NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 2

Illustration of eulerian graph

By
Prof. S.R.S Iyengar
Department of Computer Science
IT Ropar

The professor has quoted, start from a node, traverse all the edges comeback to the same node without repeating the edges, this is called the Eulerian circuit.
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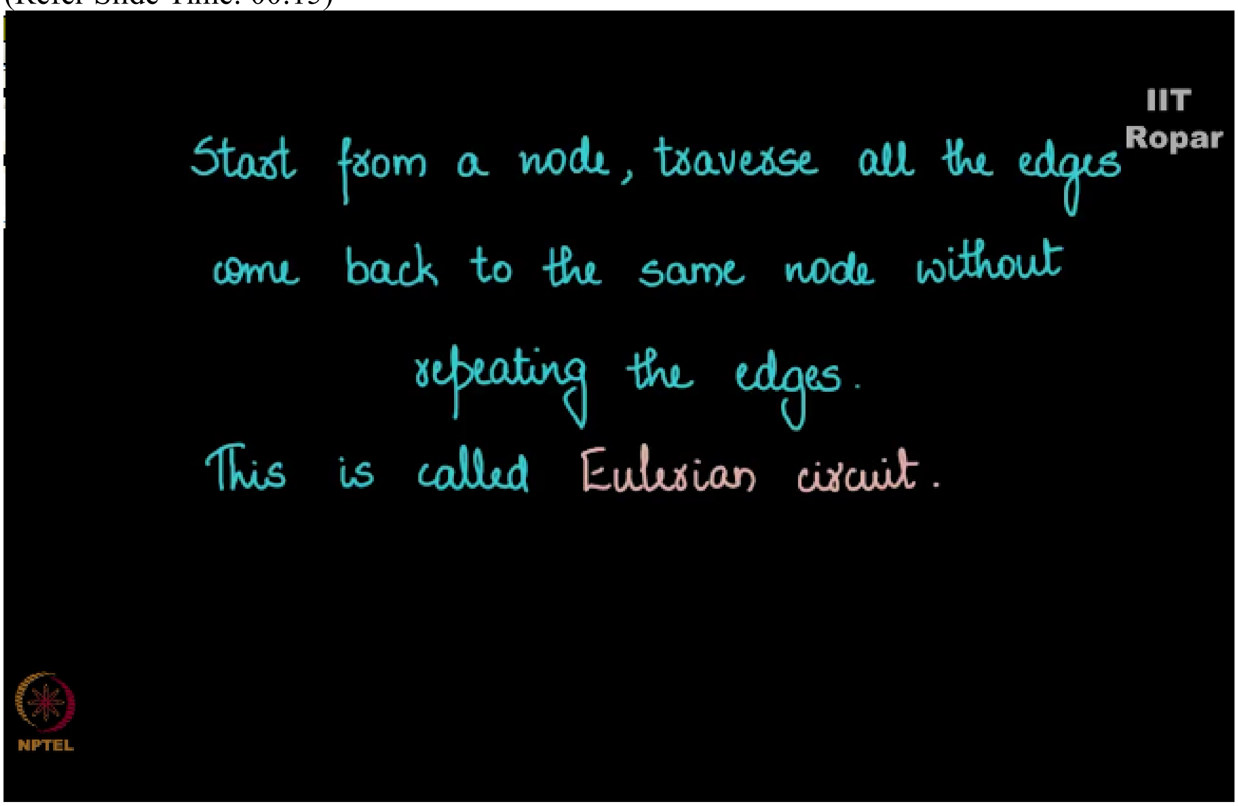

If this is the definition of Eulerian circuit you must be very clear with what is an Eulerian trial, the only difference is that you do not go back to the same node, you can start anywhere, go to all the edges and go to some other node, it is not a closed one, this is called an Eulerian trial.
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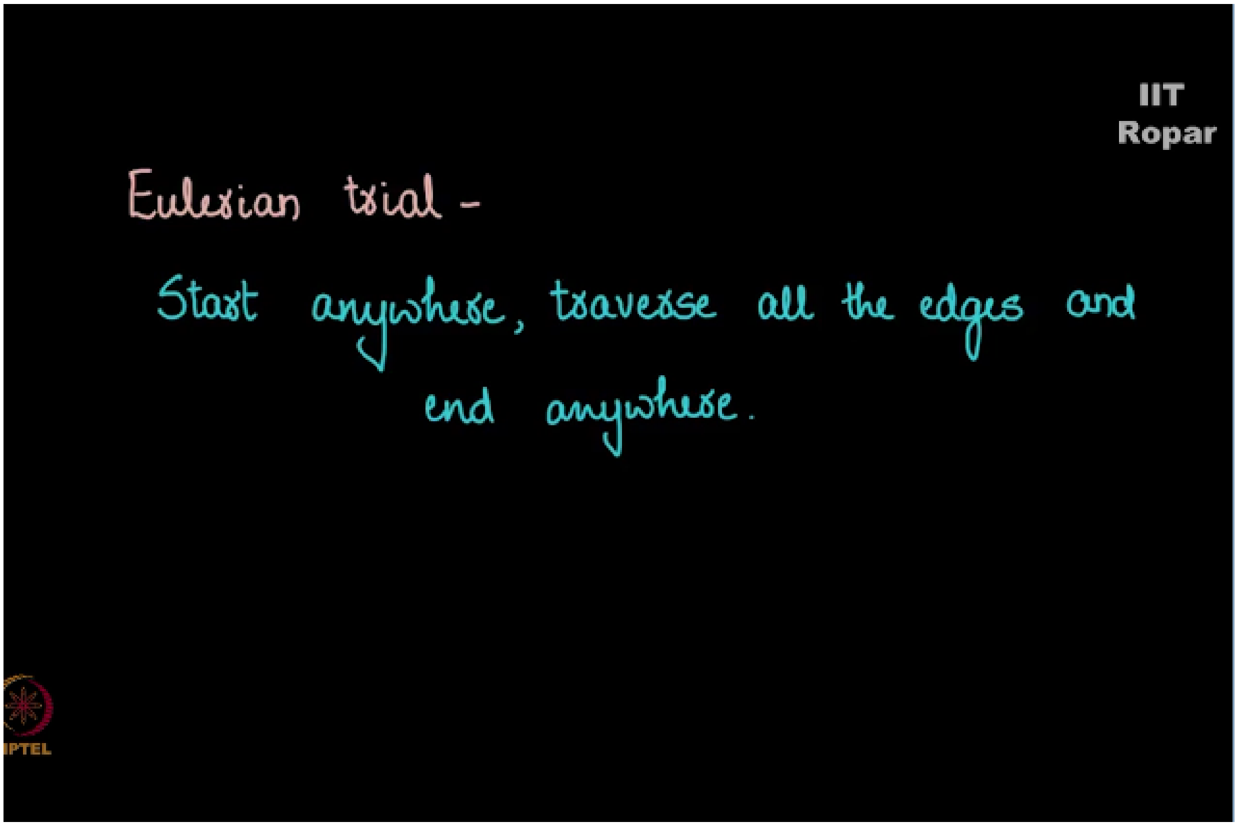

Now when is a graph called as an Eulerian graph, a graph G is called as an Eulerian graph if it contains an Eulerian circuit,
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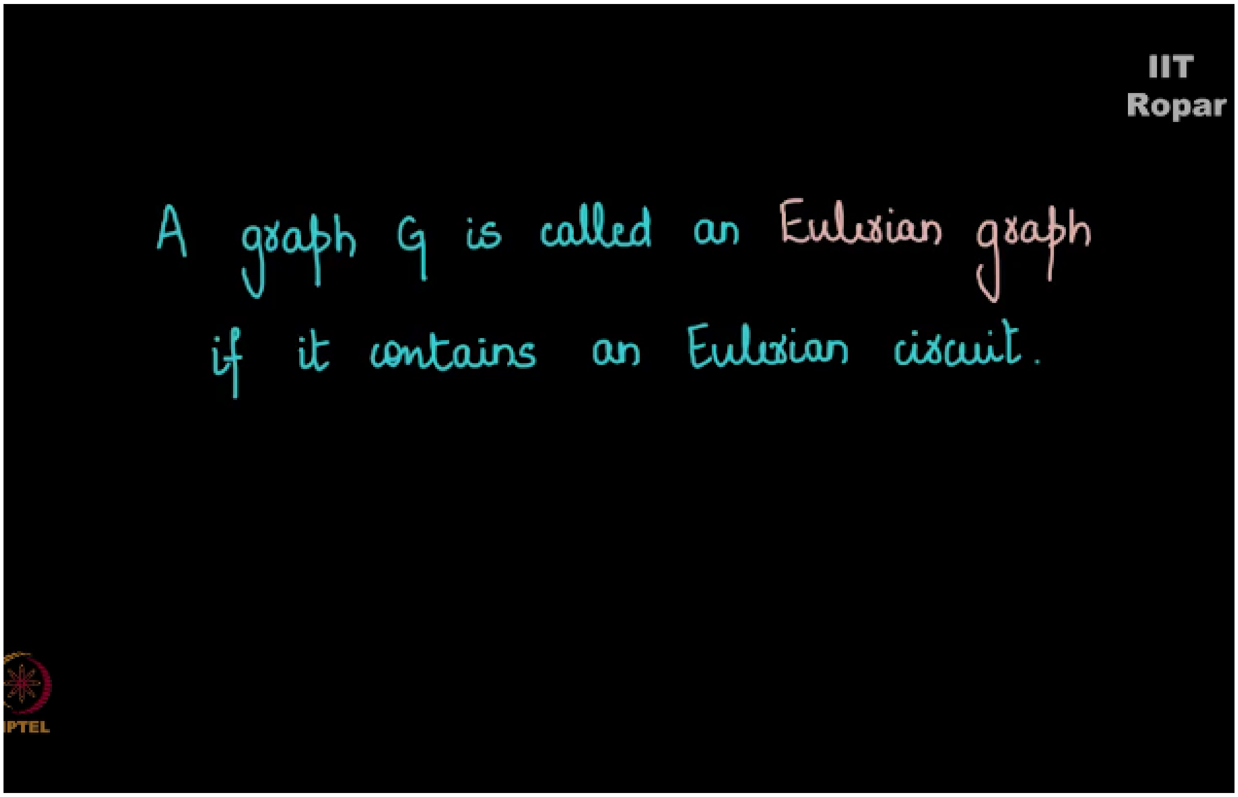
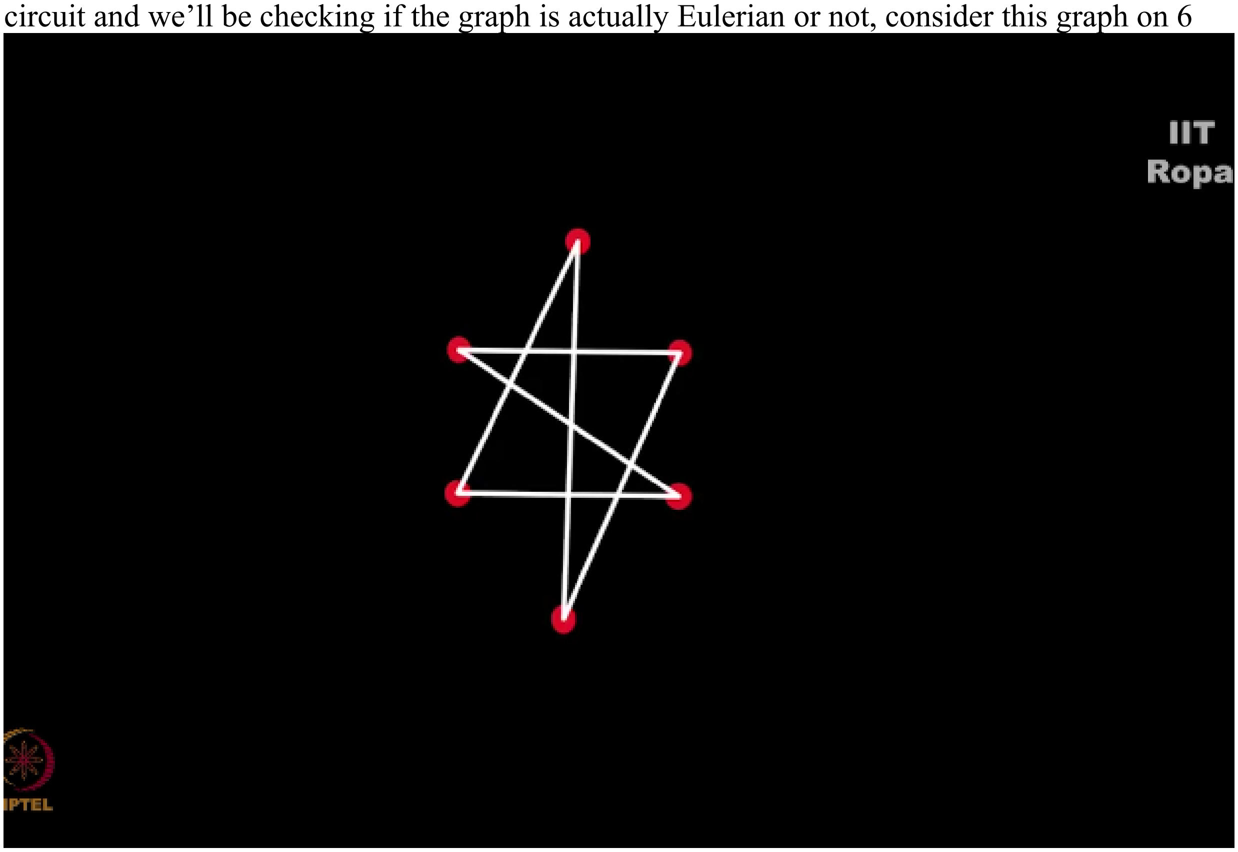


here are the few illustrations, now I'll be displaying a few graphs and we'll be taking Eulerian circuit and we'll be checking if the graph is actually Eulerian or not, consider this graph on 6



vertices, let me call this vertex A, I'll start from this vertex, okay, let me label all the vertices A, B, C, D, E, F,

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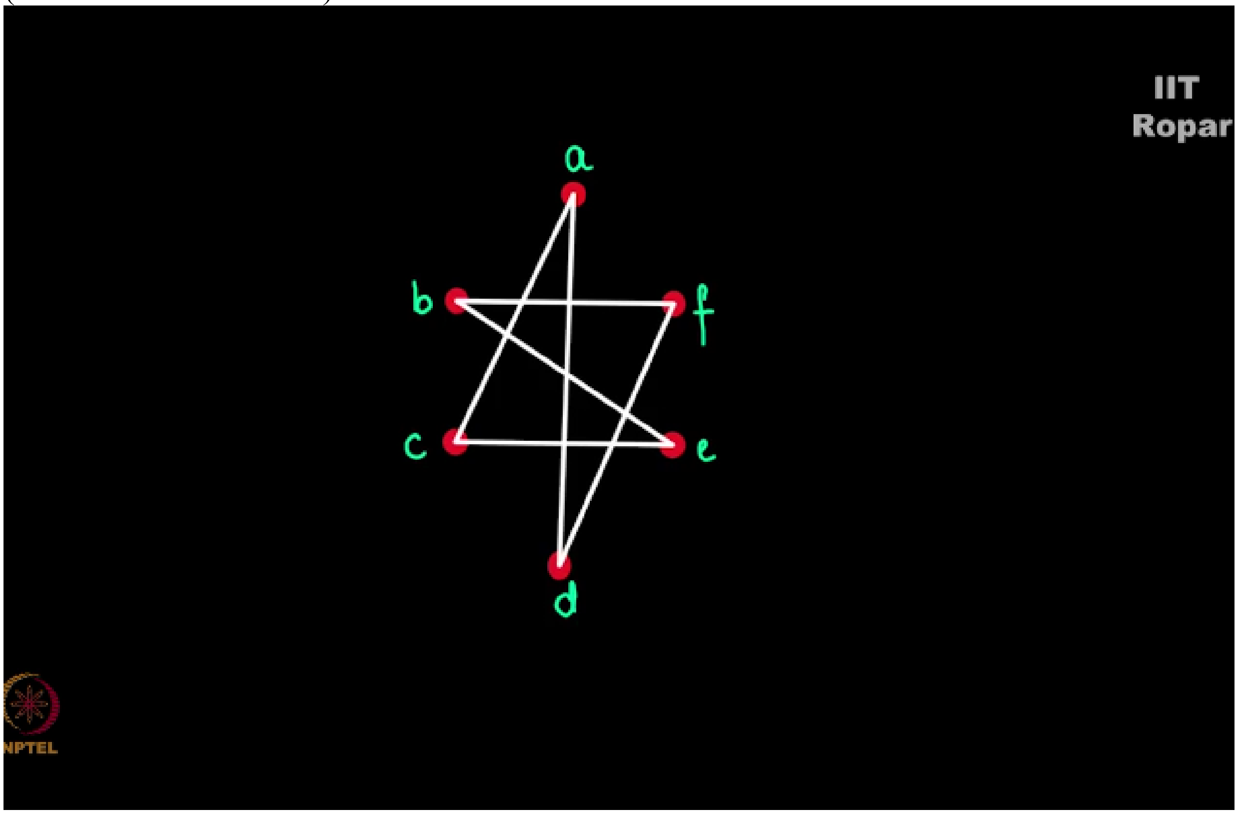

so let us start from A, check if it has an Eulerian circuit, what do I mean by that? I've to go through all the edges and come back to A, so I go from A to C, C to E, E to B, B to F, F to D, and D to A, guess we have covered all the edges and we have obtained an Eulerian circuit, this graph is Eulerian.

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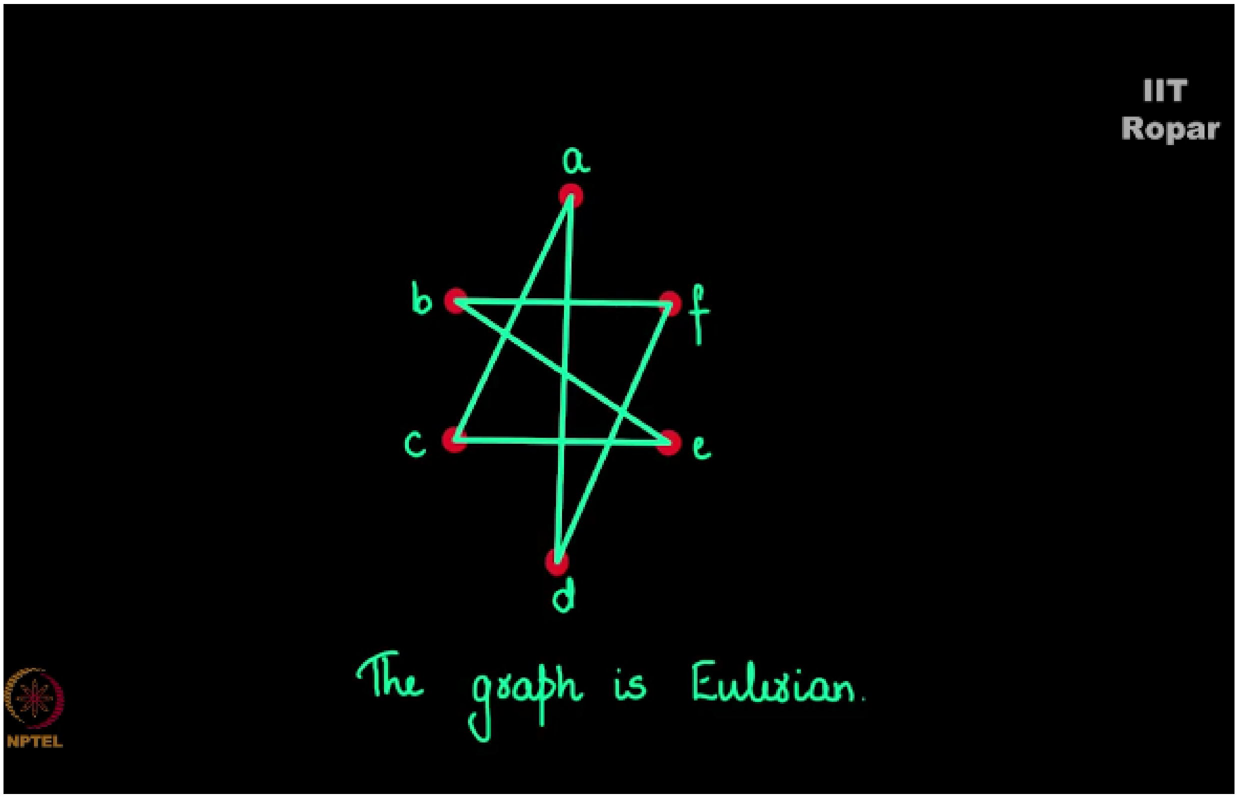

Consider this graph with the labeling A, B, C, D, E, F, G, H, I, so these are the labels,
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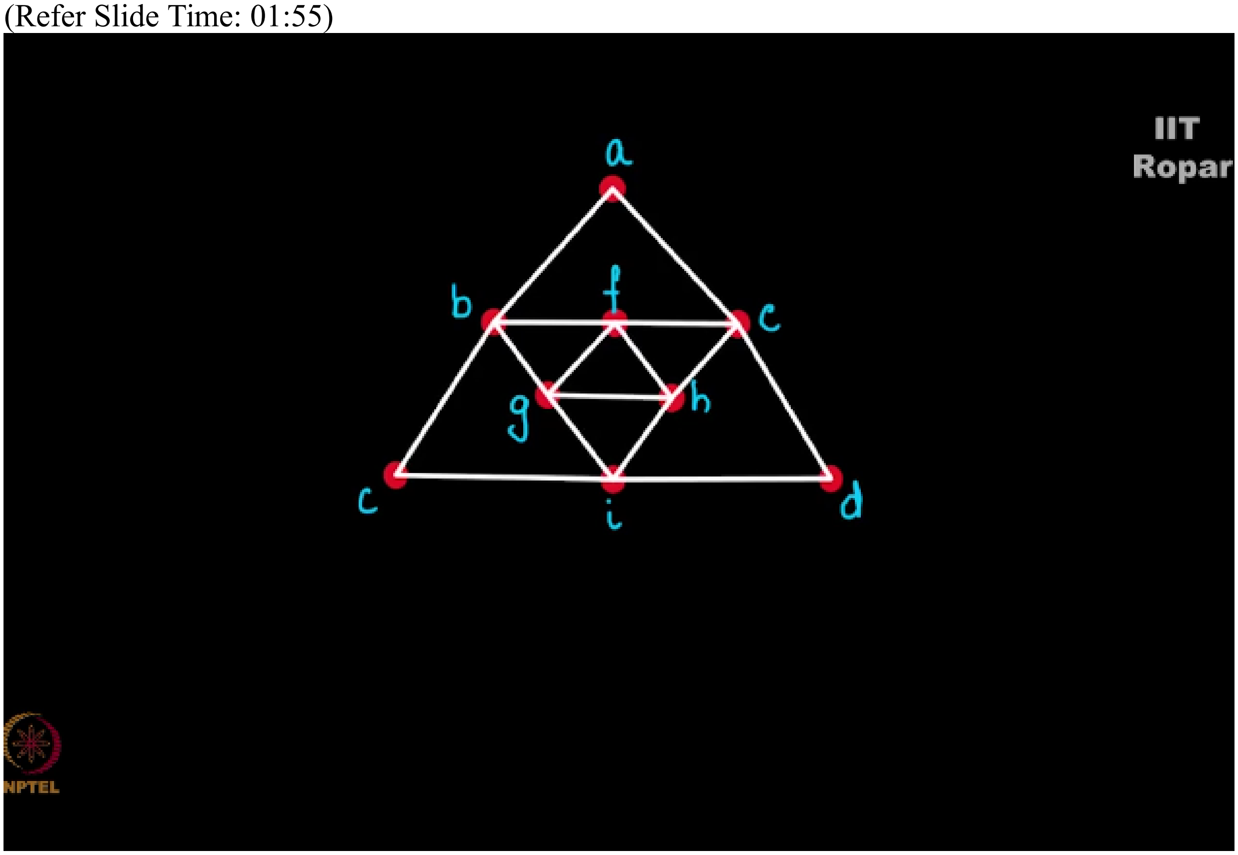

now I’m going to consider, I’m going to start from let’s say for a change vertex C, I’ll start from here, C to B, B to A, A to E, E to F, F to B, then to G, from G I’ll go to H, now H to I let me say here, and then to D, D to E, E to H, this is remaining E to H, H to F, F to G, now from G to I and I to C, so we have comeback to C again and hence this is an Eulerian graph.
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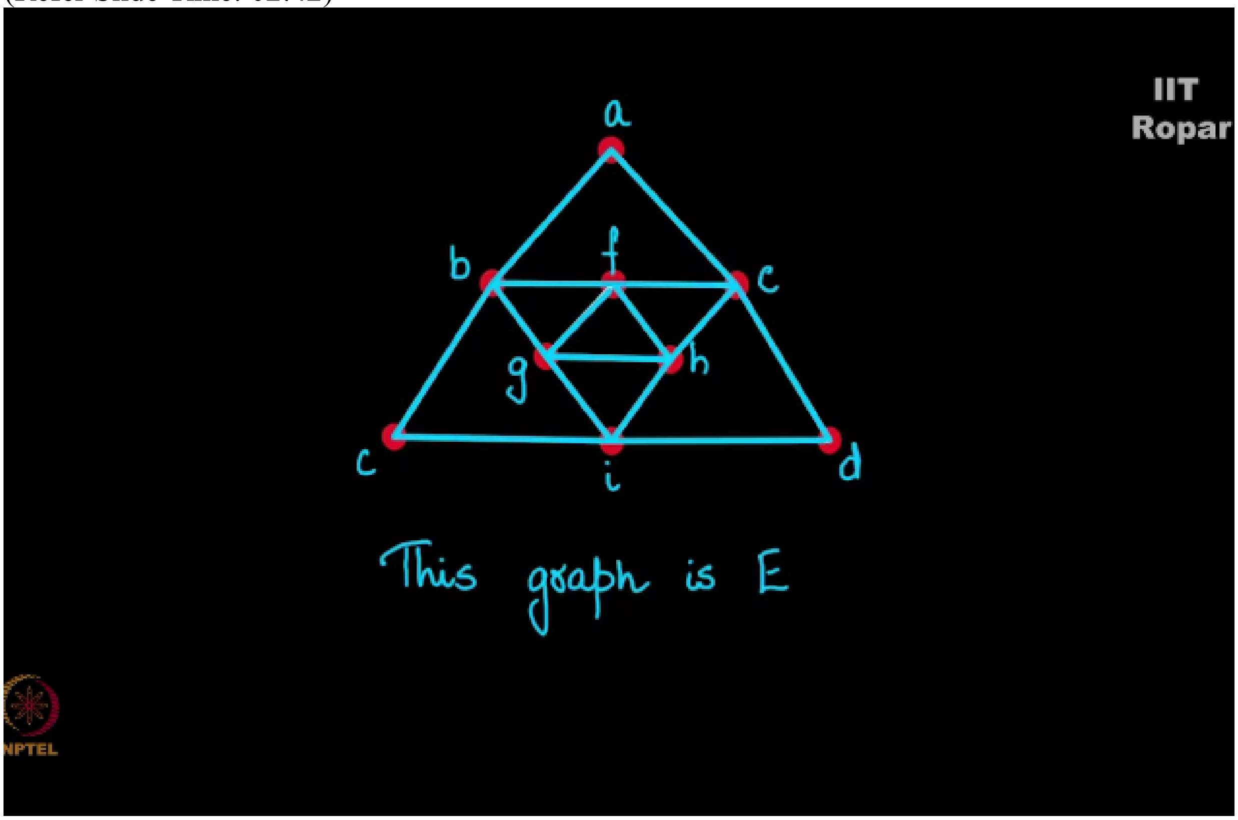

Consider this graph, do you see that this is a multi-graph, we have 2 edges for these 2 nodes,
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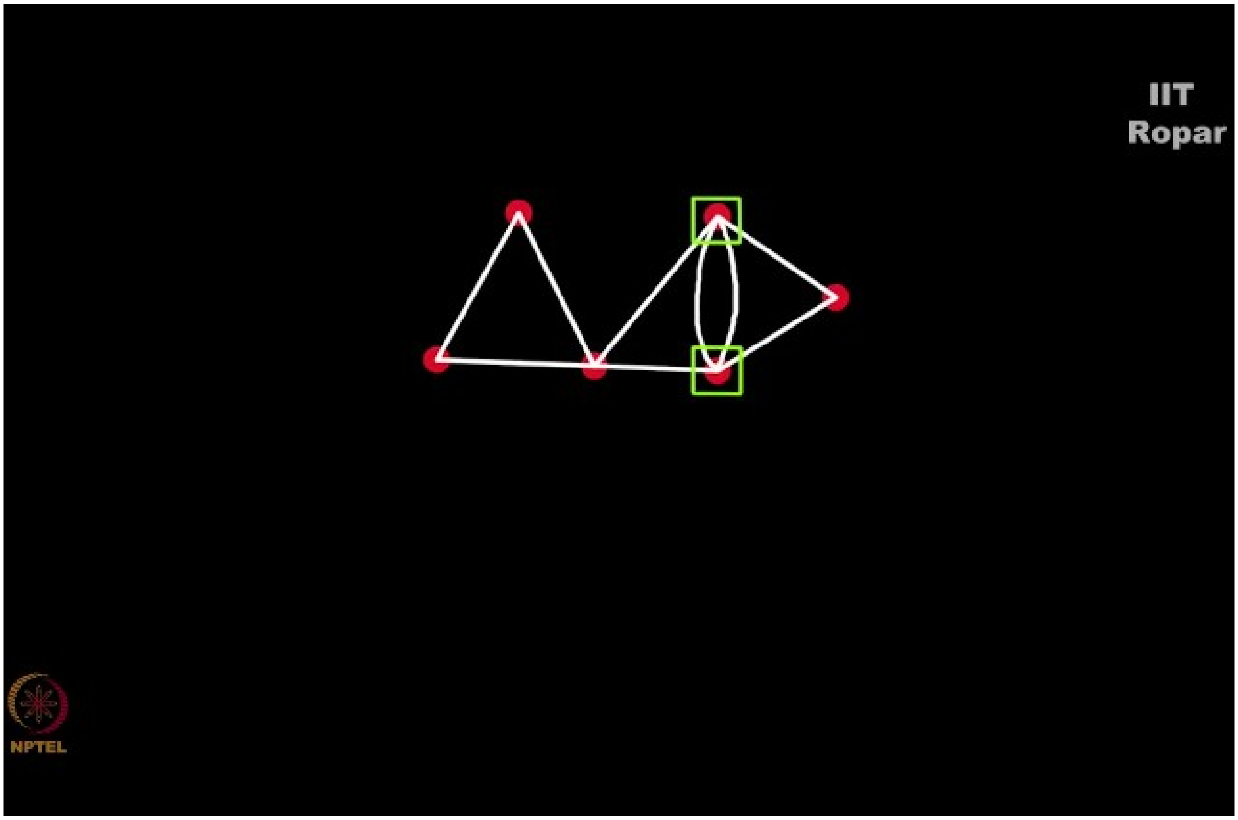

now I’ll label them as 1, 2, 3, 4, 5, and 6, I’ll start from let me say 3, 3 to 4, 4 to 5, I’m taking this path, this edge here 4 to 5, 5 to 6, 6 to 4, you see there is one more edge remaining from 4 to 5, I’ll take that route now, 6 to 4 is done, 4 to 5, 5 to 3, now it becomes easy 3 2, 2 1 and 1 3, this graph is also Eulerian.

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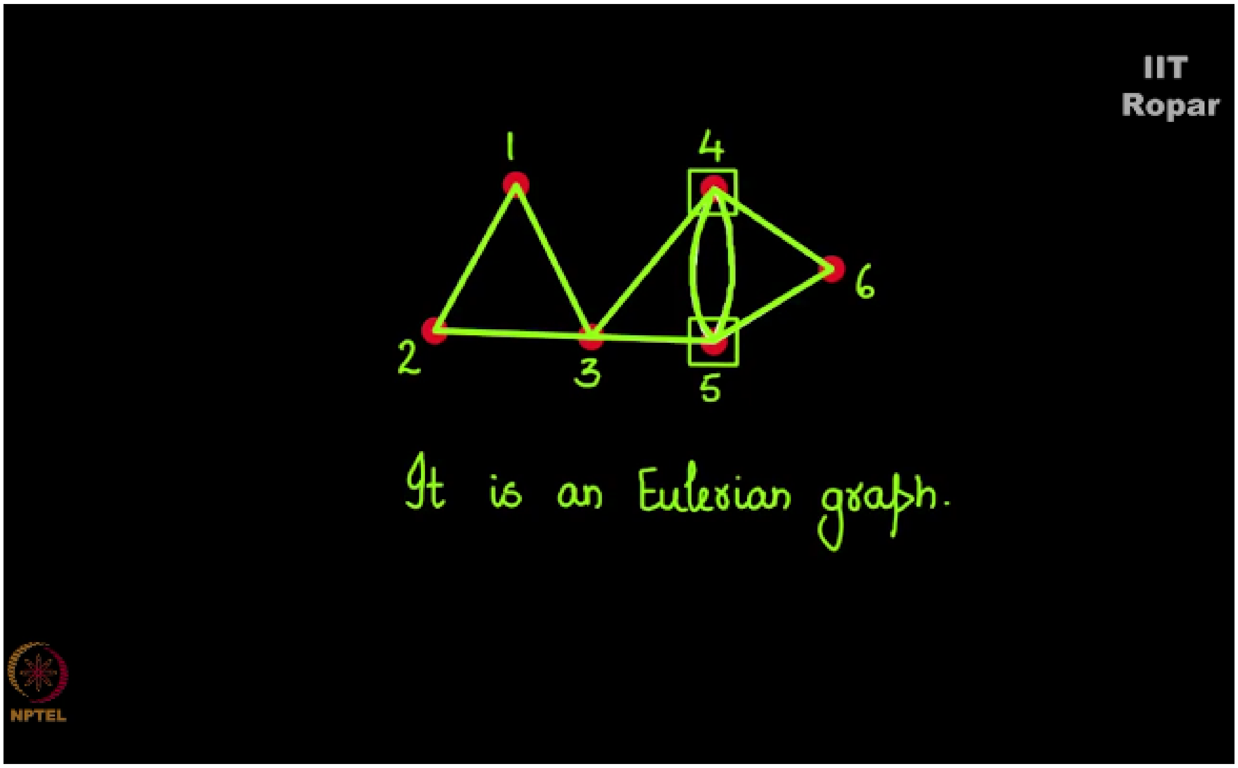

Consider this graph 1, 2, 3, 4, and 5,
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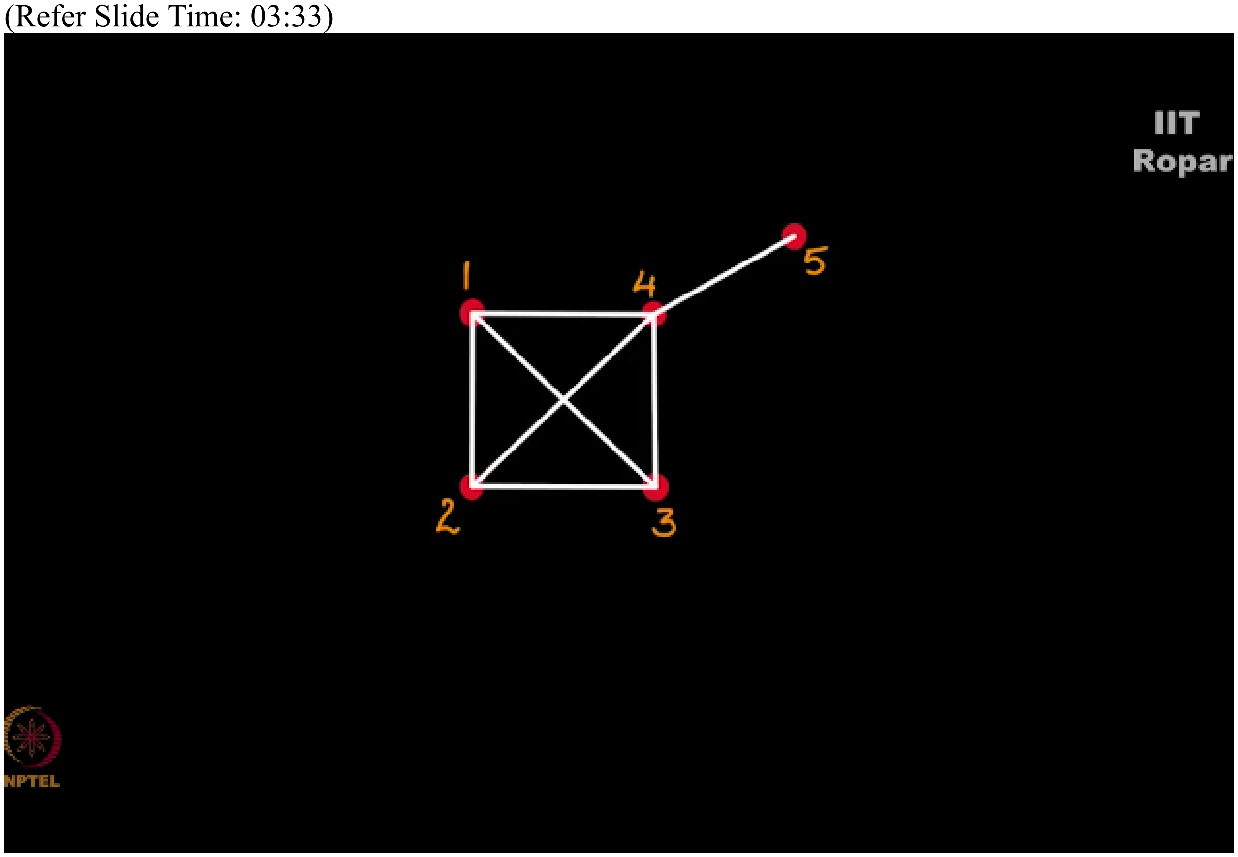

I'll start from 1, 1 to 2, 2 to 4, 4 to 5, you see I've got locked here, I've to again come back from 5 to 4 in order to comeback to this portion,

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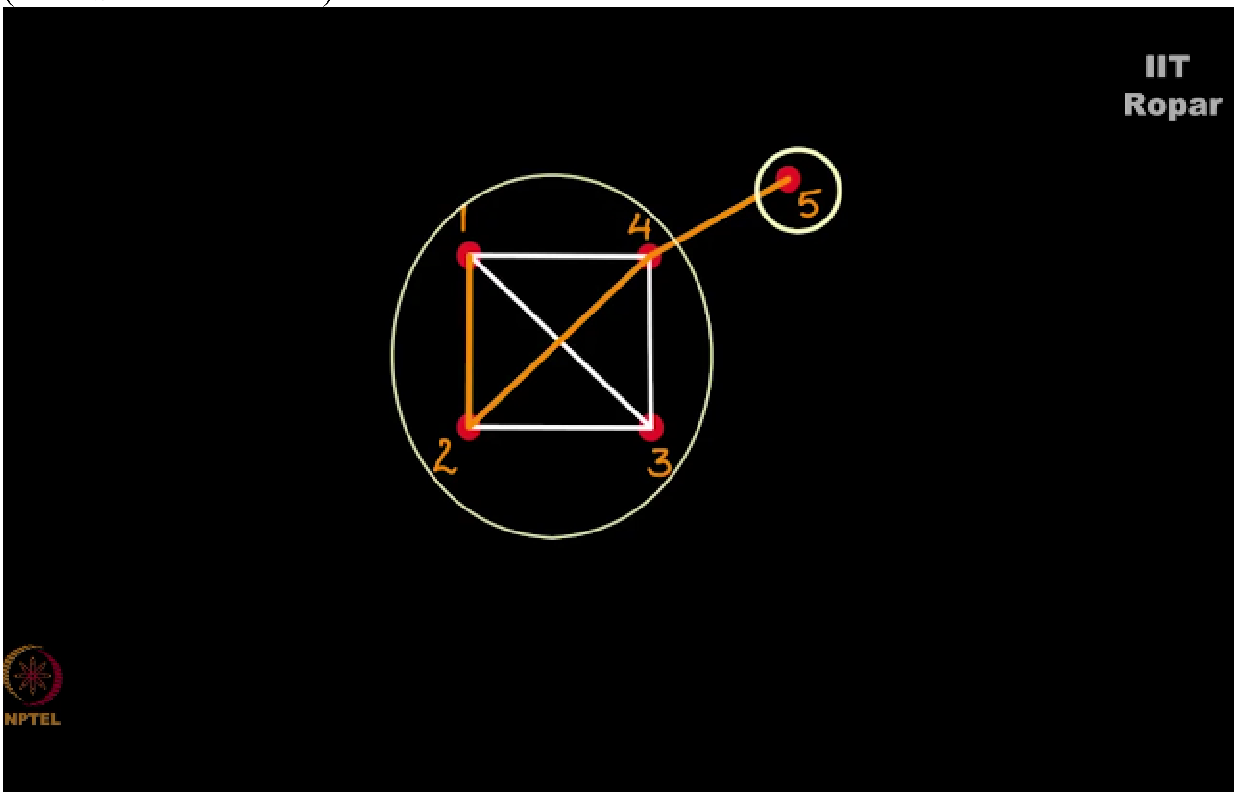

right, now can I do that? I cannot do that because I'm going to repeat this edge, and hence this graph is not Eulerian, I can stopped the process once I encounter such a situation, this graph is not Eulerian.

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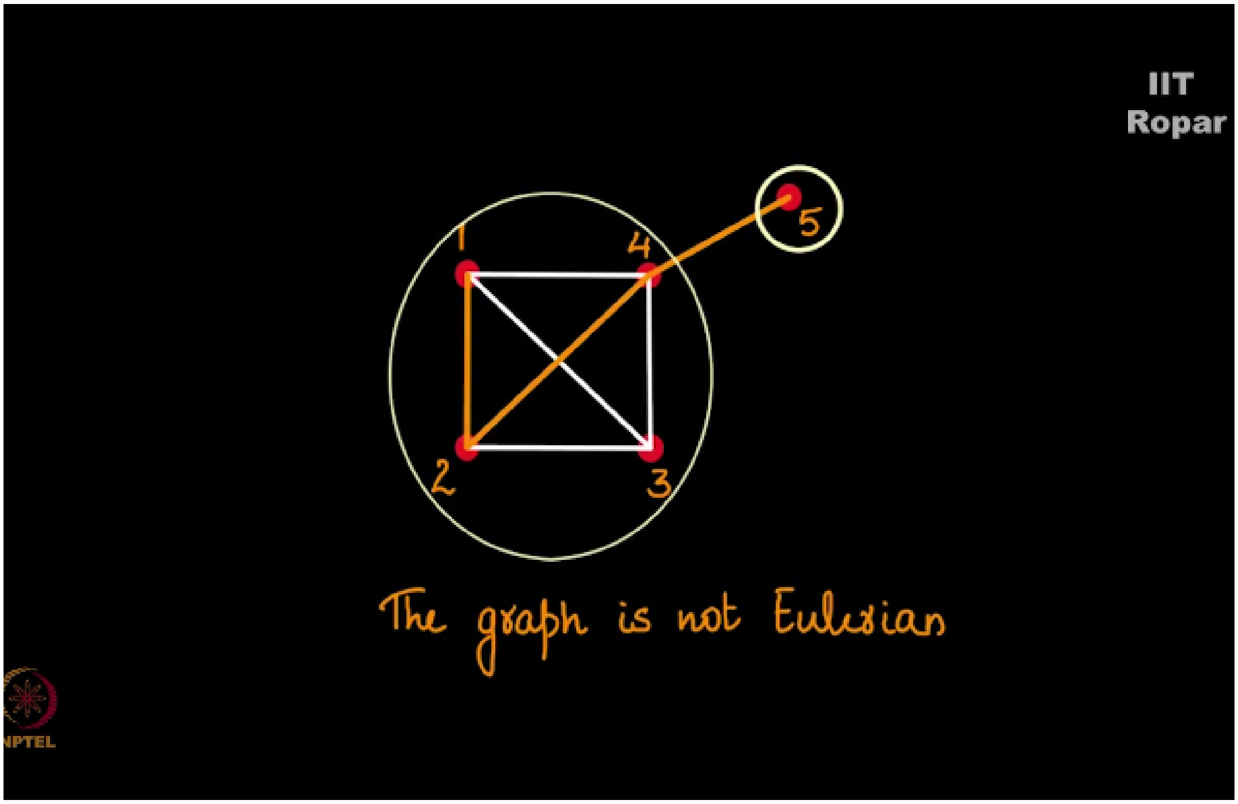

Consider this graph where we have a loop here and I'll label the vertices as 1, 2, 3, 4, 5, 6 and 7,
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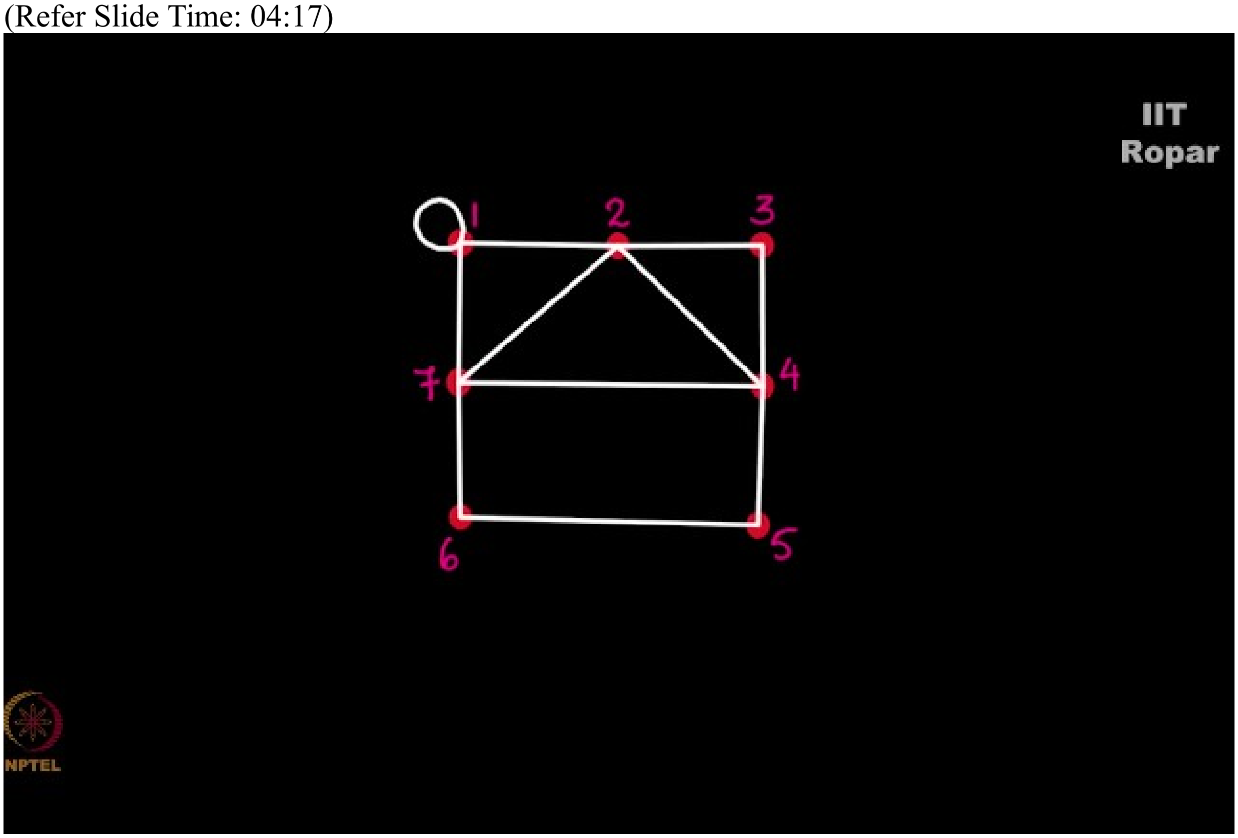

I’ll start from this vertex let me say 5, 5 to 6, 6 to 7, 7 to 4, 4 to 2, 2 to 1, 1 has a loop here so I cross the loop like this and again I’m at 1, 1 to 7, 7 to 2, 2 to 3, 3 to 4, 4 to 5, I’ve come back to 5 now, this graph is also Eulerian.

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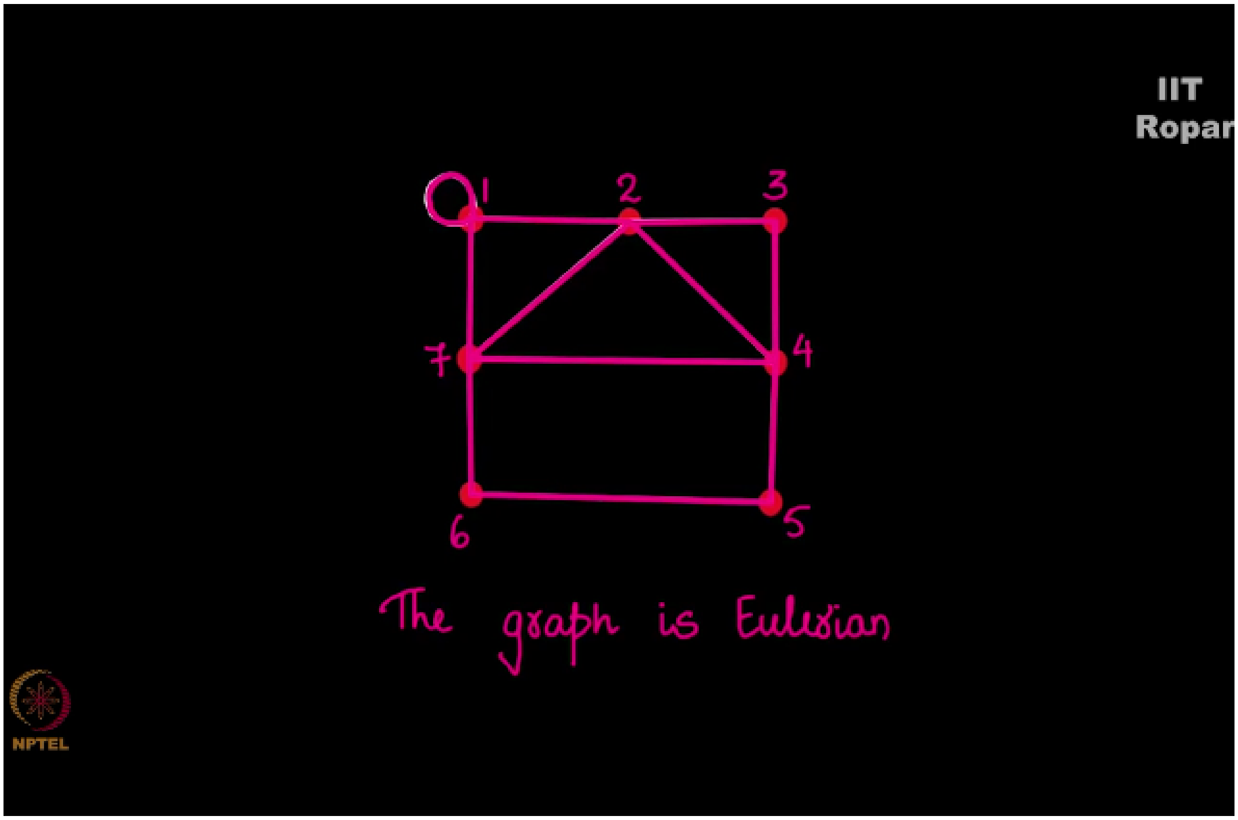

Consider this graph, I’ll start from this vertex let’s say 6 here, I’ll go from 6 to 5, 5 to 4, 4 to 1, 1 to 3, 3 to 2, 2 to 1, you see now I have to either come to 3 or to 4, which means I’ll be repeating either of these two edges.

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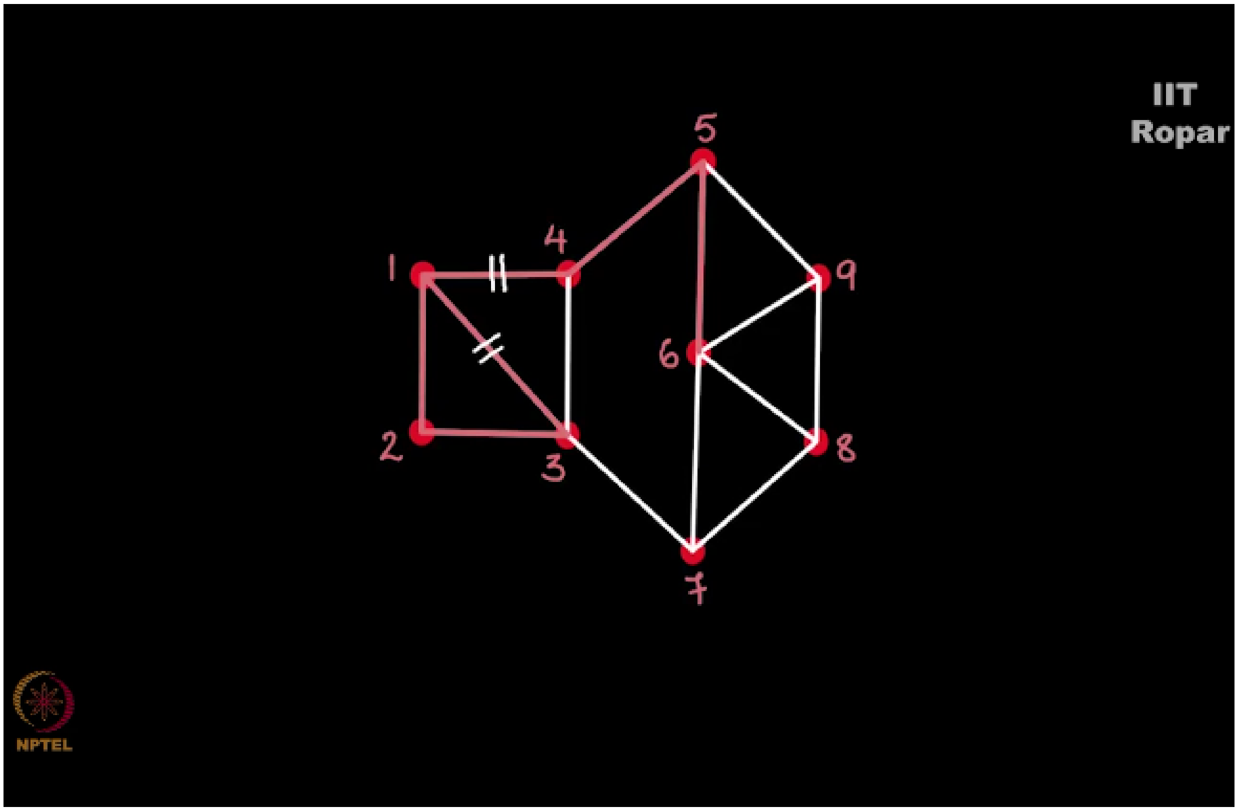

Now the rule gets violated, the graph is not Eulerian, but before I conclude let me just try if I can obtain an Eulerian circuit by starting from this vertex 9,

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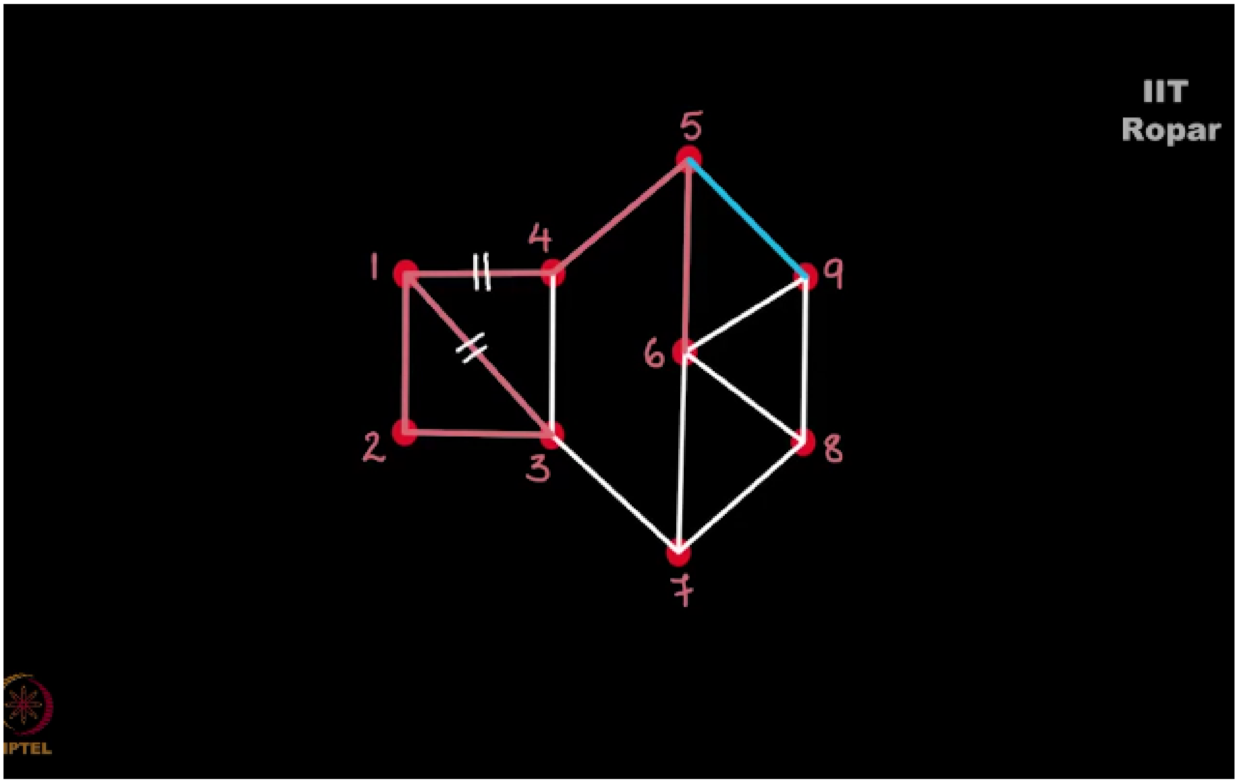

9 to 5, 5 to 4, 4 to 3, 3 to 7, 7 to 6, 6 to 8, 8 to 7, you see again here I have got locked here, I've to either go to 6 or to 3

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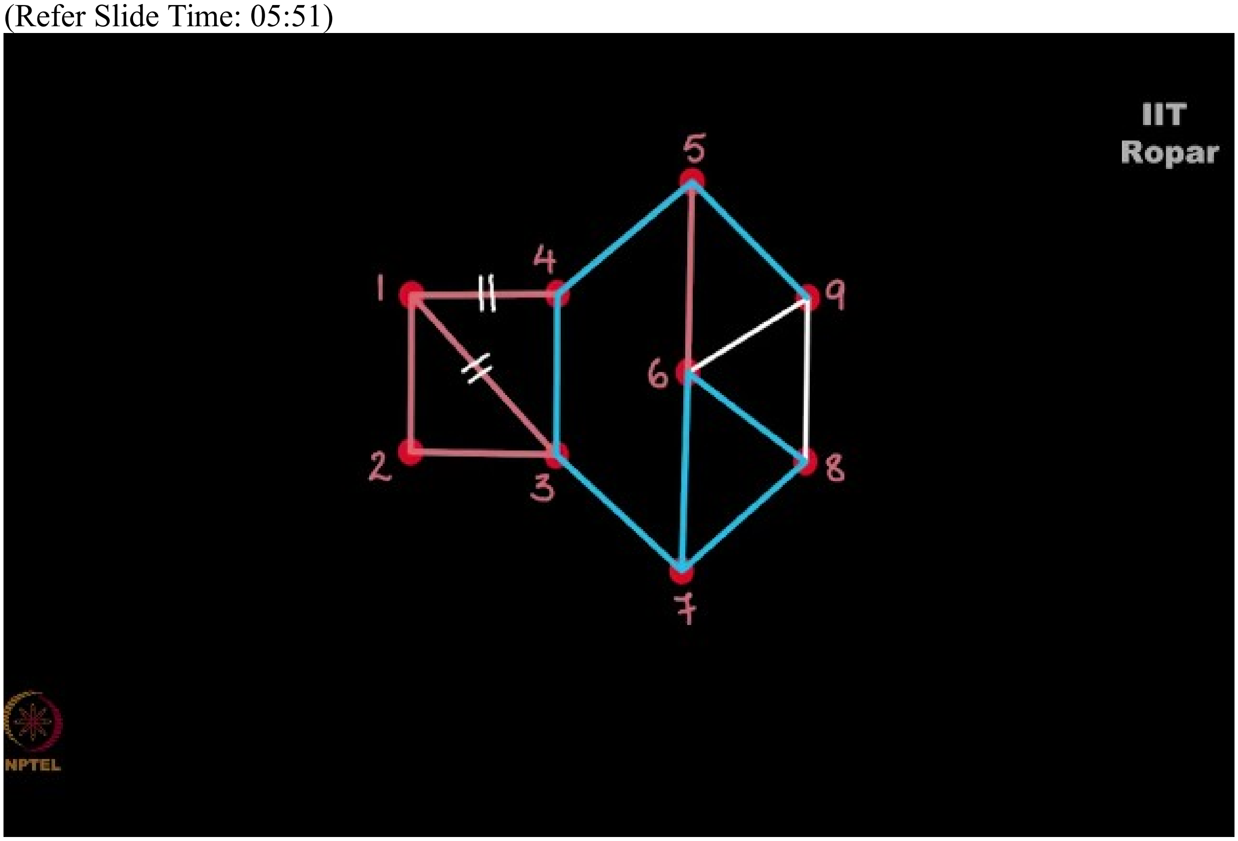

which means I'll be repeating either of these edges, again not allowed,
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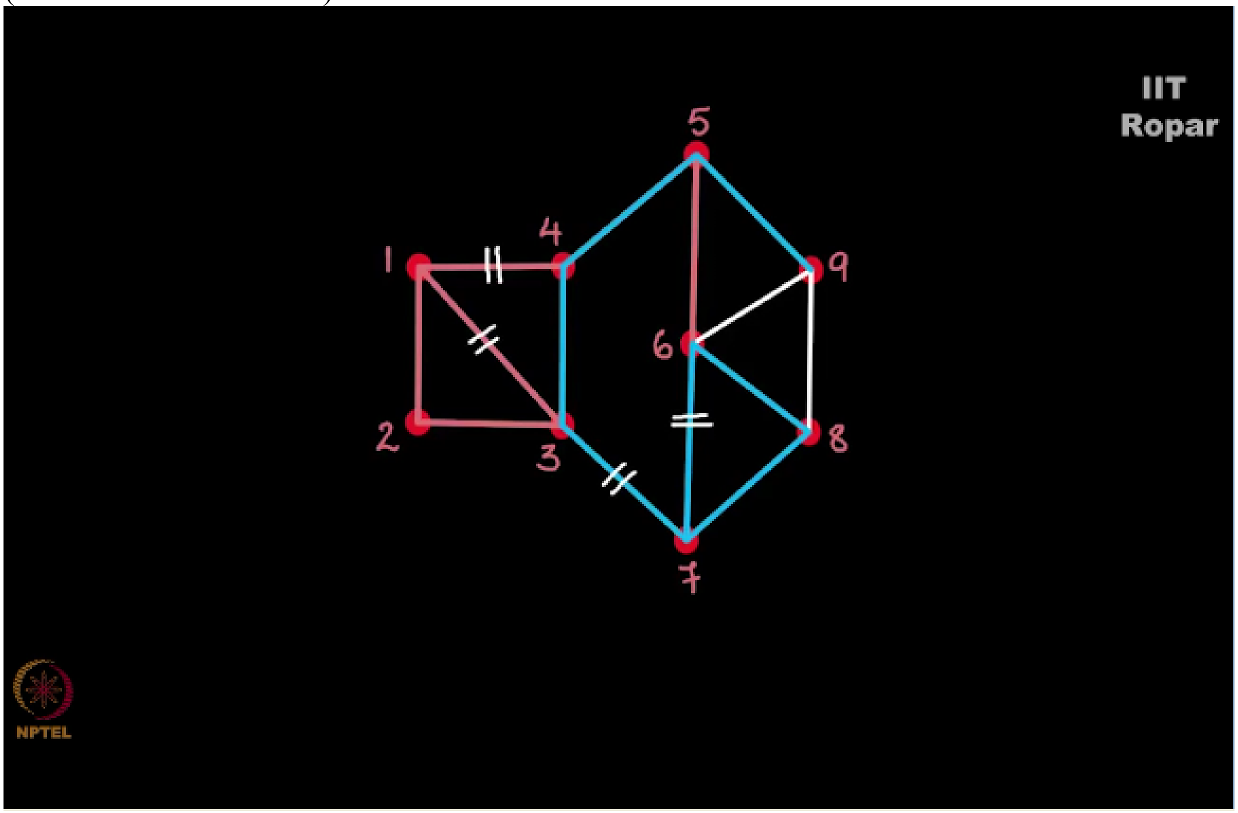

so we can conclude that this graph is not Eulerian, though you need not check it for every such vertex, once you encounter such a situation just for the first time you can conclude that the graph is not Eulerian.
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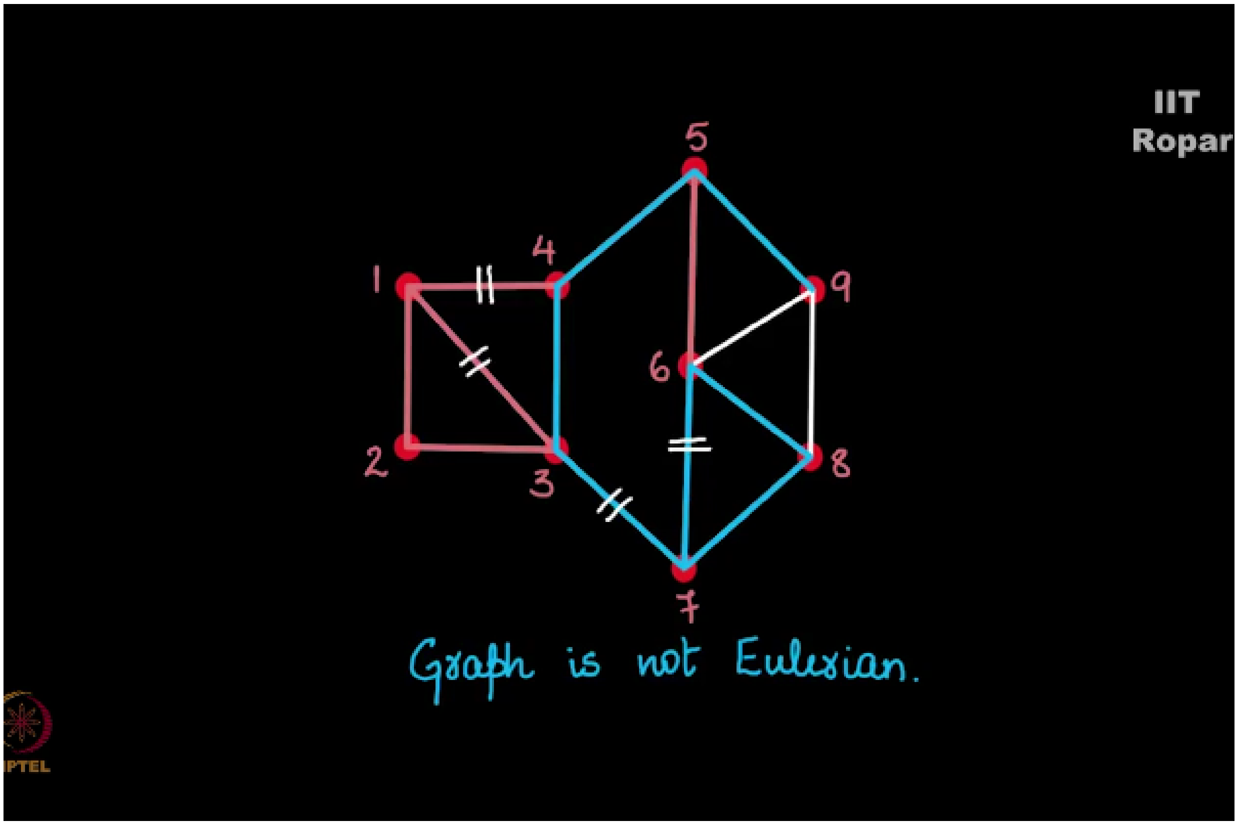

Consider this last graph, this will be the last example, I'll label it as A, B, C, D, E, F, G, H, and I,

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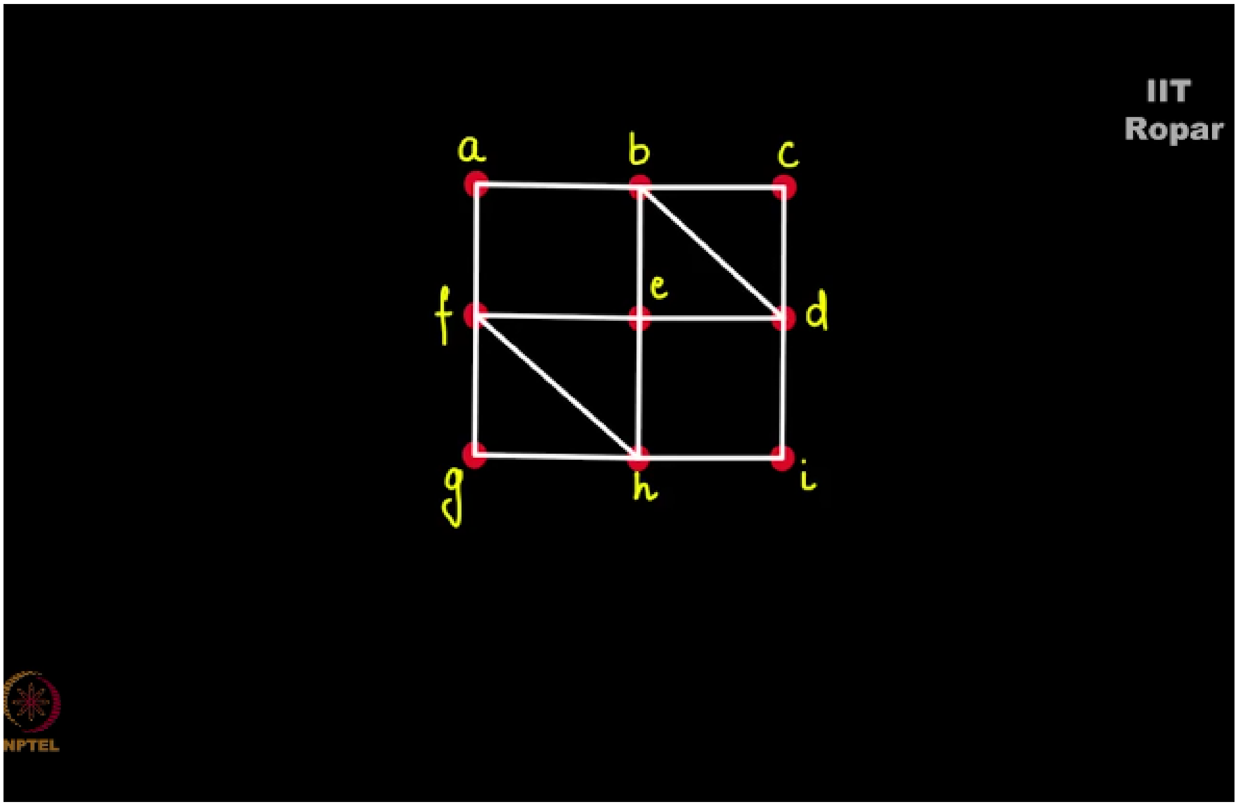

I'll start from let me say the center vertex E, E to B, B to A, A to F, F to G, G to H, H to F, F to E, E to D, D to let me say C, C to B, you see here this is remaining, so I'll take that B to D, D to I, I to H, and H to E, so we have reached back the vertex E by taking all, by covering all the edges we have reached E and hence the graph is Eulerian.

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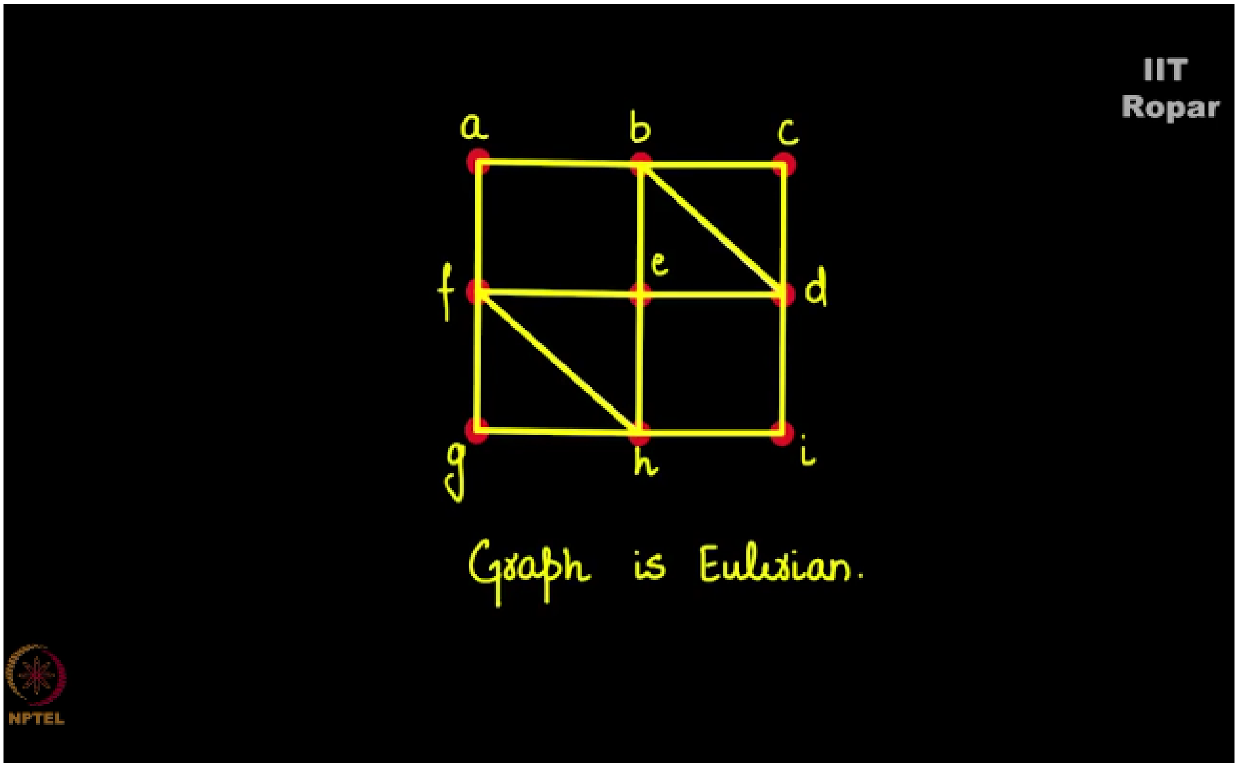
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Graph is Eulerian.
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