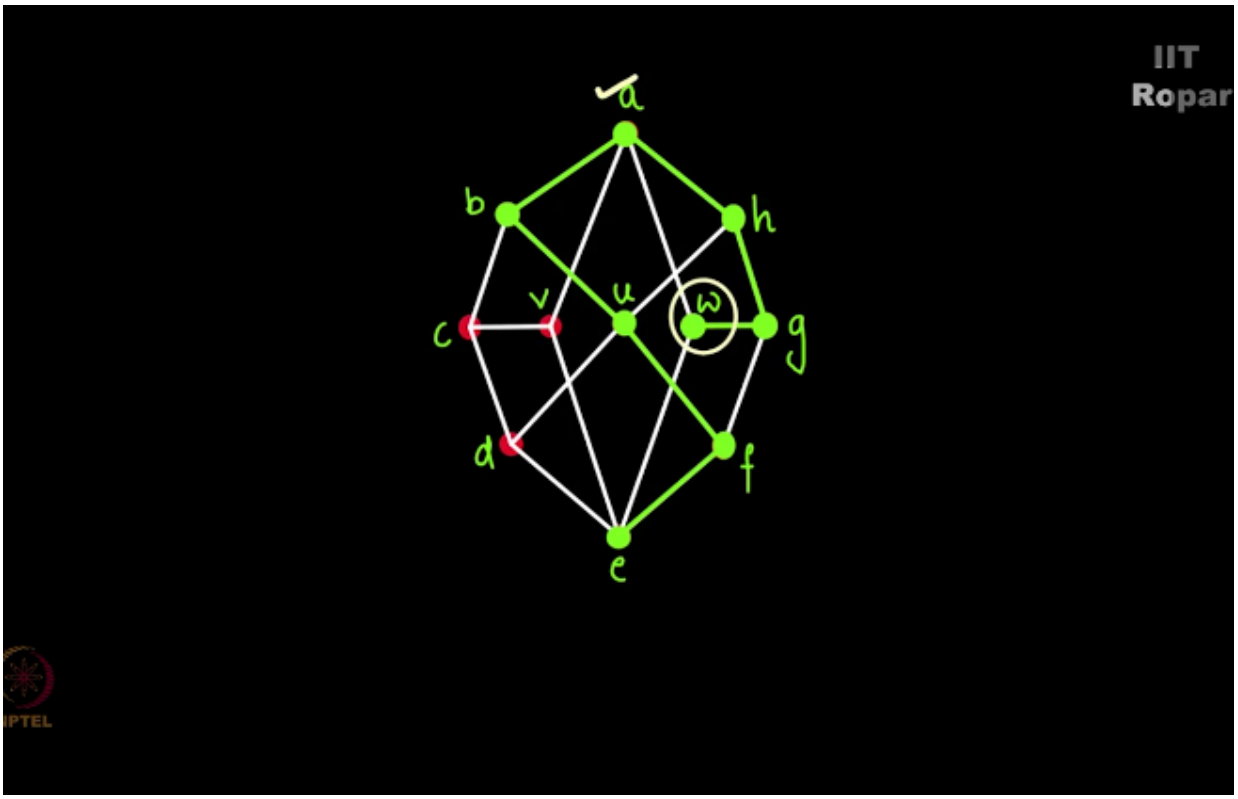


I'll start from this vertex, let's say E, I'll go from E to F, F to U, U to B, B to A, A to H, H to see I cannot go to U now, because I've already covered it, the only option remaining is G because I cannot go to A neither, H to G, G to W, W to oh I'm locked, because I've finished A, (Refer Slide Time: 02:24)



and I just came from G and E is also over, if I go to either of these vertices,

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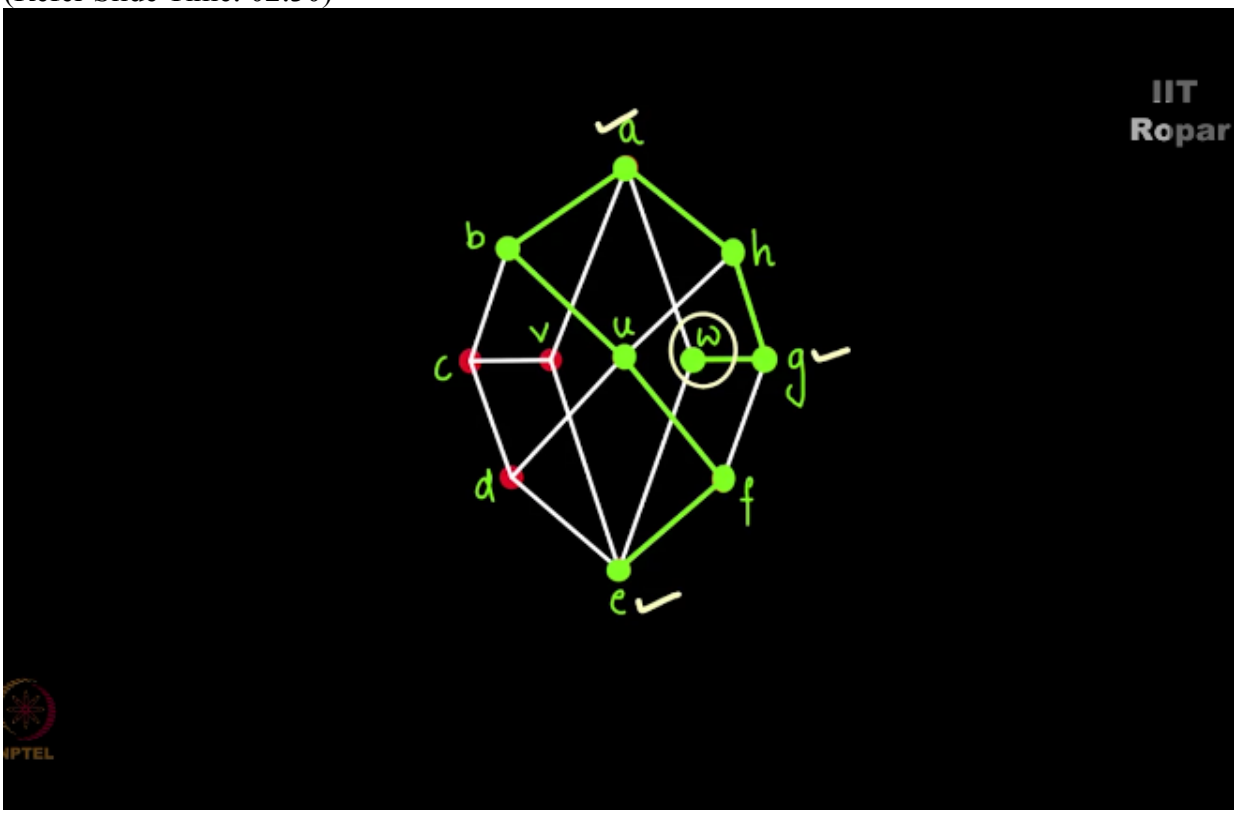

the vertices get repeated which violates the rule and hence I'm locked here, so I cannot find a Hamiltonian path now or a Hamiltonian cycle.

So let us see if I can start from C and obtain a cycle, C to V, V to A, A to W, and then to G, G to H, H to U, U to D, D to E, E to F, yeah,
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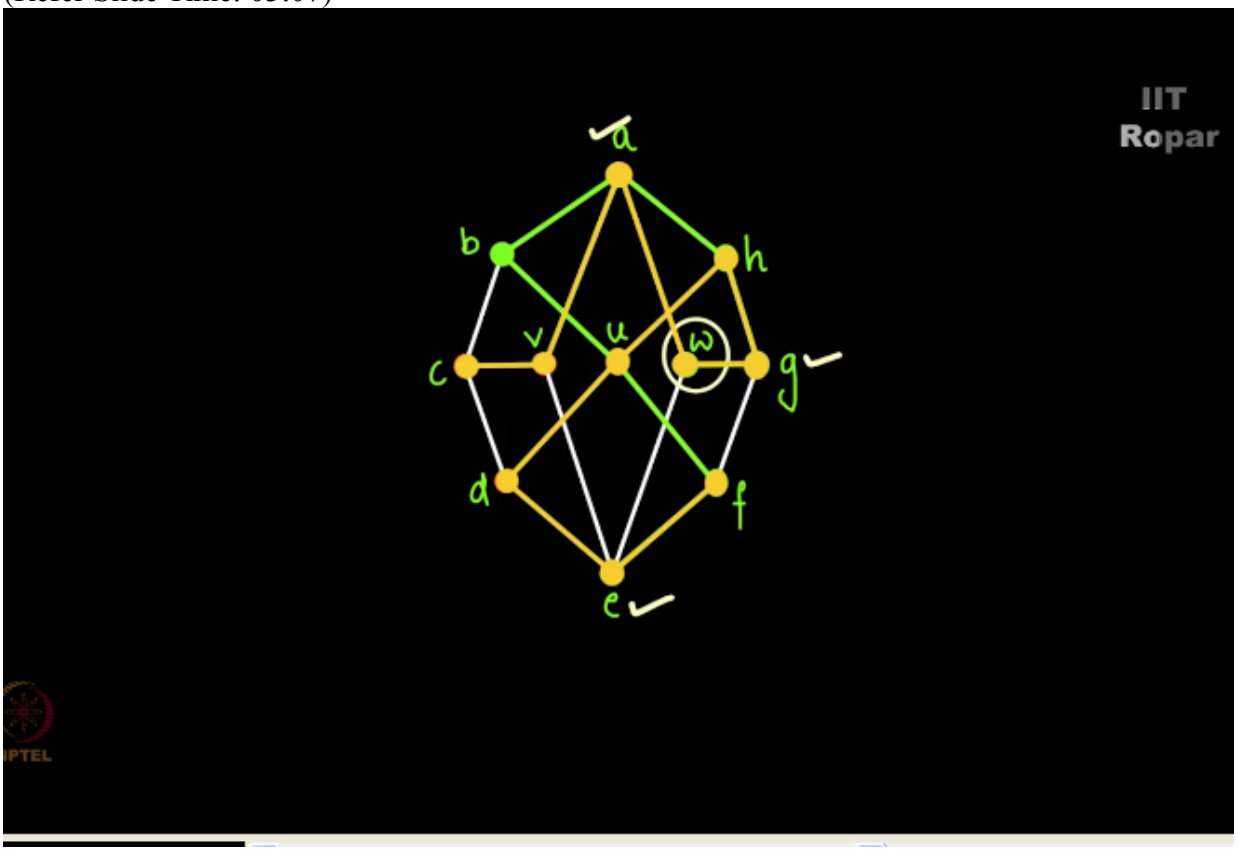

again F I cannot go neither to U nor to G from F, because I've finished both of them, I have visited both of them, I'm locked again, I haven't yet visited B and there is no choice for me to reach , and hence I cannot obtain a Hamiltonian cycle here, looks like the graph is not Hamiltonian.
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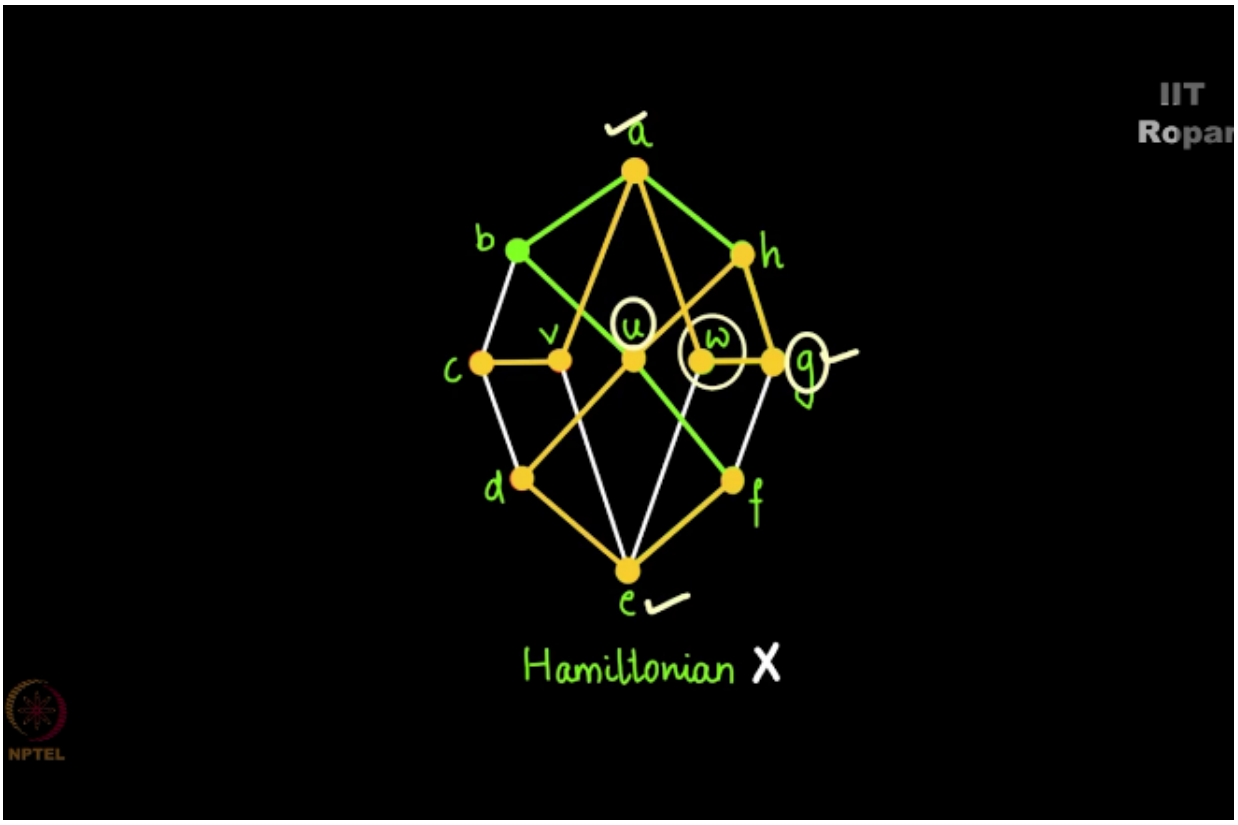

Consider this graph, complicated graph on so many vertices, let us see if we can get a Hamiltonian cycle here, I'll start from this vertex D,

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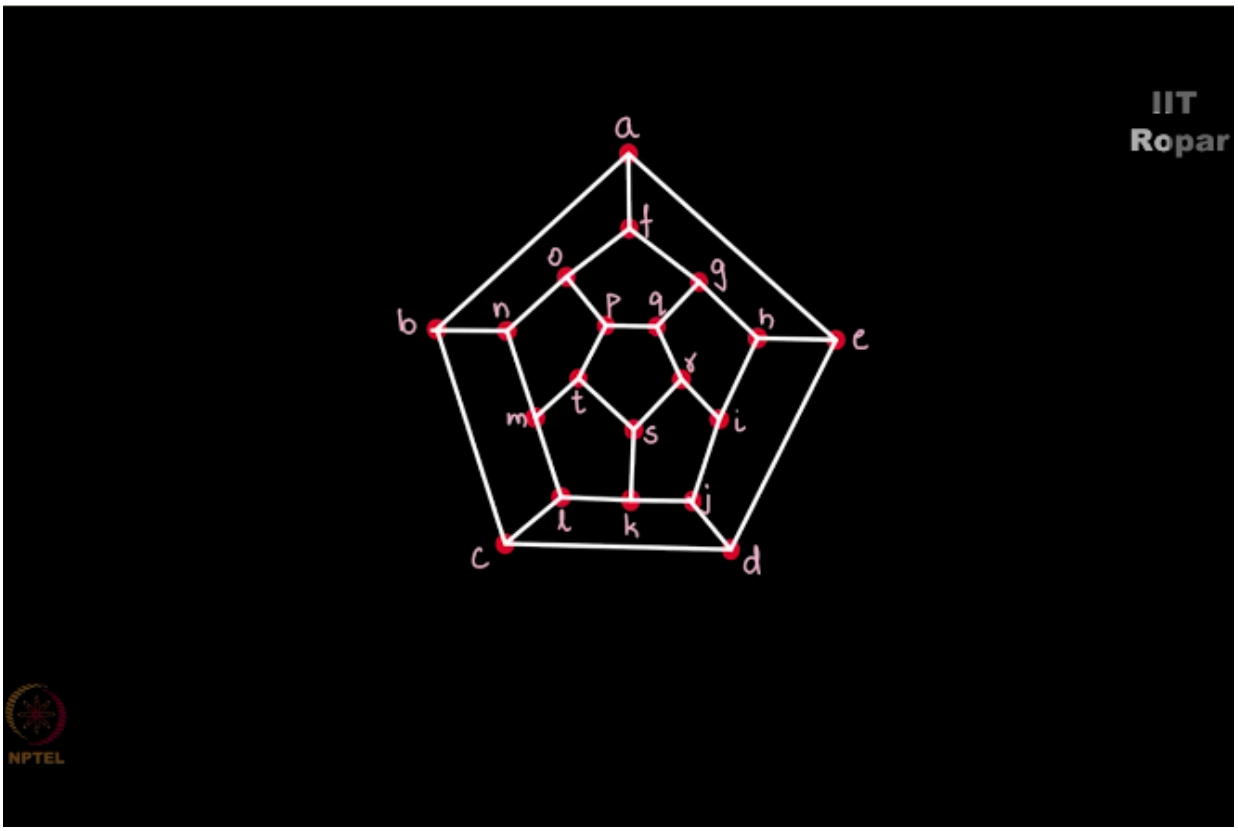

now I go to E, H, then to I, then to J, then to K, then to L, then to M, then to N, O, you see I'm covering this inner cycle now,

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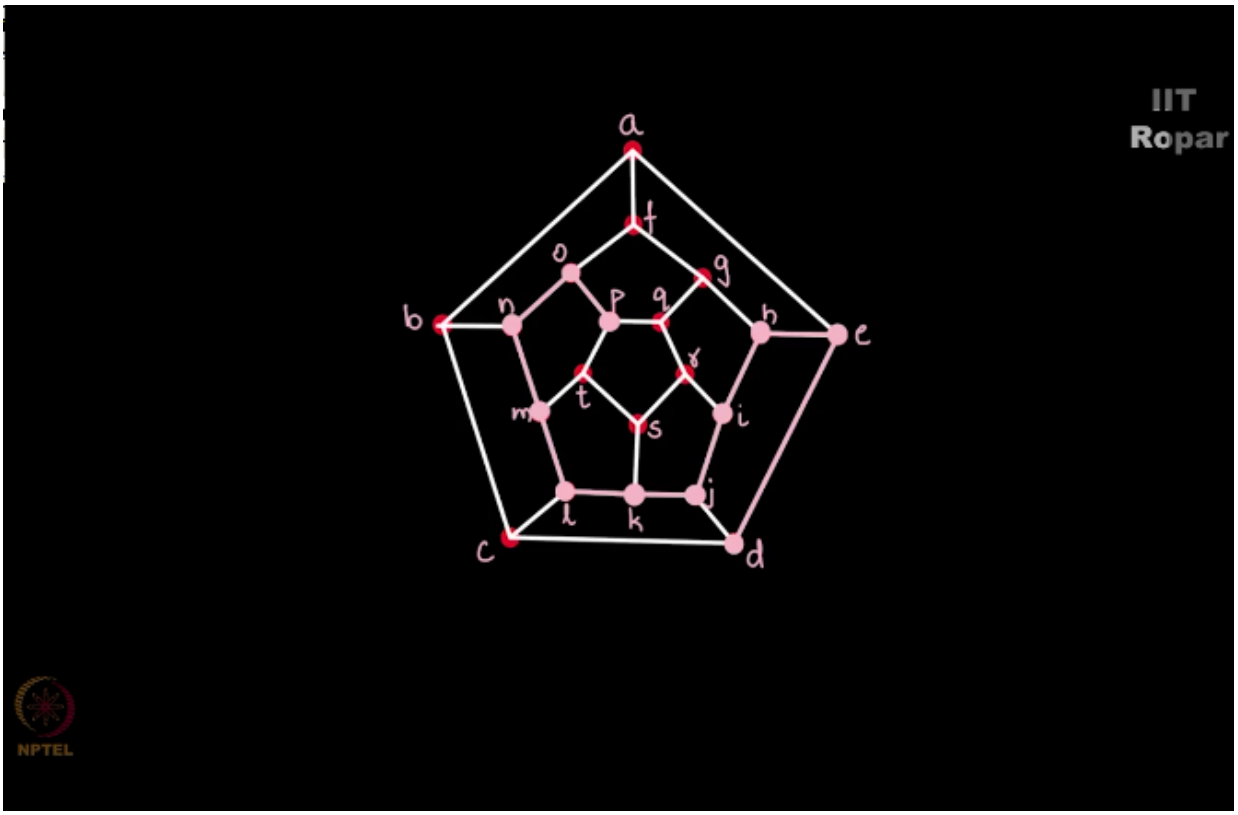

O, P, T, S, R, Q, G, F, A I have come out,

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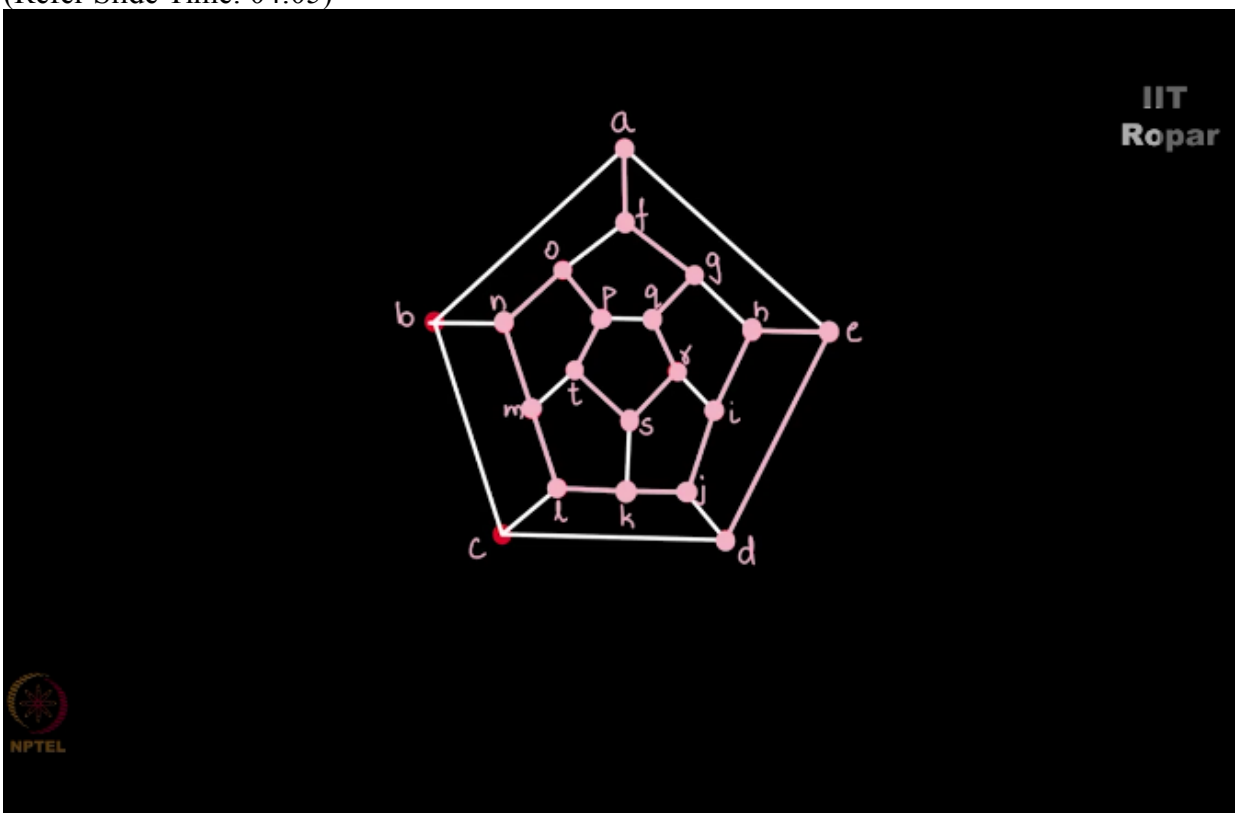

B, C, and then P, yes we have finished our Hamiltonian cycle,

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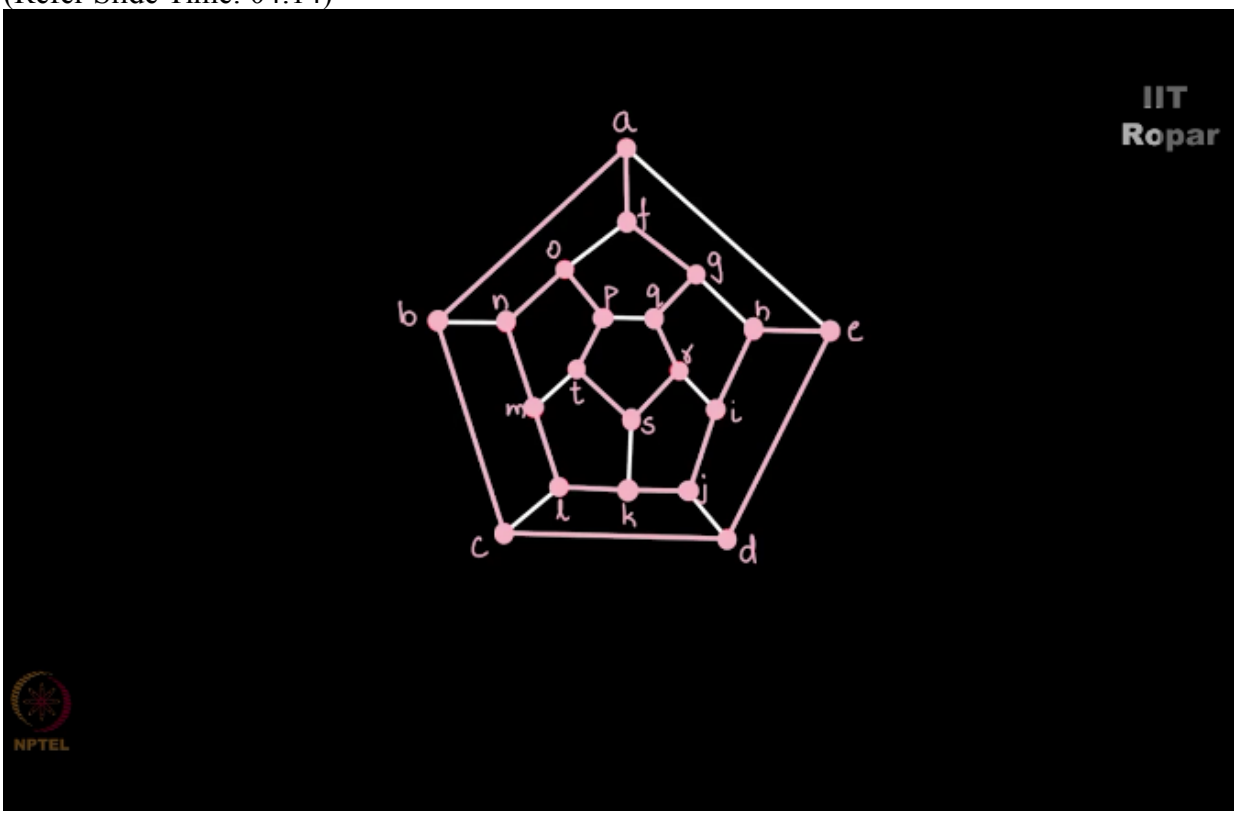

I've gone through all the vertices, if you observe keenly we have left out so many edges in between, but still we have obtained, we have covered all the vertices and we have obtained a Hamiltonian cycle,

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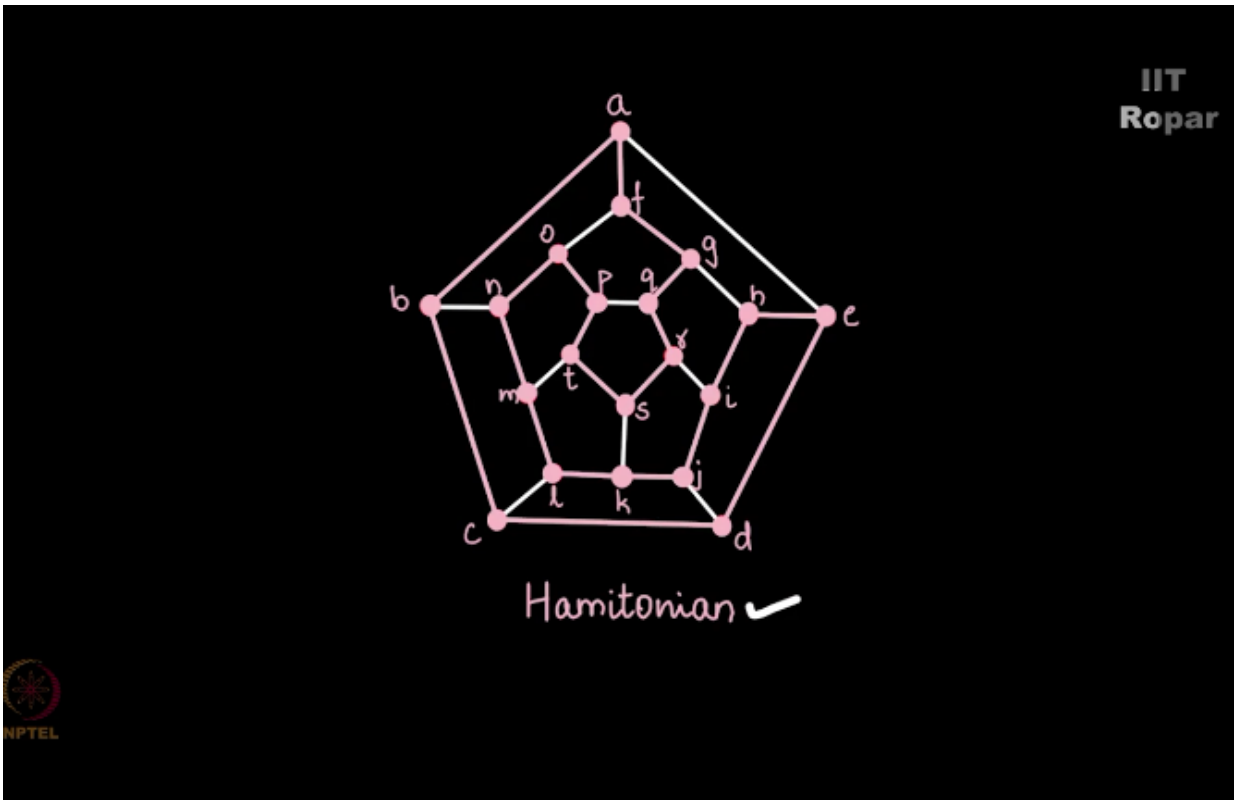

and hence this graph is Hamiltonian.

Consider this graph on 7 vertices,
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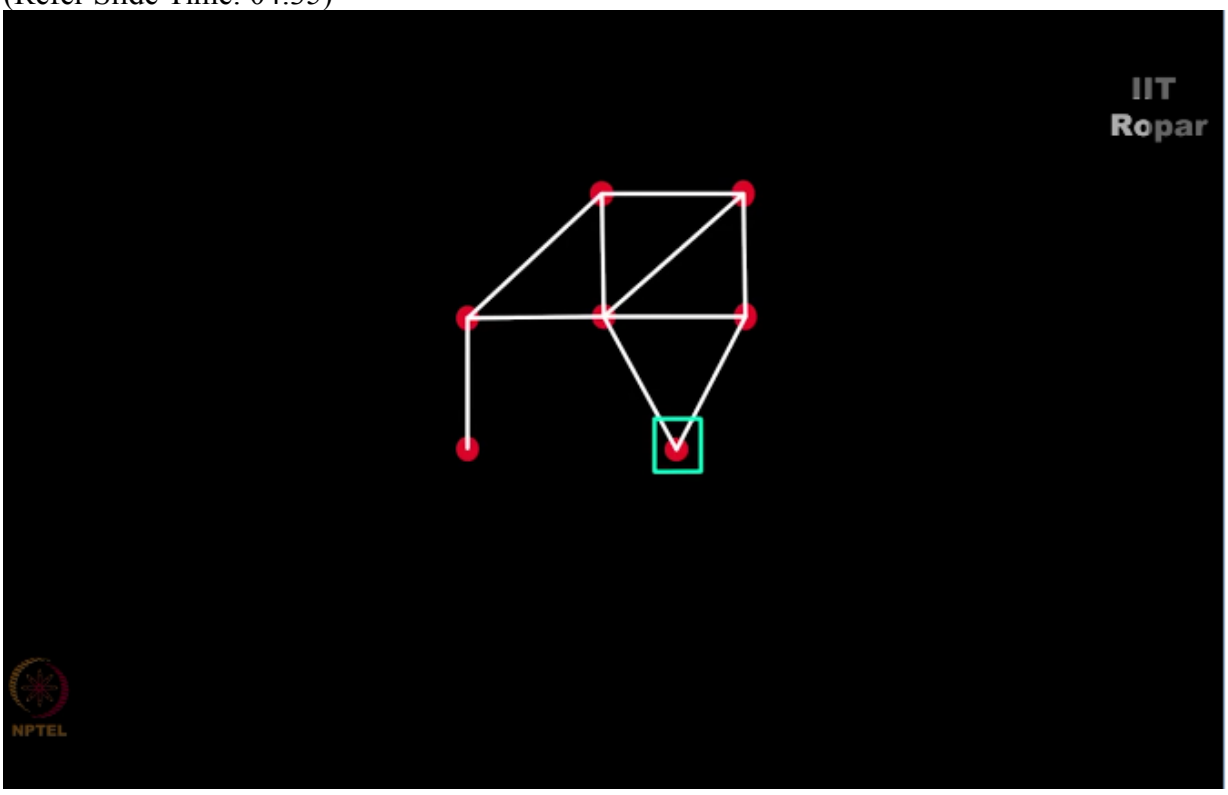

I start from this vertex, then let me name it as A, I'll name all the vertices A, B, C, D, E, F and G,
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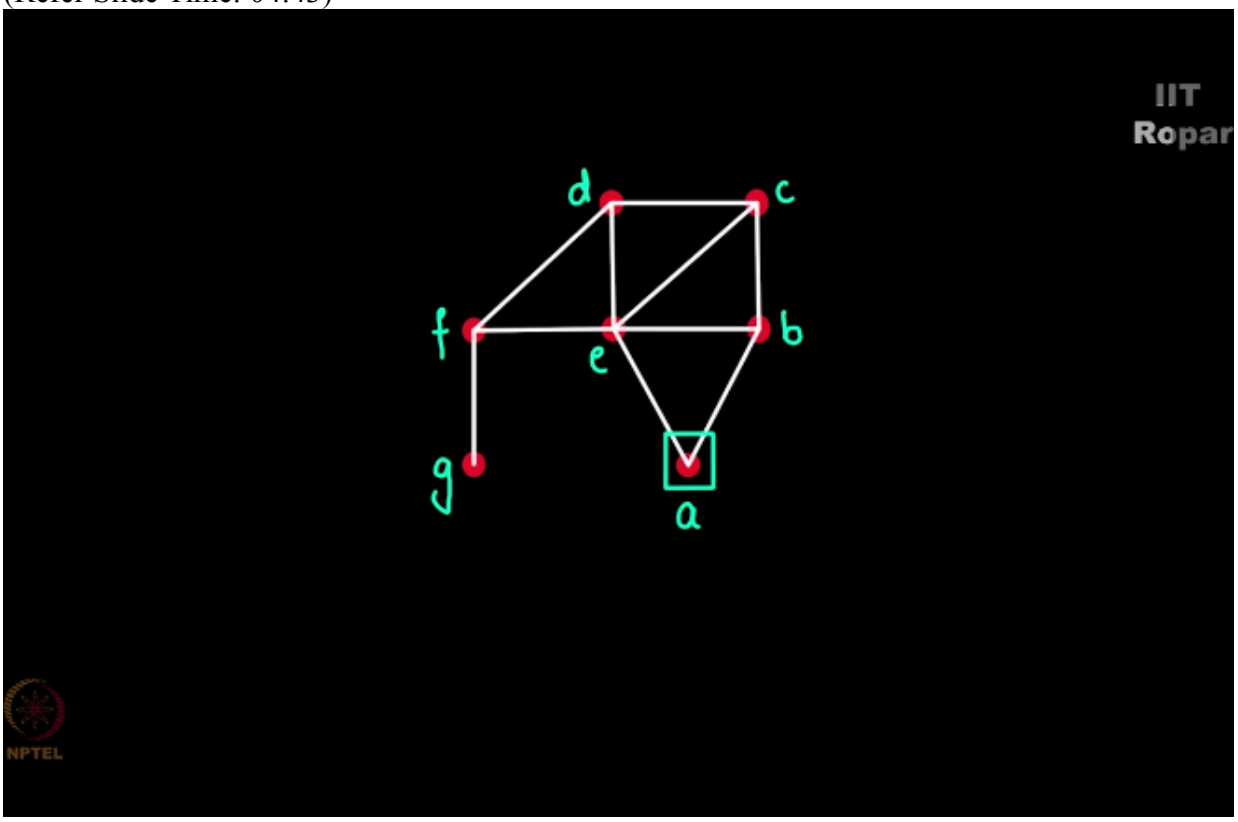

now I start from A, go to B, go to C, D, E, and then to F, now I come to G,
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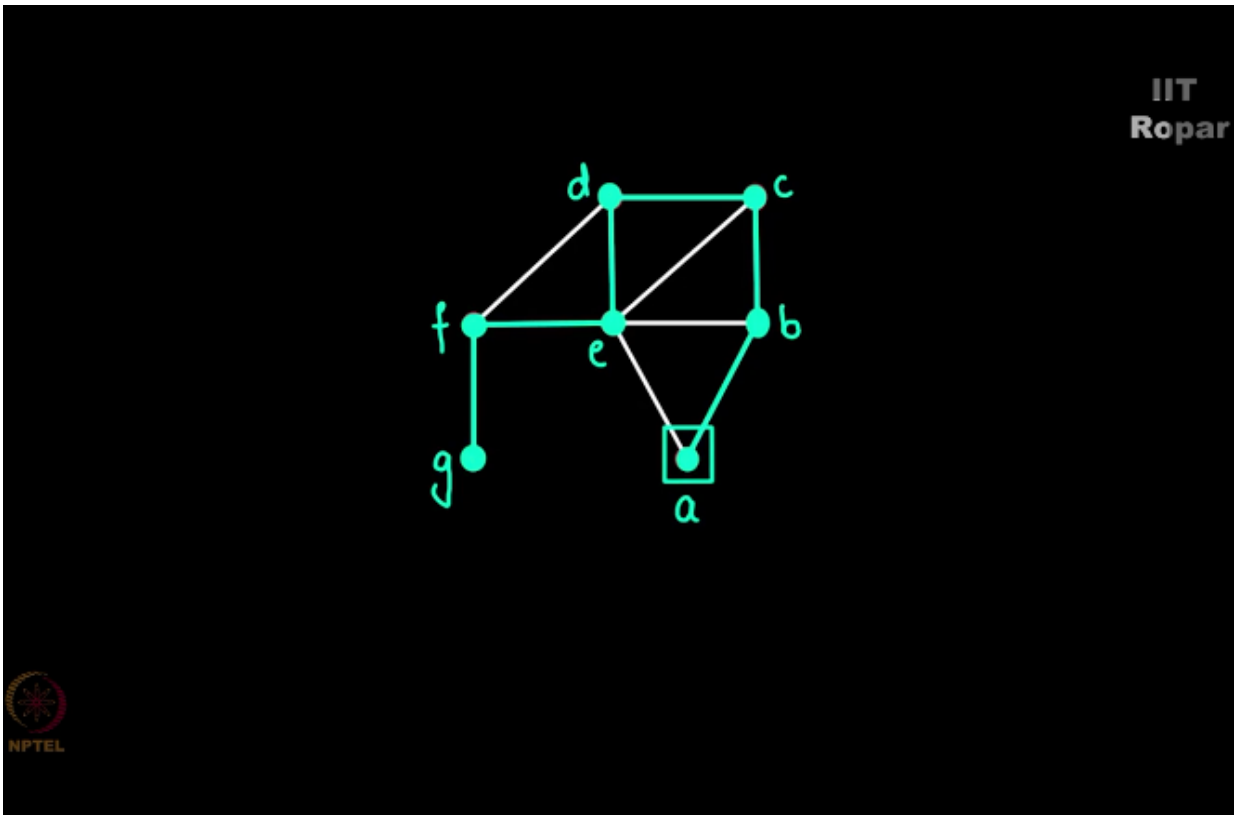

but I'm locked at G because I can neither go to A, there is no edge here, neither can I go back to F, again to E there is no edge, and I'm locked at G and hence this graph is not Hamiltonian.

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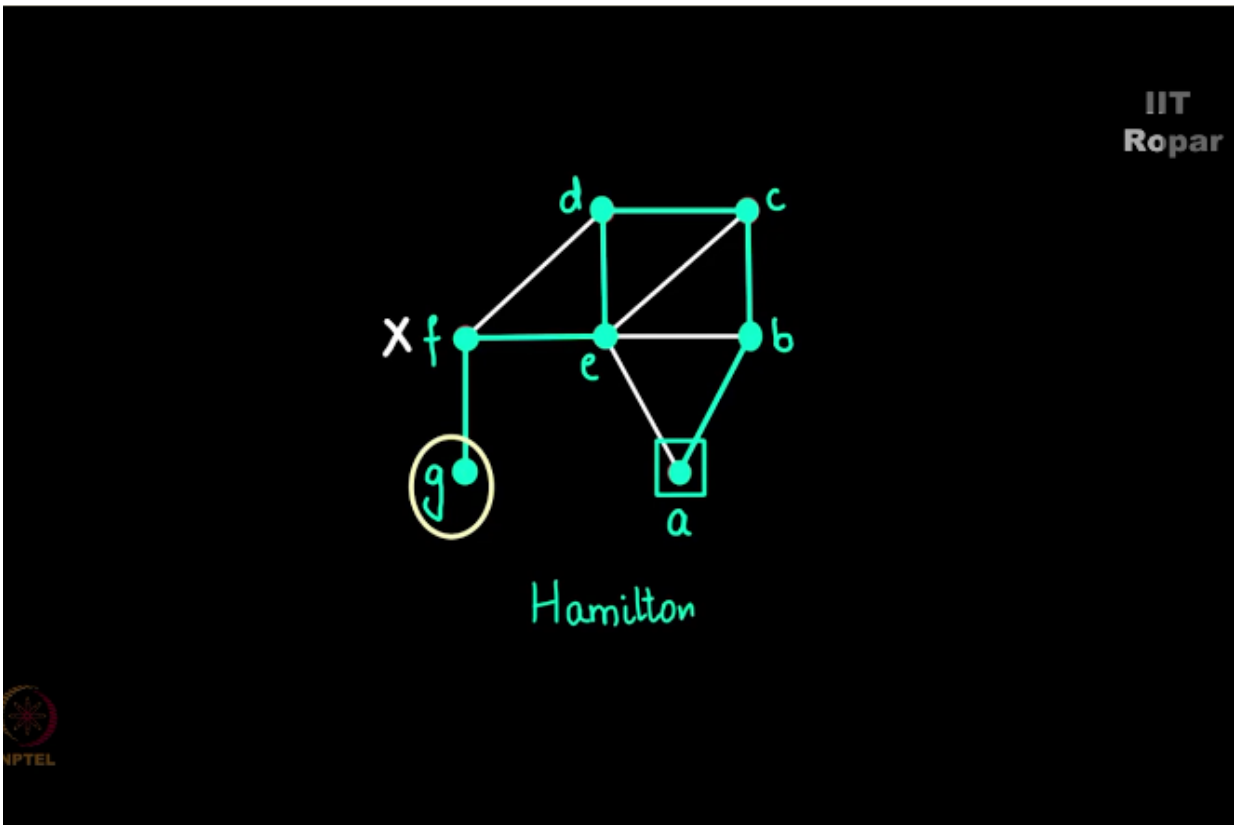

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