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NPTEL ONLINE COURSE

Discrete Mathematics

Let Us Count

Problems on Binomial theorem

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So that was a lot of learning on binomial theorem, multinomial theorem and all the properties of binomial theorem. Let us now solve some problems on binomial theorem.

The first one. Expand A plus B whole to the 6. This is quite a direct problem. You just have to apply the formula here. So let me do it now. A plus B whole to the 6 after applying the formula will give me 6C0, A to the 6, B to the 0 plus 6C1 A to the 5 into B plus 6C2 A to the 4 B square plus 6C3 A cube B cube plus 6C4 A square B to the 4 plus 6C5 A into B to the 5 plus 6C6 A to the 0 B to the 6 and after simplifying all the combinations that is 6C1, 6C2 etcetera, and simplifying we get A to the 6, 6C0 is 1 and therefore we get A to the 6 here plus 6C1 is 6 so it becomes 6 into A to the 5 B plus 6C2 gives me 15 into A to the 4 B square plus 6C3 is 20 A cube B cube plus 6C4 is 15 and hence I'll write that as 15 A square B 4 plus 6C5 is same as 6C1 therefore it is 6A into B to the 5 plus 6C6 is 1. So as you can see here the coefficients go on increasing and at the point they become the maximum and again decrease. So this was with A plus B whole to the 6.

Now let us go on to the next question. Expand 1.04 whole to the 4. you might be thinking why am I even asking this question because it's very simple to calculate this on a calculator but let us see the beauty of binomial theorem and use it to expand 1.04 whole to the four. So 1.04 whole to the four I can write it as 1 plus 0.04 whole to the four. Now it becomes very simple. You can directly apply the formula here. Please do remember this is in the form 1 plus X whole to the n so as I had told this becomes 4C1 1 to the 4 plus 4C0 1 to the 4 plus 4C1 0.04 plus 4C2 into 0.04 whole square plus 4C3 into 0.04 whole cube plus 4C4 into 0.04 whole to the four. It's all 1 here and therefore I am skipping it. I'm not telling it because in every term we have the powers of 1 this gives me 1 plus 4 into 0.04 plus 4C2 gives me 6 so 6 into 0.0016 plus 4C3 is same as 4C1 which is 4 into 0.04 cube is 0.00064 plus 4C4 is 1 into 0.04 whole to the four gives me this number. Now after multiplication and simplification I will get this as 0.16985856. So let me round off this to two decimal places and therefore I will get 1.04 whole to the four as 1.17. So you must be thinking it can be easily done on the calculator but did you see the beautiful application of binomial theorem here. It can be used for such approximations.

So the next question. Find the fourth term in the expansion of x cube by 2 minus 2 by x square whole to the nine. So the term seems to be complicated but the problem is very simple. So we need to find the fourth term in this expansion. So we have been asked to find a specific term and not all the terms. So as you remember the r^{th} term was given by nCr minus 1 A power n minus r plus 1 B power r minus 1. So what is r here? r is 4 and let me write the fourth term as T4 I just denote it like that. So T4 is equal to, so n here is 9 as you can see so 9Cr minus 1 which gives me 9C3 into A power what is n minus r plus 1 n is 9 and r is 4 so we get A power 6 B to the r minus 1 is B cube. Now so what does A and B here? A is x cube by 2 and B is 2 by x square. Yes. So let me substitute them and what do I get as the answer, 9C3 gives me 9 factorial by 3 factorial into 6 factorial into x cube by 2 whole to the 6 into, please note B is -2 by x square into -2 by x square whole cube. So after simplifying 9C3 I'll get it as 84 into x to the 18 by 64 into minus 8 by x to the 6. I have just multiplied the powers here. So this gives me after simplification minus 21 by 2 x cube. You can do the calculation yourself. So the answer is the fourth term in the expansion of this gives me minus 21 by 2 x cube. I hope it was clear. If it is not please pause the video here and there and watch it again.

Now the next question. Determine if the expansion of x square minus 2 by x whole to the 18 will contain a term containing x to the 10. The question is not difficult. Let me just explain. So the expansion of x square minus 2 by x whole to the 18. We have to find out if in the expansion of this there is a term containing x to the 10. So it is very similar to the previous problem. So what is Tr? Tr as we know this is the formula. Now let me apply the formula for this. So how can I get -- how will I find out if there is a term containing X to the 10. So what I mean here is Tr must have X to the 10 somewhere, right. So how will I get X to the 10? Okay. Let me apply the formula first. n is what? n is 18, r we don't know so let us keep it as it is, 18C r minus 1 into A will be x square so it is x square to the power 18 minus r plus 1 into B is -2 by x so it is -2 by x whole to power r minus 1. Now simplifying this 18C r minus 1 into x to the 38 minus 2 r into minus 2 to the power r minus 1 by x power r minus 1. As you can see this is in the form x power m by x power n and I can write it as x power m minus n. I'll simplify this. It becomes 18C r minus 1 x to the power 39 minus 3 r into minus 2 power r minus 1. Now so this is the expansion I have. I don't know what is r. So if there is a term containing x to the 10 then x to the 10 here we'll have some coefficient like this and that x to the 10 can be equated to x to the power 39 minus 3r, I am equating the like terms on the right-hand side and the left-hand side and therefore x to the m is equal to x to the n and this implies x to the m minus n is equal to is 1 if X is not 0. So this is the result we know earlier and therefore I can write this as 38, sorry 39 minus 3r is equal to 10. simplifying this gives me r is equal to -29 by 3 which is not possible because we know that r must be an integer and hence the expansion of x square minus 2 by x whole to the 18 does not contain a term containing x to the 10. I hope it was clear.

So let us move on to the next question. Evaluate 96 the whole cube. Again here it becomes very easy to calculate this on a calculator but let me explain how this can be done using binomial theorem. So as I had used binomial theorem for approximation, I can use it here too in this way. 96 can be written as 100 minus 4. This is very simple. So 96 cube will be 100 minus 4 the whole cube. This is the hint for this question and now it becomes very simple just applying the binomial theorem 3C0 into 100 cube into 4 to the 0 plus 3C100 square into minus 4 to the 1 plus 3C2 into 100 power 1 into minus 4 square plus 3C3 into 100 power zero into minus 4 cube. So now let me simplify all of this. This is very simple and I will get it as, it's a long number. 884736. I leave the intermediate steps to you to solve it and I'm just giving the last answer for you to verify.

Now find the middle term in the expansion of 3x minus 4 whole power six. So a lot of things to keep in mind here. First of all we need to find the middle term and the next one is n is even here. As we had seen earlier if n is even there is exactly one middle term and therefore we need to find it but what is the middle term? Which term is the middle term? That is given by n plus 2 by 2, n is 6 and therefore it is 8 by 2 which is 4. Therefore the fourth term in this expansion is the middle term. So now we need to find even the 4th term here. So directly applying the formula let me denote the fourth term as T4. So T4 is equal to nCr minus 1 into A to the n minus r plus 1 into B to the r minus 1. So when I substitute I get T4 is equal to 6C3 into 3x the whole cube into minus 4 the whole cube. I hope it is clear because r is equal to four here and applying it appropriately gives me this expression. Now simplify; 6C3 gives me 6 factorial by 3 factorial into 3 factorial into this is 27x cube into minus 64. So after calculation this gives me minus 34560x cube. So this is the middle term in the expansion of 3x minus 4 the whole power 6.

So that is it.

Next question. What is the coefficient of x square yz in the expansion of x plus y plus z whole to the four? So here comes the multinomial theorem. So how do we apply it? As I had told you we need to find the coefficient of x square yz here so we have three terms. So let me write it in general. The coefficient of x to the n1 into y to the power n2 into z to the power n3 is given by n factorial by n1 factorial into n2 factorial into n3 factorial. So this is the generalized coefficient.

Now applying it here what do I get coefficient of x square yz is given by n is 4 so 4 factorial by 2 factorial into 1 factorial into 1 factorial which gives me on simplification 12 as the answer.

The last question. What is the coefficient of x1 square x cube x4 whole cube into x5 in the expansion of x1 + x2 + x3 + x4 + x5 whole to the 7? So this is a huge term and we have to apply multinomial theorem here. So on the similar lines as the previous problem the coefficient of x1 square we don't have x2 here into x3 into x4 cube into x5 gives me n is 7 here so we have 7 factorial by 2 factorial into 1 factorial into so there was no x2 term and hence it was x to power 0 so I am not writing a zero factorial here. I'm directly going to x3 so x3 is power is 1 and therefore it has 1 factorial into 3 factorial into 1 factorial. I hope this was clear and on simplification this gives me 7 factorial by 2 factorial into 3 factorial which is 420.

Okay. So these were some of the problems on binomial coefficients and multinomial coefficients. So far we have seen what is binomial theorem, multinomial theorem, the coefficients, how to find a particular term here a specific r^{th} term or a K^{th} term, how to find the middle term and we have solved several problems. So now we'll be moving on to the next concept in the next video.

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