NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 2
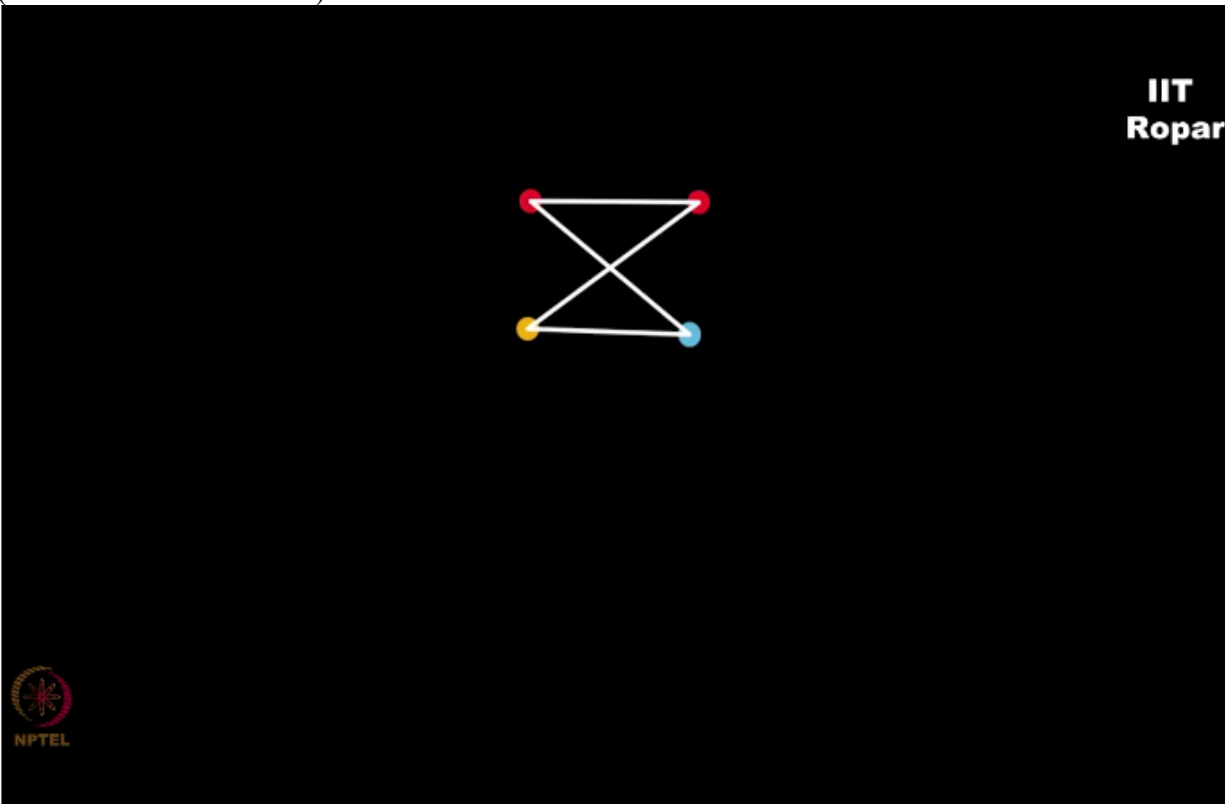
Bipartite graphs - A puzzle

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Here is a puzzle for you all, look at this take a bipartite graph,
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do you see this bipartite graph? It has a 4 cycle in it and this is the 4 cycle.
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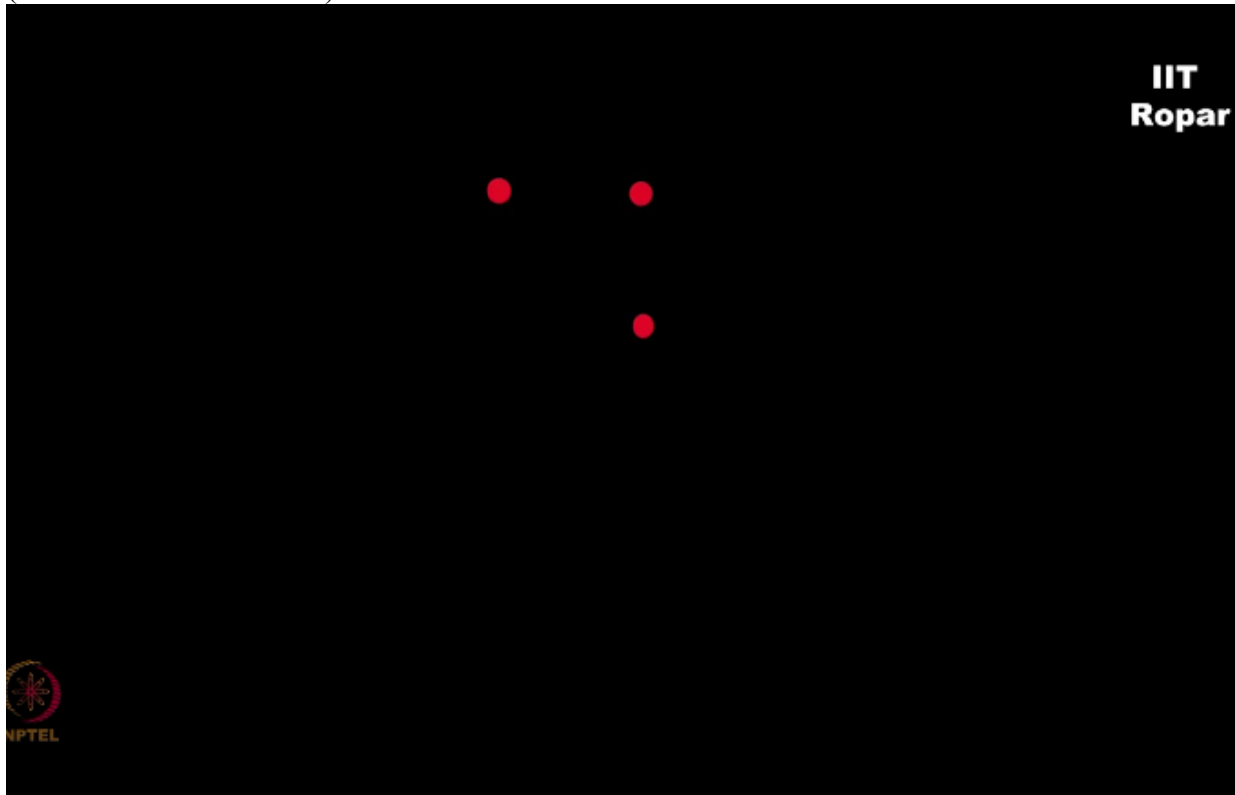Now can you find a 3 cycle in a bipartite graph?
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Can you find a 3-Cycle in a Bipartite Graph?

Let us try, if in case there was a 3 cycle a triangle in a bipartite graph one node will be this side, the other two nodes will be that side,

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if all three nodes are on the same block, it's the same block then edges will be within the partition, that's not allowed you see bipartite graph means edges are always across, it is never within, so you see

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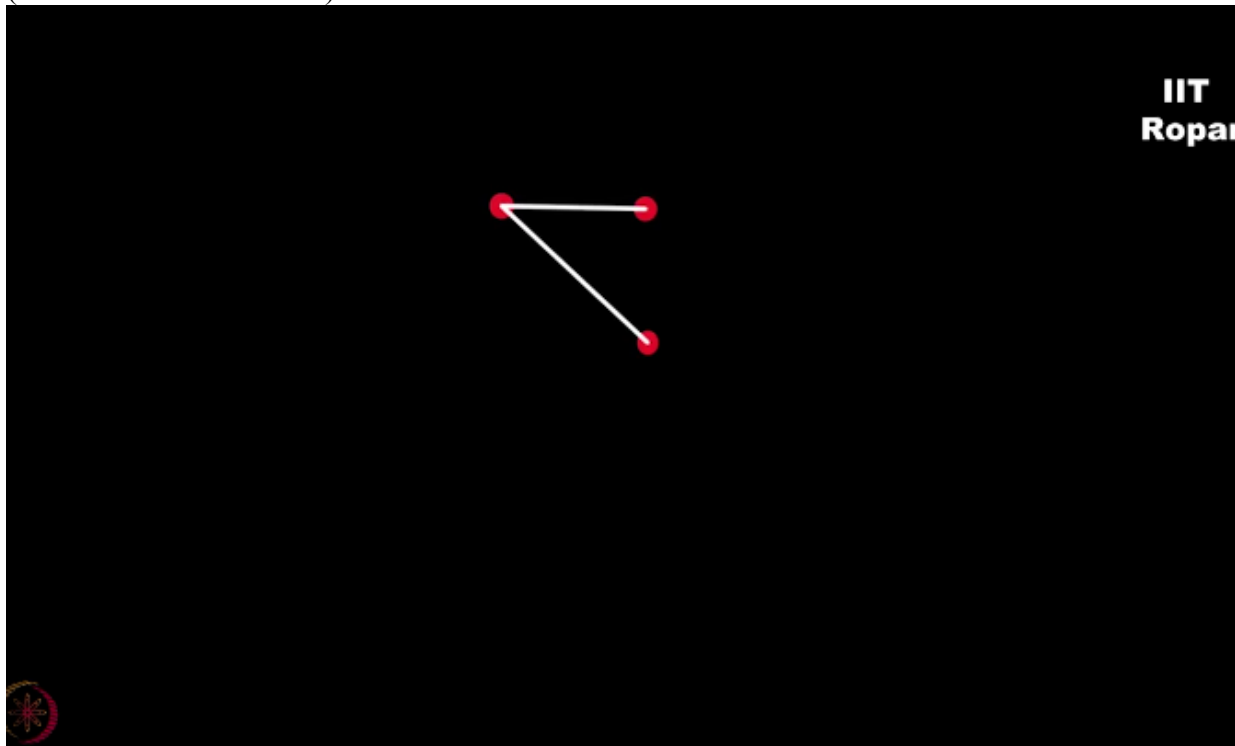
no matter how hard you try you'll not be able to find a triangle in a bipartite graph, because it's impossible for you to have one as it forces for an edge to appear within the partition, okay, it may take some time to sink in, but yes you sort of see that triangles are not present.
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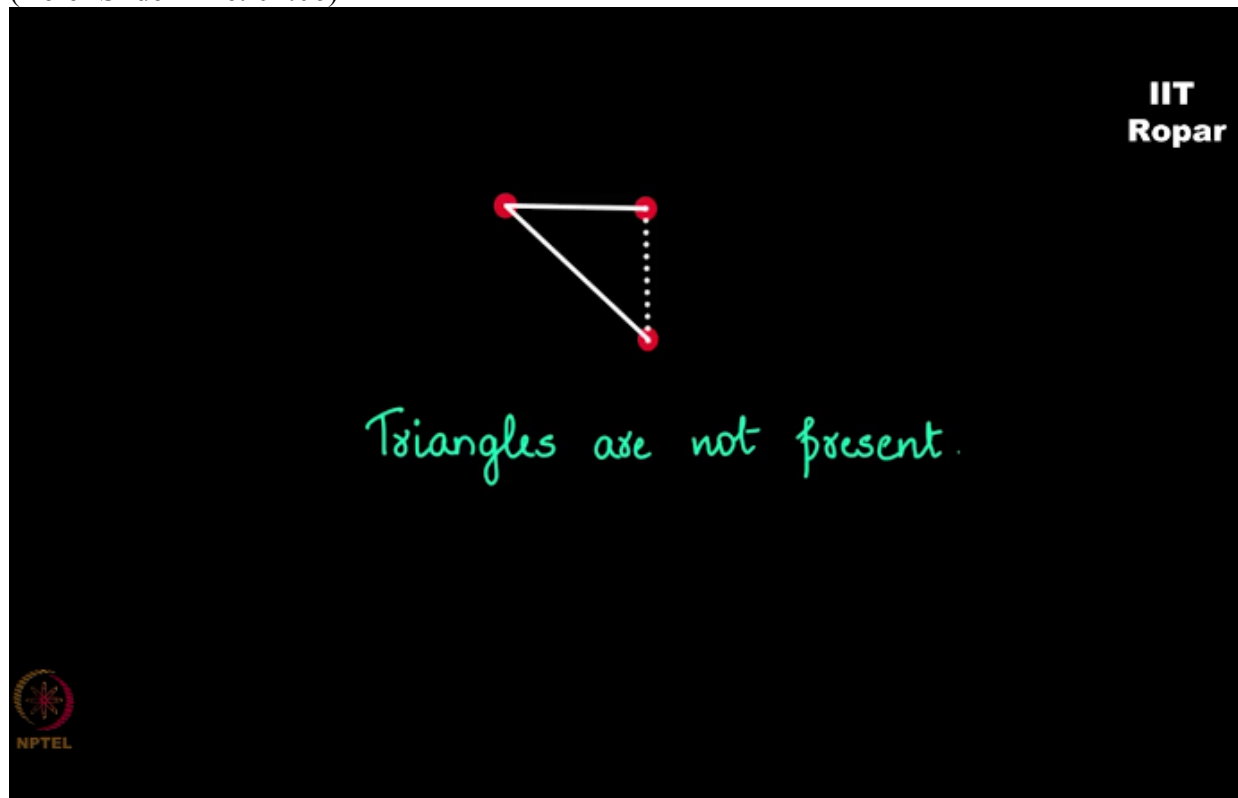
Now let me extend this and ask the next question, do you think a C4 is possible? A cycle 4, 4 cycle, is it possible? Yes, it's possible, look at this, right, it's very much possible, this is a bipartite graph with 4 vertices and this is a 4 cycle here, is a 5 cycle possible? The answer is no,
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Do you think C_4 is possible?

Is a 5-cycle possible?

NO

in fact odd cycles are never possible, it is impossible to have a bipartite graph with odd cycles, now why is that?

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Think about it, here goes a very nice story proof for this theorem, bipartite graphs cannot have odd cycles, why? A cycle always starts from a node, goes through all the nodes and comes back to the same node you see, you can take a path like that, you can traverse a cycle by starting from a node and coming back to the same node.
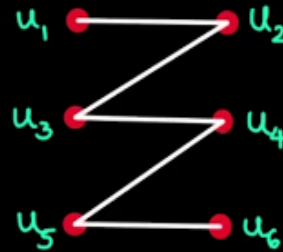
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Now if a bipartite graph well to have an odd cycle, let me say that the odd cycles starts from here, let's say from the vertex U, U1 let's say, U2 should be on the other wing on the right wing, why? Edges are always across and not within, and U3 will always be this side, on the left side, U4 will be this side, U5 will be this side, U6 will be this side,

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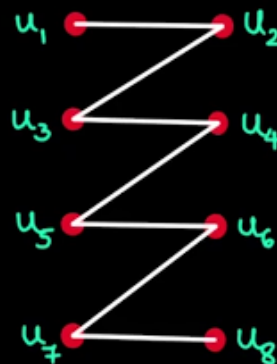
now when you come to U6 you have exhausted odd number of vertices, again U7 will be on the left side, fine, but U8 will be on the right side,
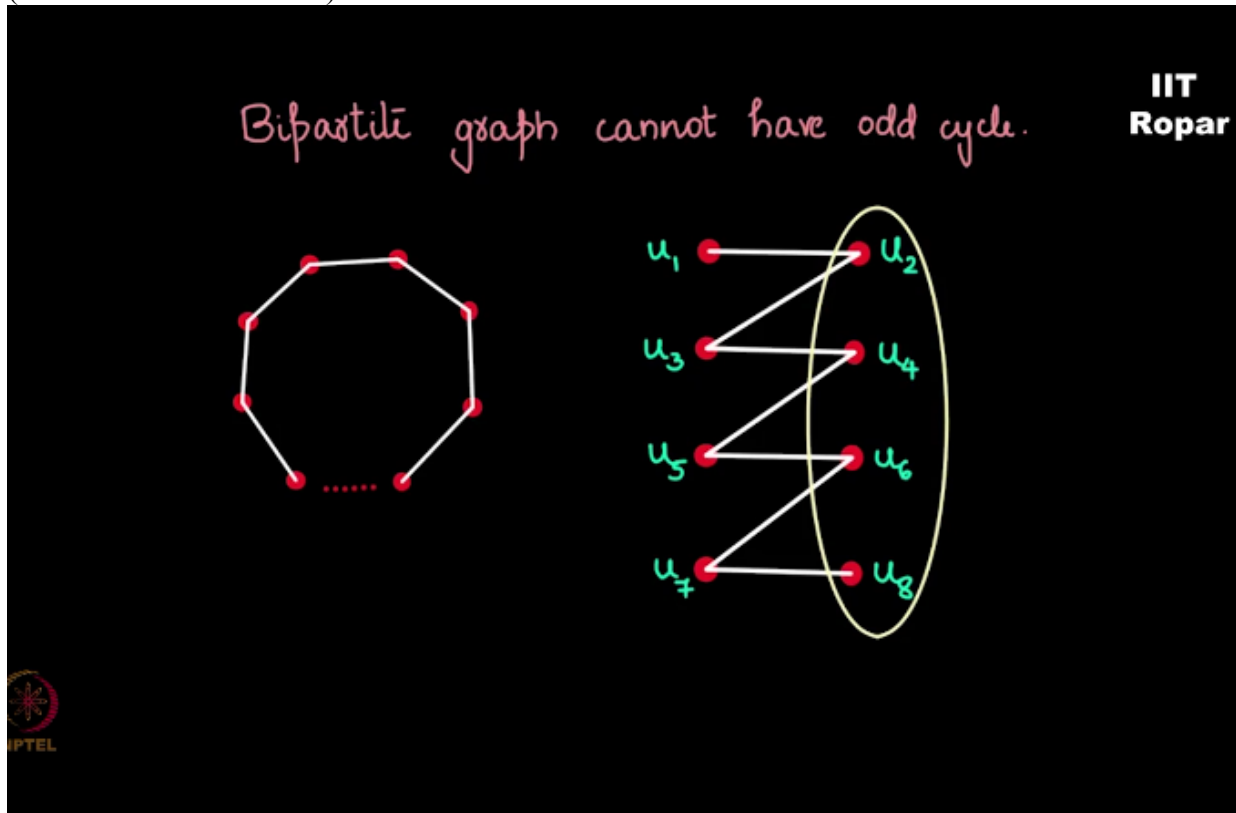
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so you observe that if you want an odd cycle you will be forced to end up in the right wing, you start from the left wing but you end up in the right wing, which means you can never complete a cycle, you see that's the reason why?
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In a bipartite graph you can never have an odd cycle, the reason is as simple as the reason why you cannot have a triangle, think through it, you all will understand.