NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 2

A condition for Eulerian trail

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let us now see a cute result which follows from what the professor has taught in the previous video, it was a lengthy theorem let us now see a cute result based on that, let G be a connected graph with exactly two vertices of odd degree, then there exists an open Eulerian trail.
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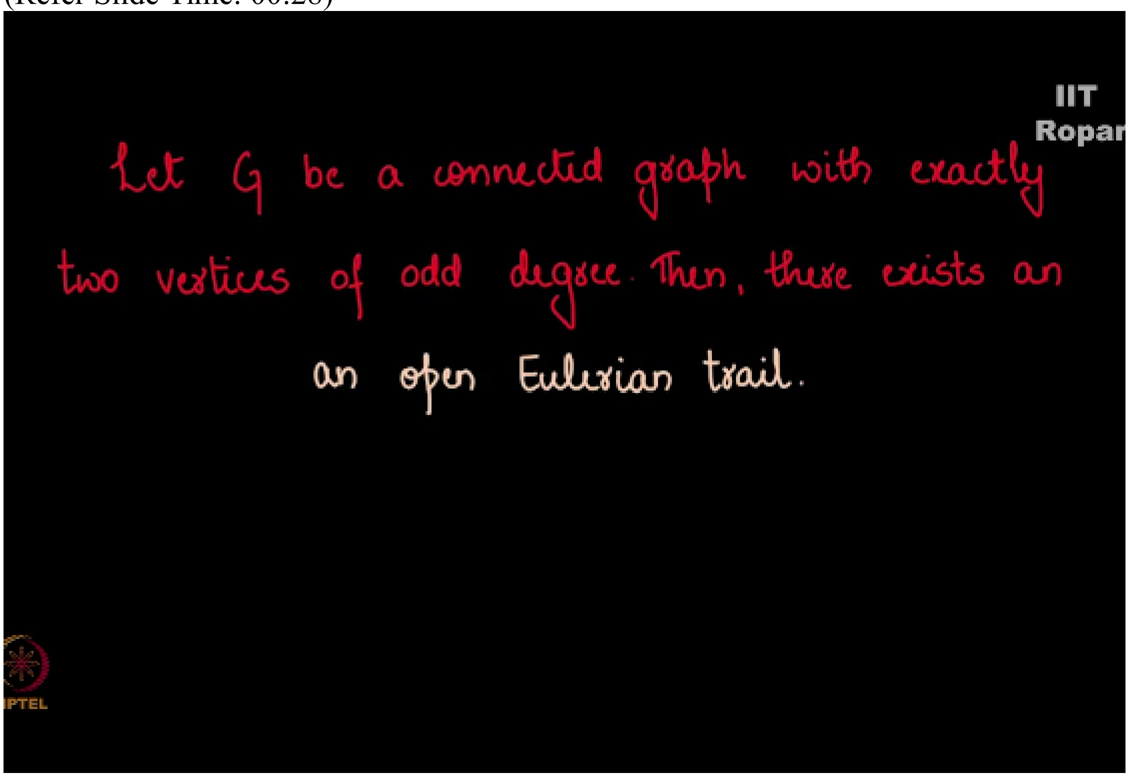

Now let me explain the theorem if G is a connected graph with exactly two vertices which are having odd degree, then it says that there is an Eulerian trail in it, what did we see earlier? We saw that if G is a connected graph with all vertices having even degree, then the graph is Eulerian and vice-versa, but now we say that if there are only two vertices of odd degree then still we can find an Eulerian trail, how do we prove this? Consider this to be G, there are two vertices here let me say A and B which are having odd degree.

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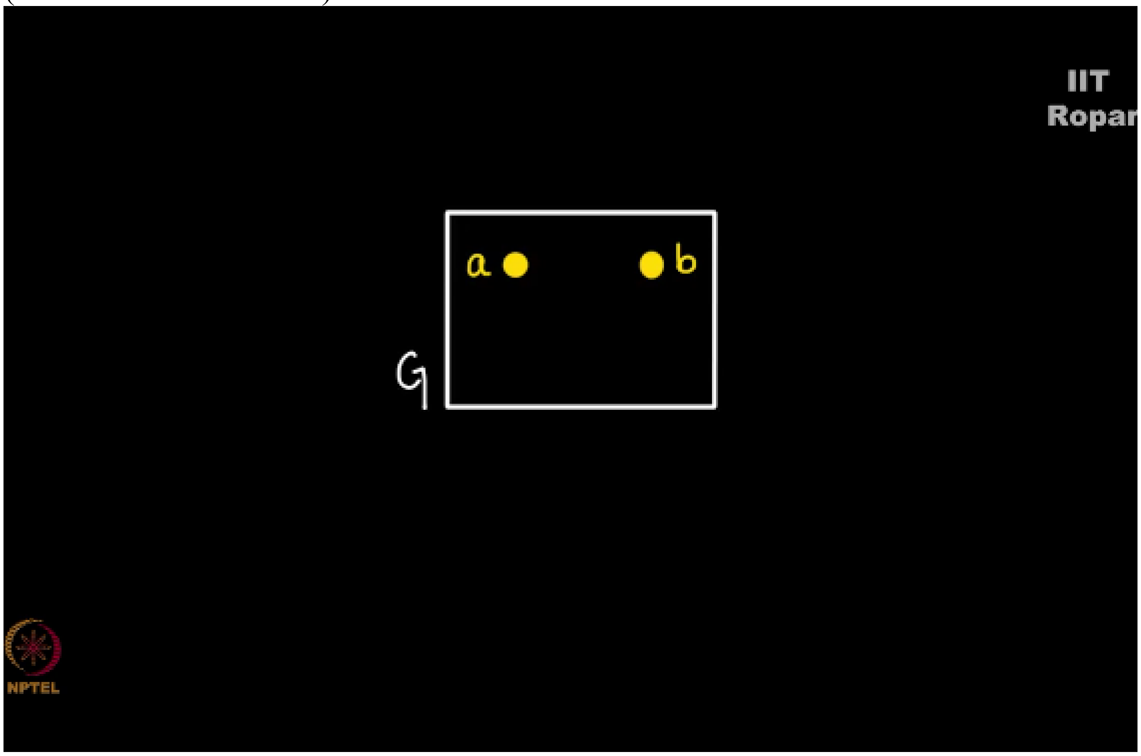

Now what I do is, I introduce a new vertex let me say Z, and then I connect Z to both A and B, do you remember what the professor had taught was on the same lines as these,

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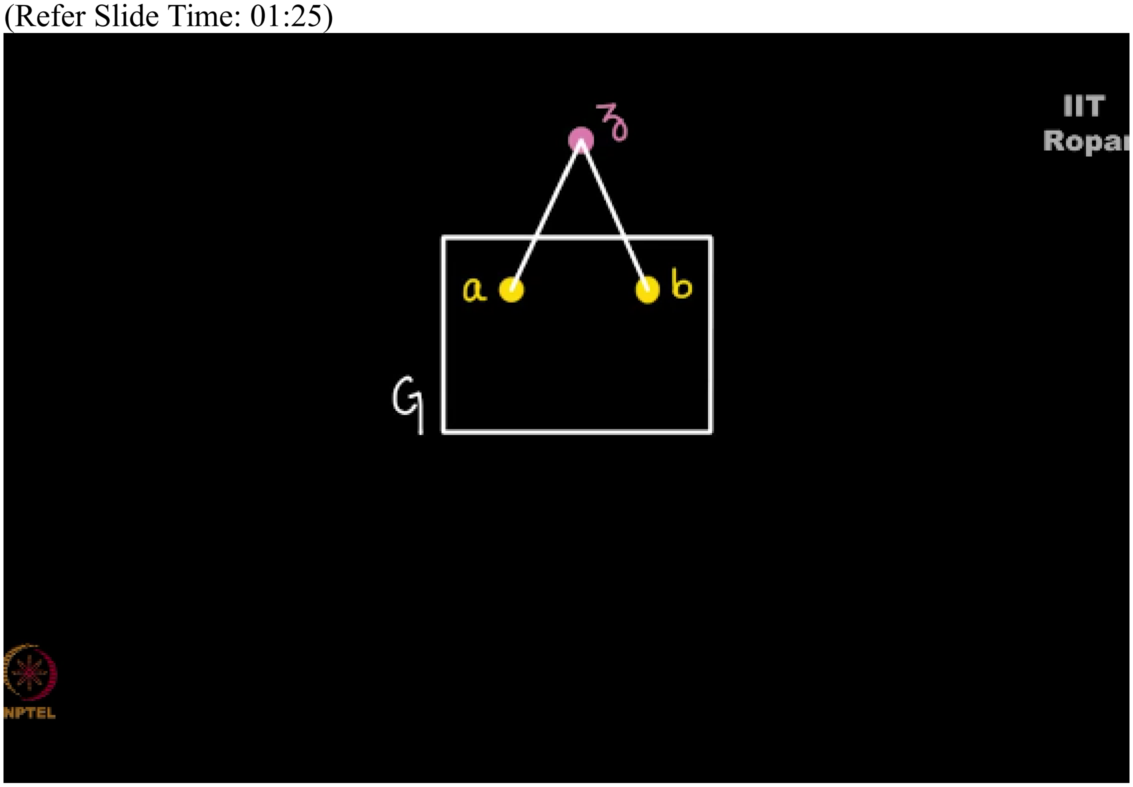

we introduced a new vertex even there, so I introduced a new vertex here let me call that as Z, connected to both of them, now what does the degree of A and B happen to be?

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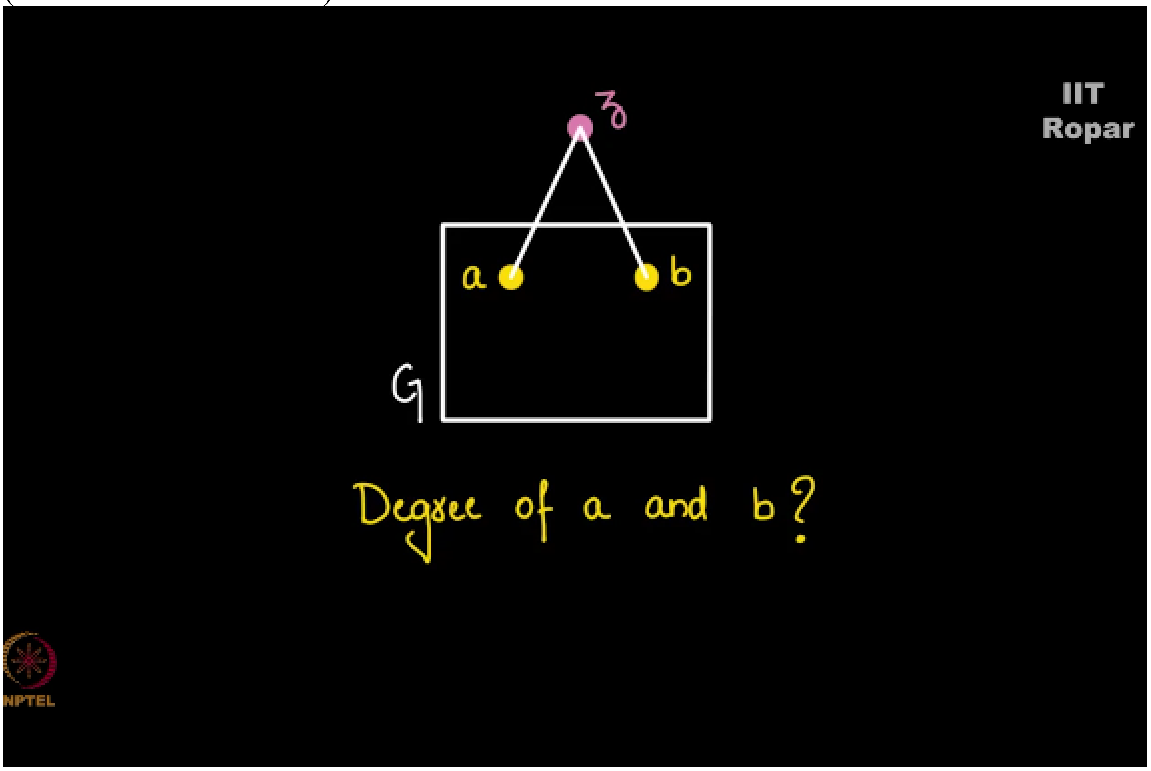

Since it is connected to Z, both of them we have added a new edge it was earlier odd and plus one becomes even,

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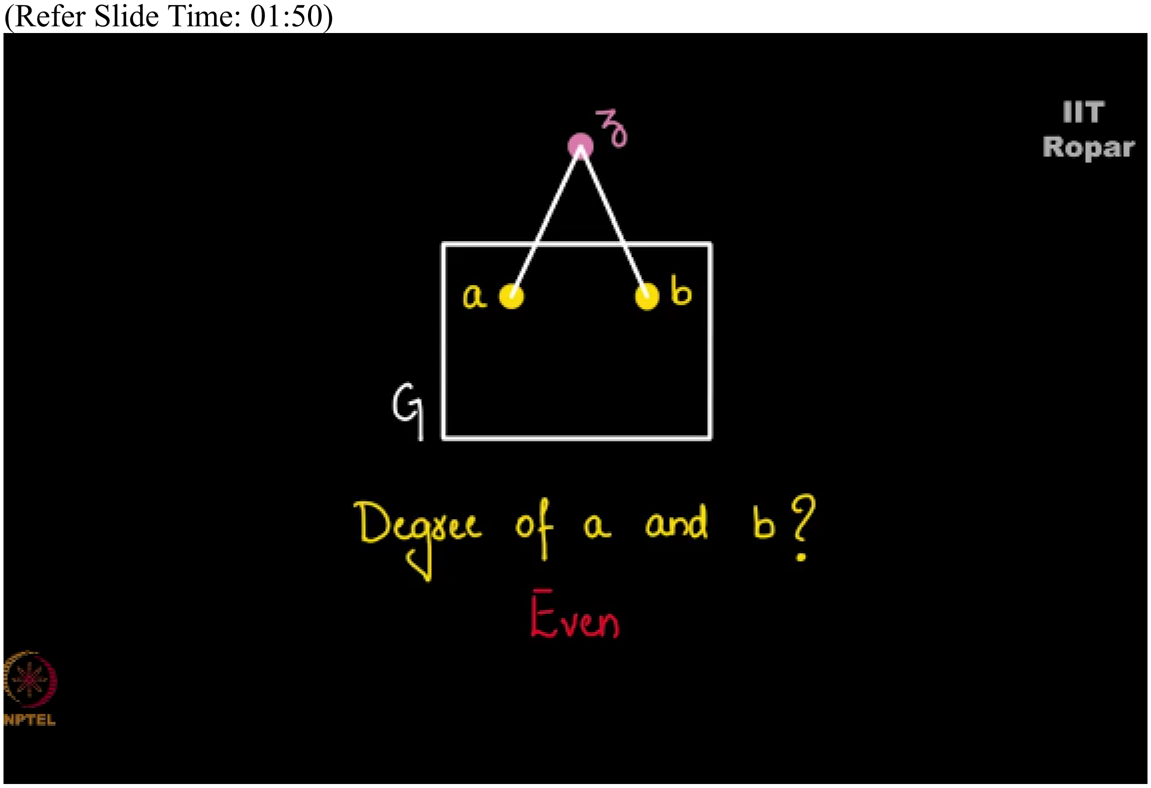

so the degrees of A, B and Z all three of them have an even degree.

Now I start from Z, I reach A, I go to the next vertex let me say X and so on and I reach B, and then I go to Z,

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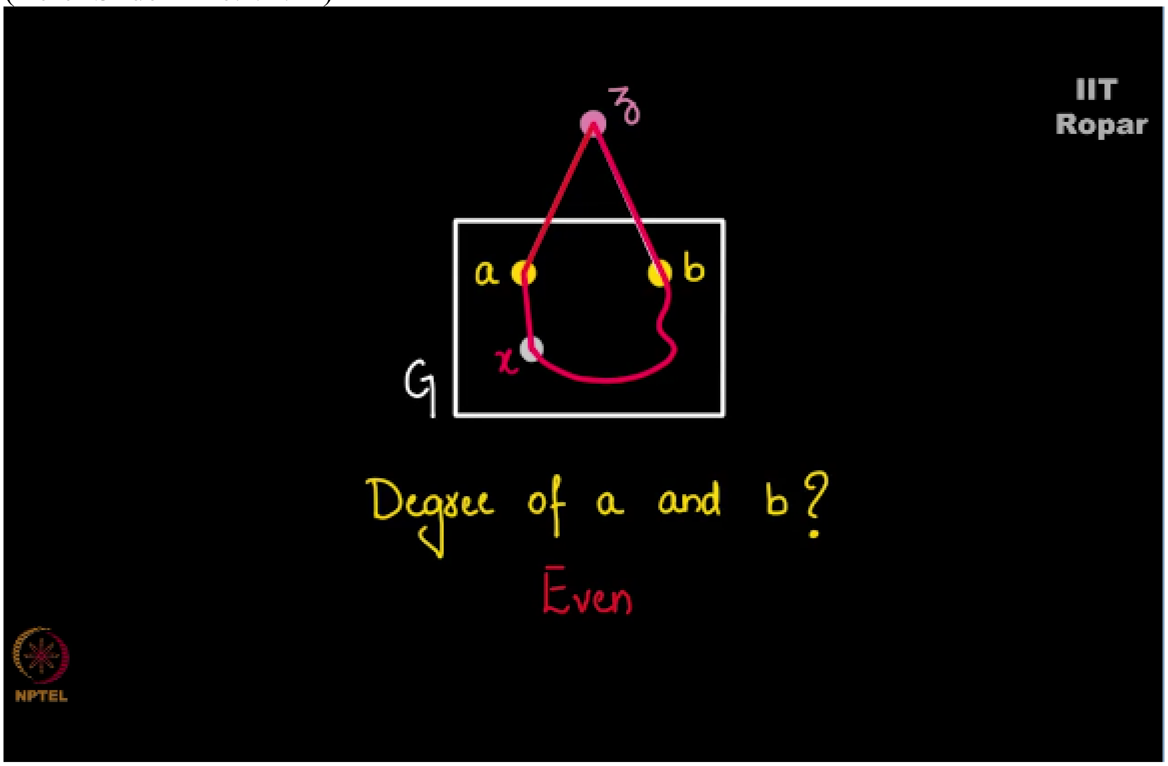

do you see that I have obtained an Eulerian circuit here,

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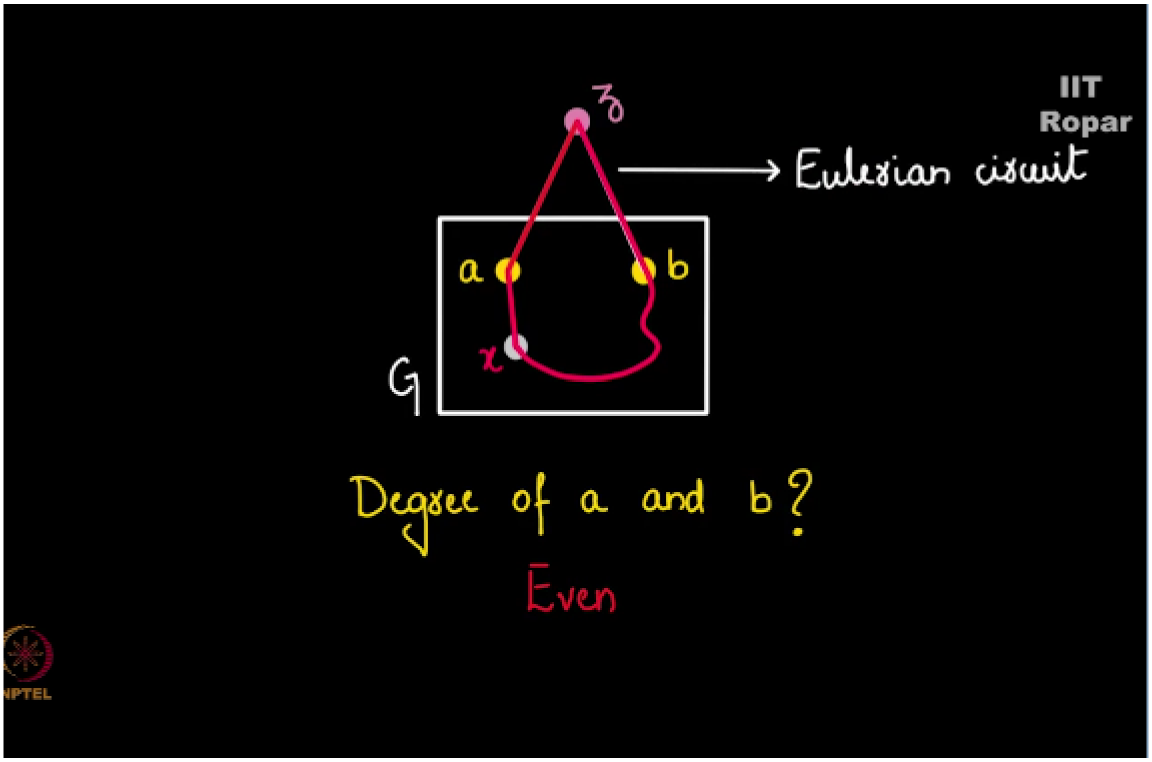

but this is in the new graph which we had constructed let me call that as H, H is the new graph which we have constructed using an extra vertex at Z, so we have obtained an Eulerian circuit in the new graph H.

Now look at this, A, X so on up to B is an open Eulerian trail,
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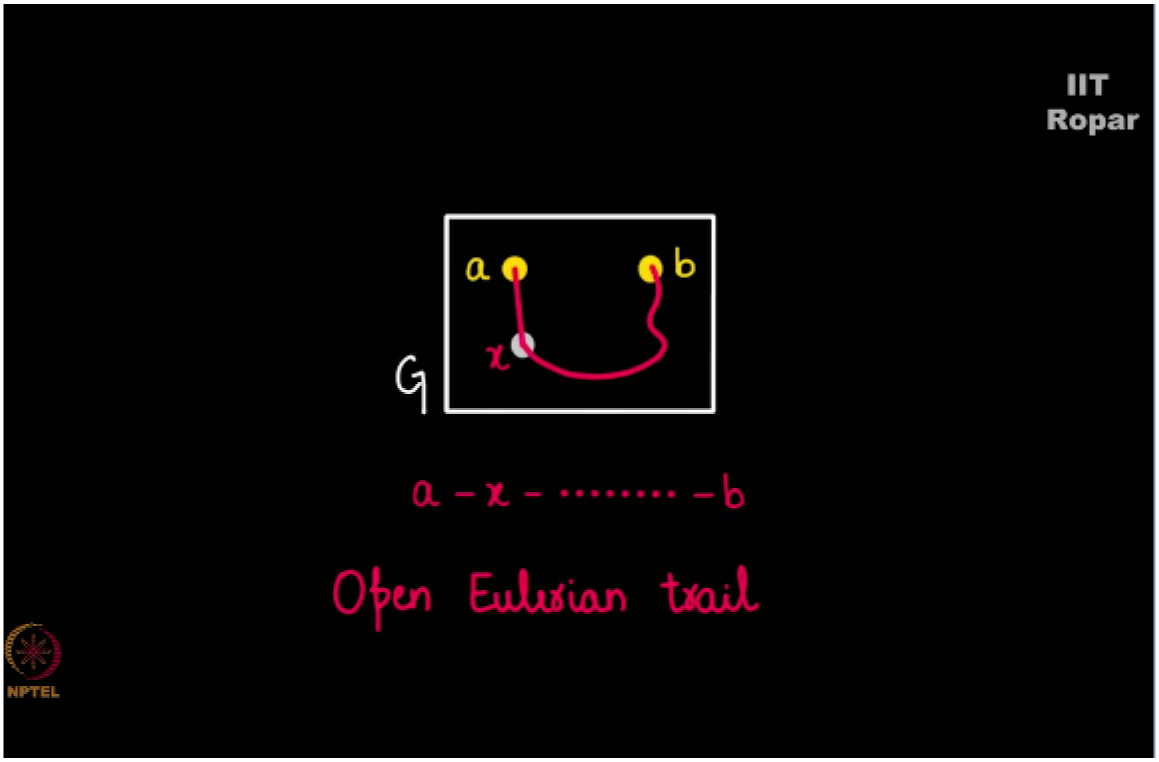

do you see that now I have removed this vertex Z, and hence even these two edges which were connected to A and B, this results in an Eulerian trail in G,

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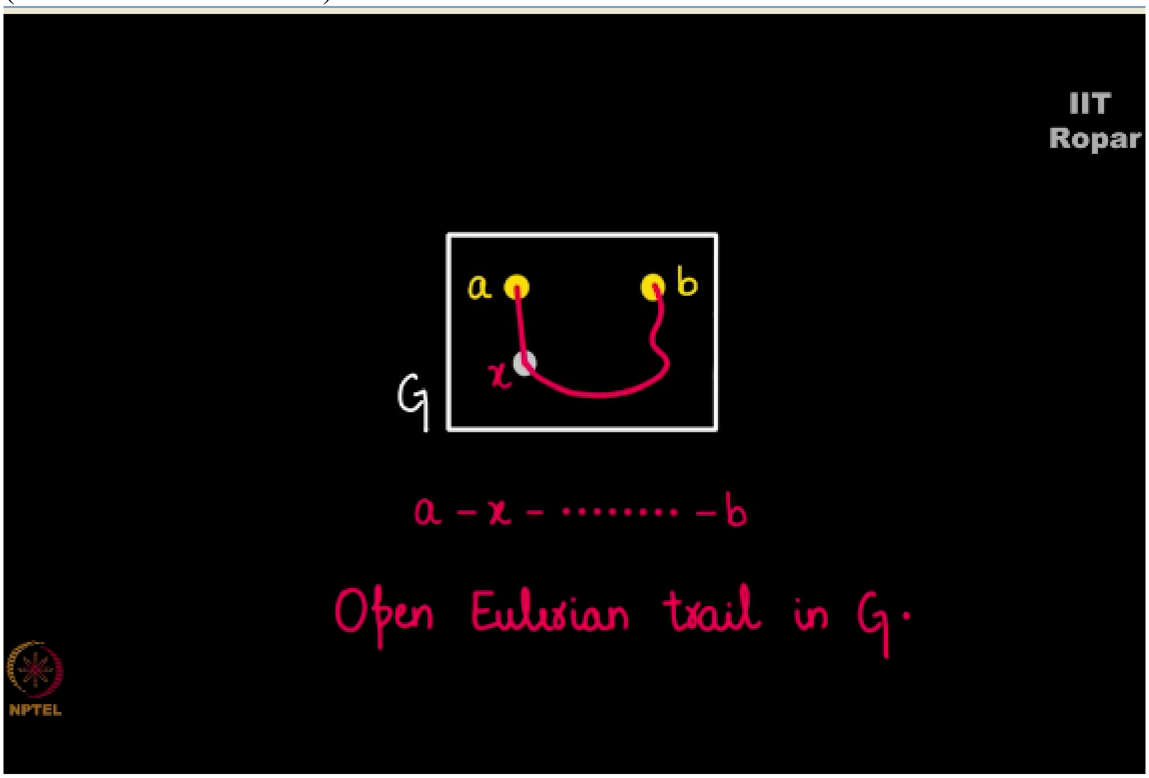

hence we have obtained what we were aiming at.

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