NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 2

Which is more? Connected graphs or disconnected graphs

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Let me take all possible graphs on 10 nodes,
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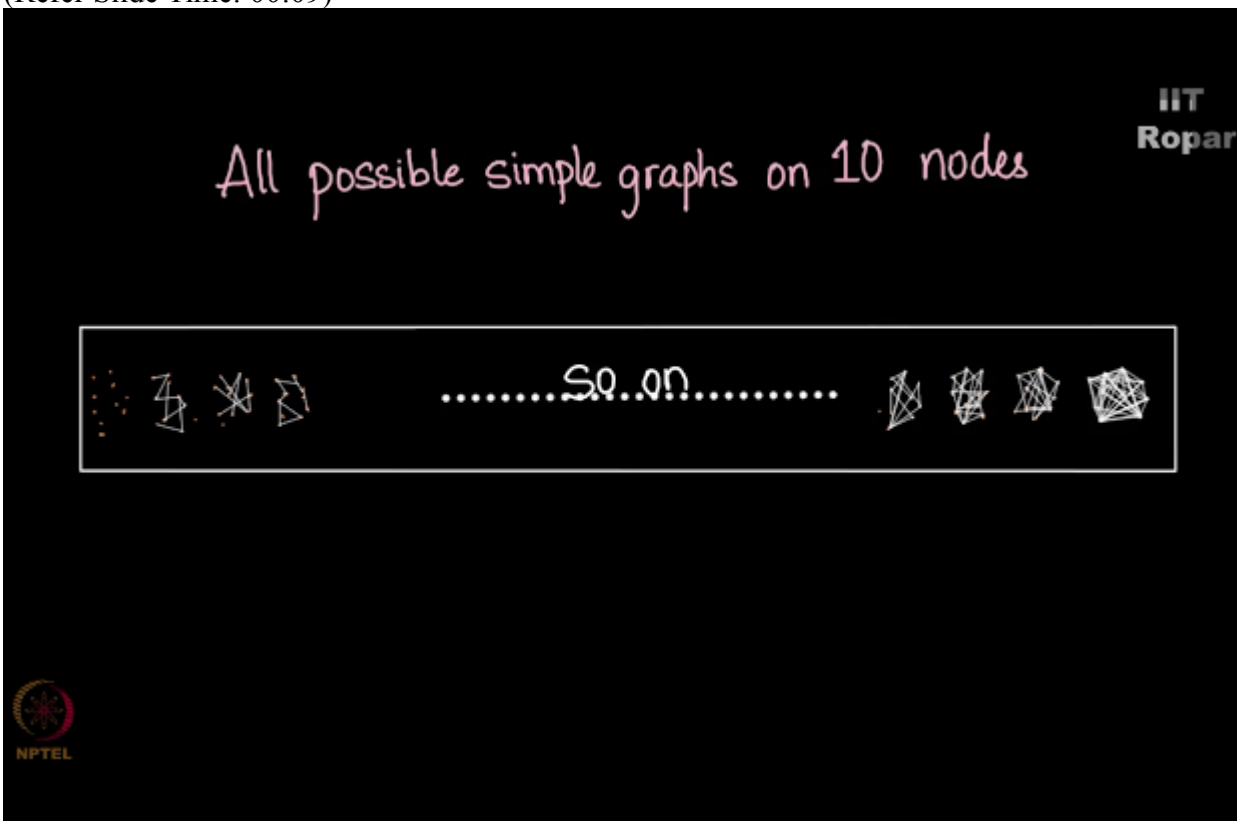

and I'll ask this question, I'll go to every connected graph on 10 nodes and paint it blue, why?
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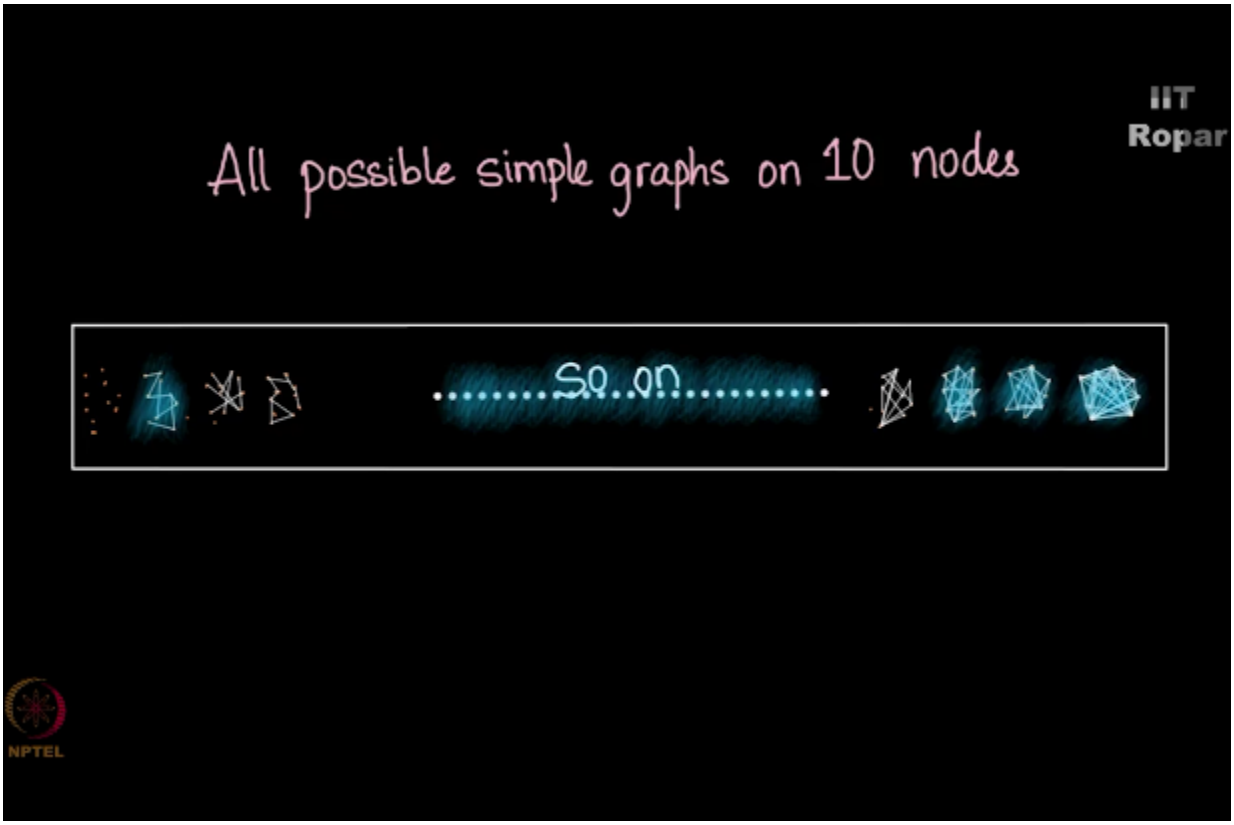

Just for fun, and if the graph if disconnected I paint it red,

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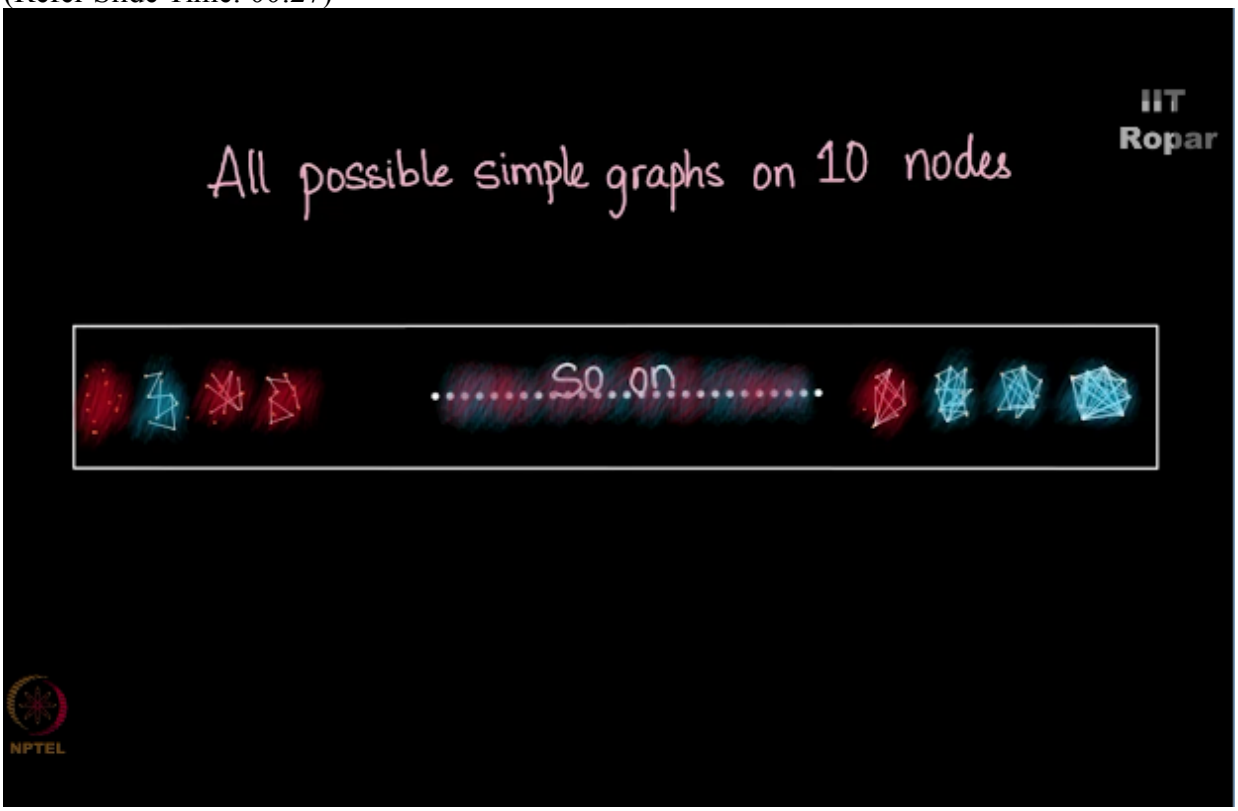

you see there are so many people here, how many possible graphs are there on 10 nodes, 2 to the 10 choose 2, right of this graphs, some are red, some are blue, what's the red graph? A graph that is disconnected, what's a blue graph? A graph that is connected, now can you tell me, are blue graphs more than red graphs, or red graphs more than blue graphs?
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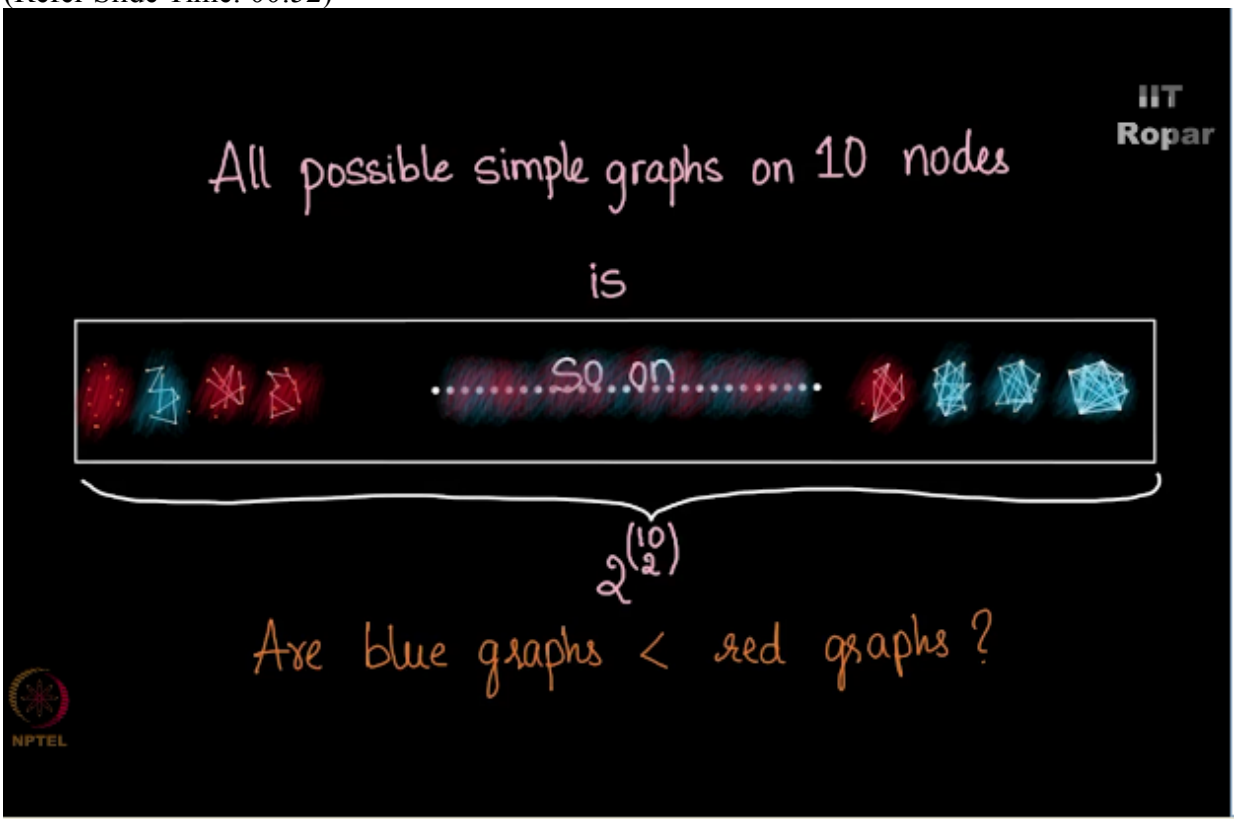

All I am asking you people is this theorem, show that there are more connected graphs than disconnected ones, this is the result, now you know what this translates to in the example that I gave you the color example, there are more blue graphs than the red graphs let us see how?
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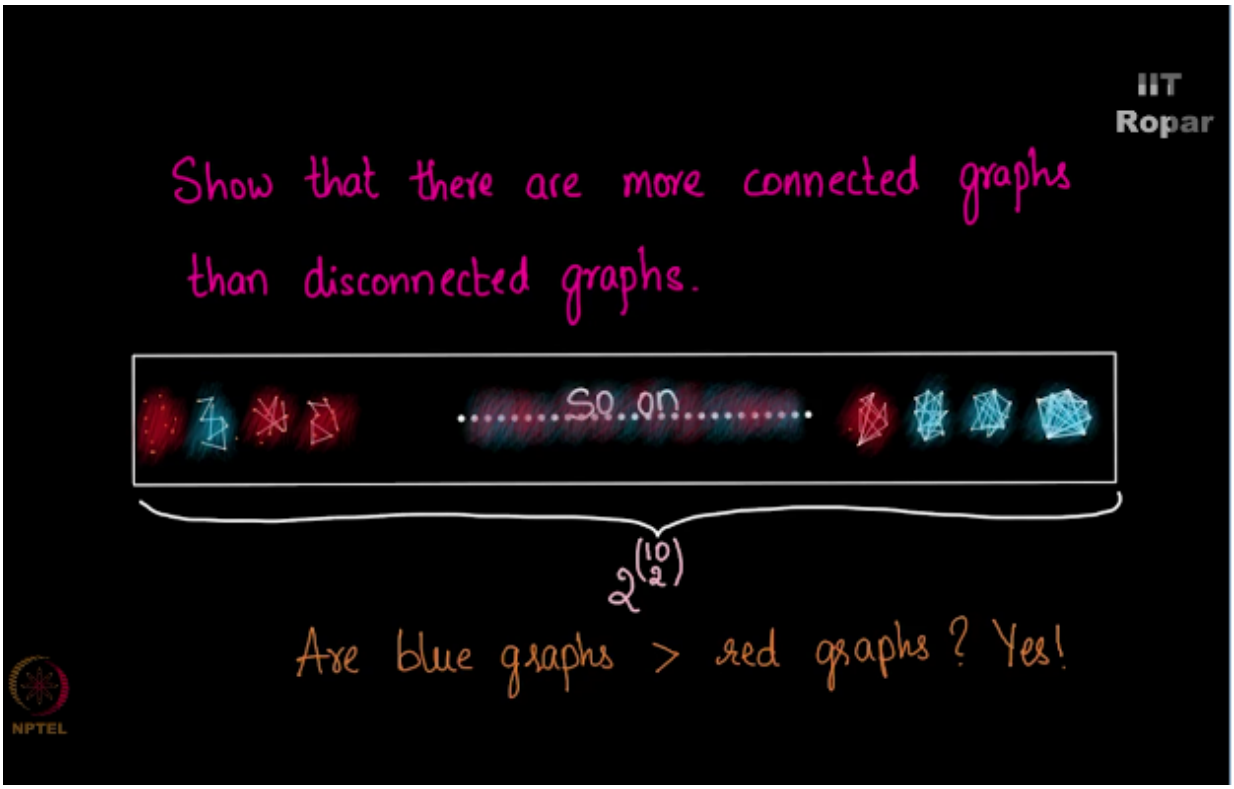

Before I go any further I'll give you a quick analogy that is easy on your brain, assume there is this city, full of only adults and every woman there is a married to a unique man,
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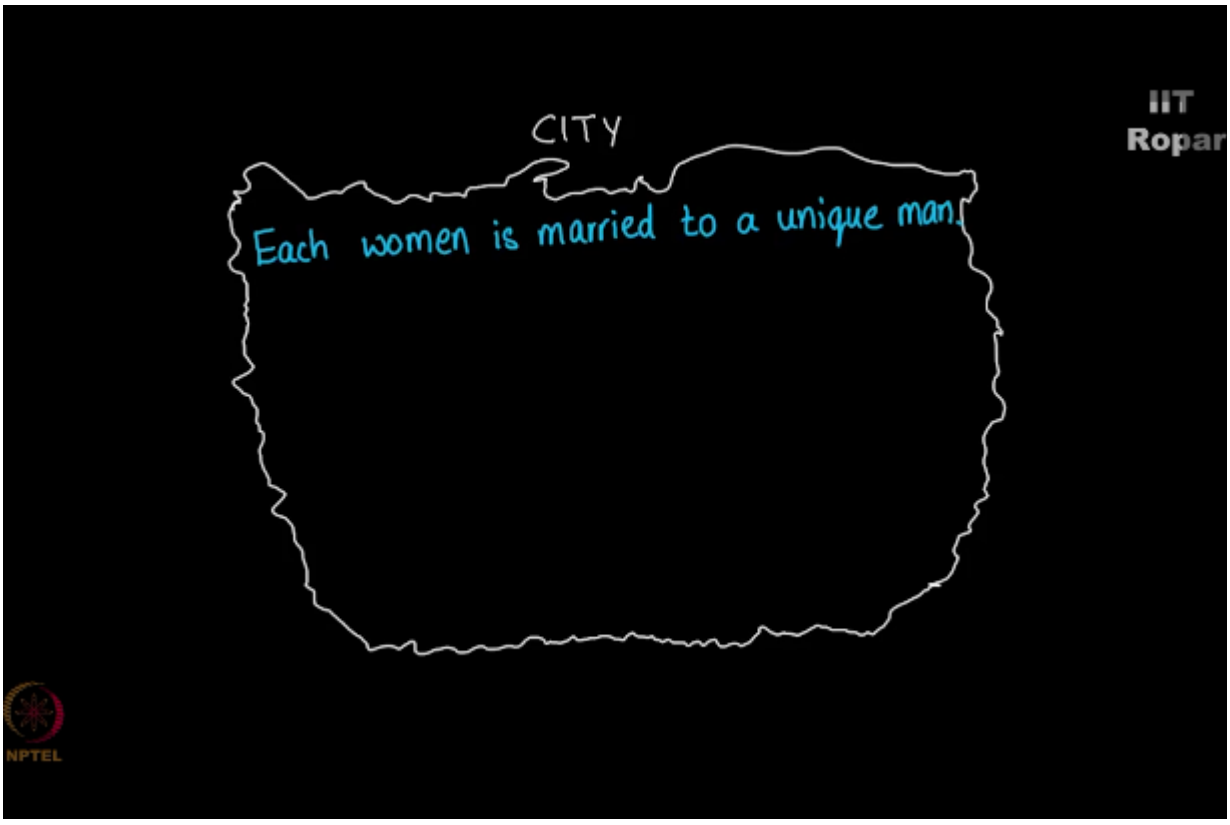

what can you say about men here, please note all I'm saying is every woman is married to a unique man I'm not saying every man is married to a unique woman, by unique I mean let's assume that marriage can only happen between two parties, right, okay, meaning a person cannot be married to two people, right, okay, so a woman is married to a man, which means if I write all the woman in this city on my left wing and write all the men in this city in the right wing, I'll see lines like this depicting partnership,

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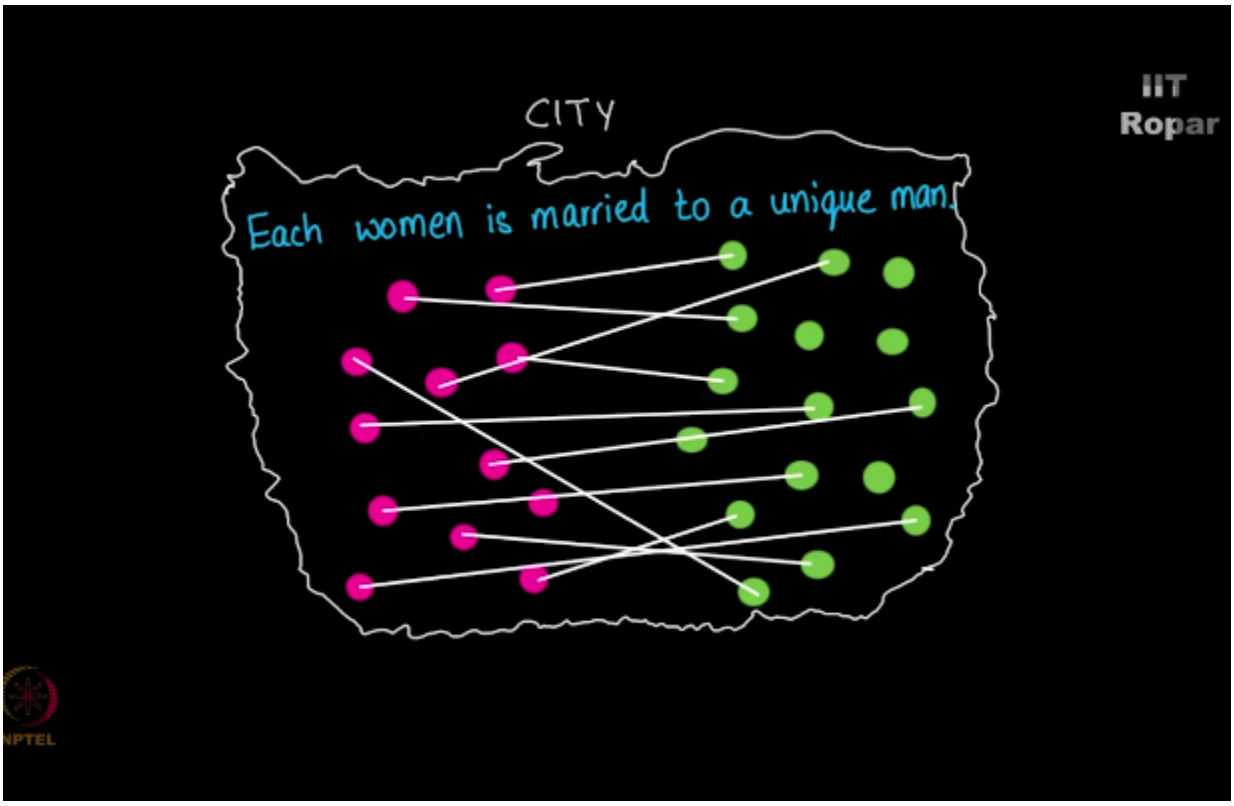

right this may or may not tell me anything about the left over men in the city, there could be some people left over amongst men who are not at all married, right, this says, the statement says, which statement? That in this town every woman is married to a man, statement says that the number of woman are less than or equal to number of men,

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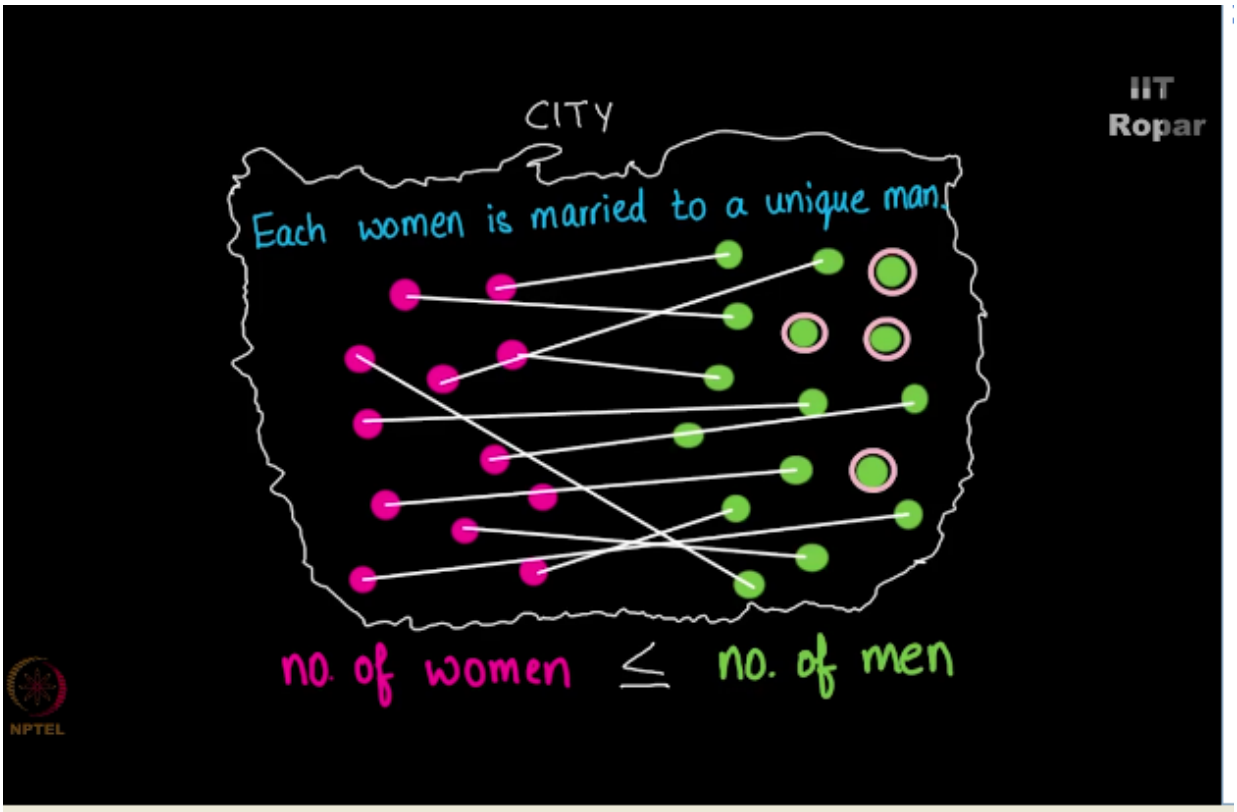

hey, please listen what I am saying is pure common sense, I'm using a very complicated example probably but all I'm saying is if I get some chocolates and distribute this chocolates one each to the classroom, chocolates get over, which means the number of people in the classroom is at least as many as the number of chocolates that I brought to the class, correct.

Now let us take this analogy and then show a proof for this theorem, there are more connected graphs than disconnected ones, put all the disconnected graphs on the left wing, they're equivalent to the woman in our example, okay, and all corrected graphs on the right wing, okay,
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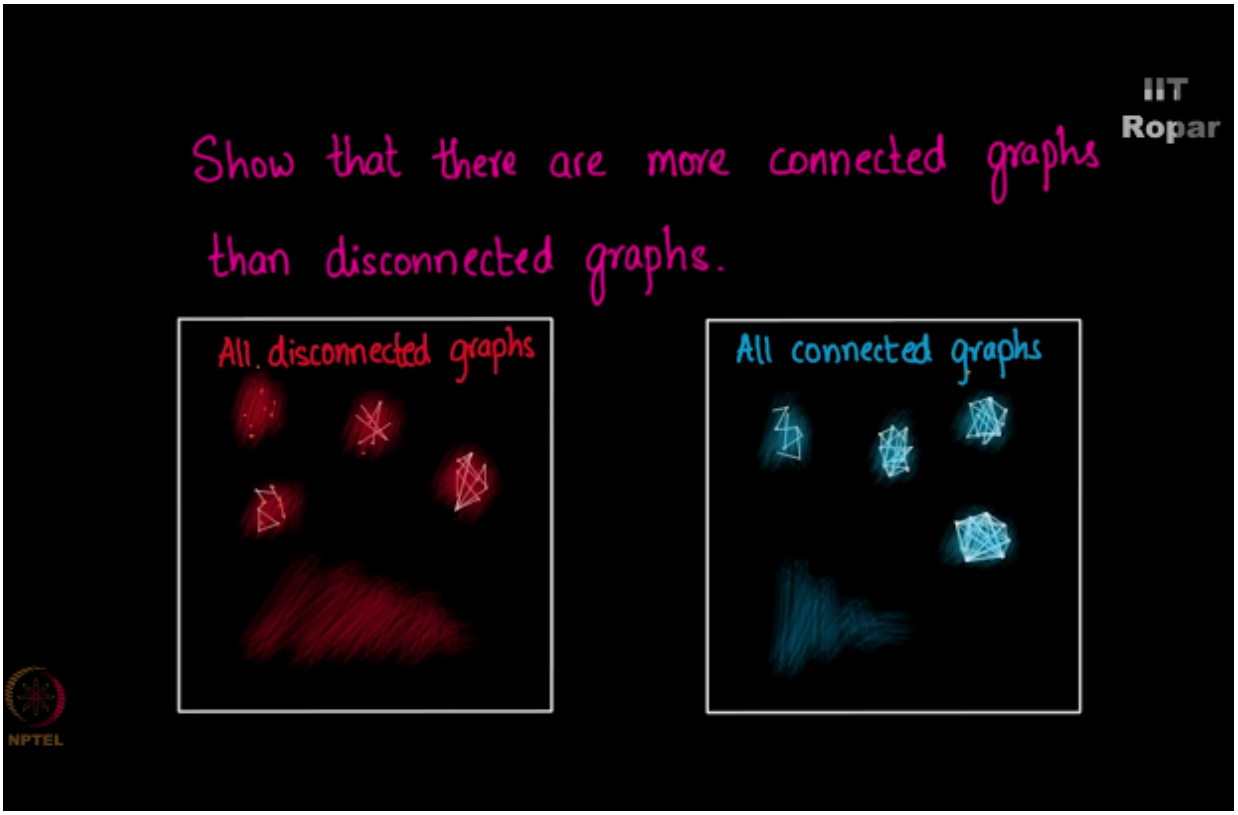

and you join a disconnected graph to a connected graph if the connected graph that you are joining is a complement of this disconnected graph, look at this,

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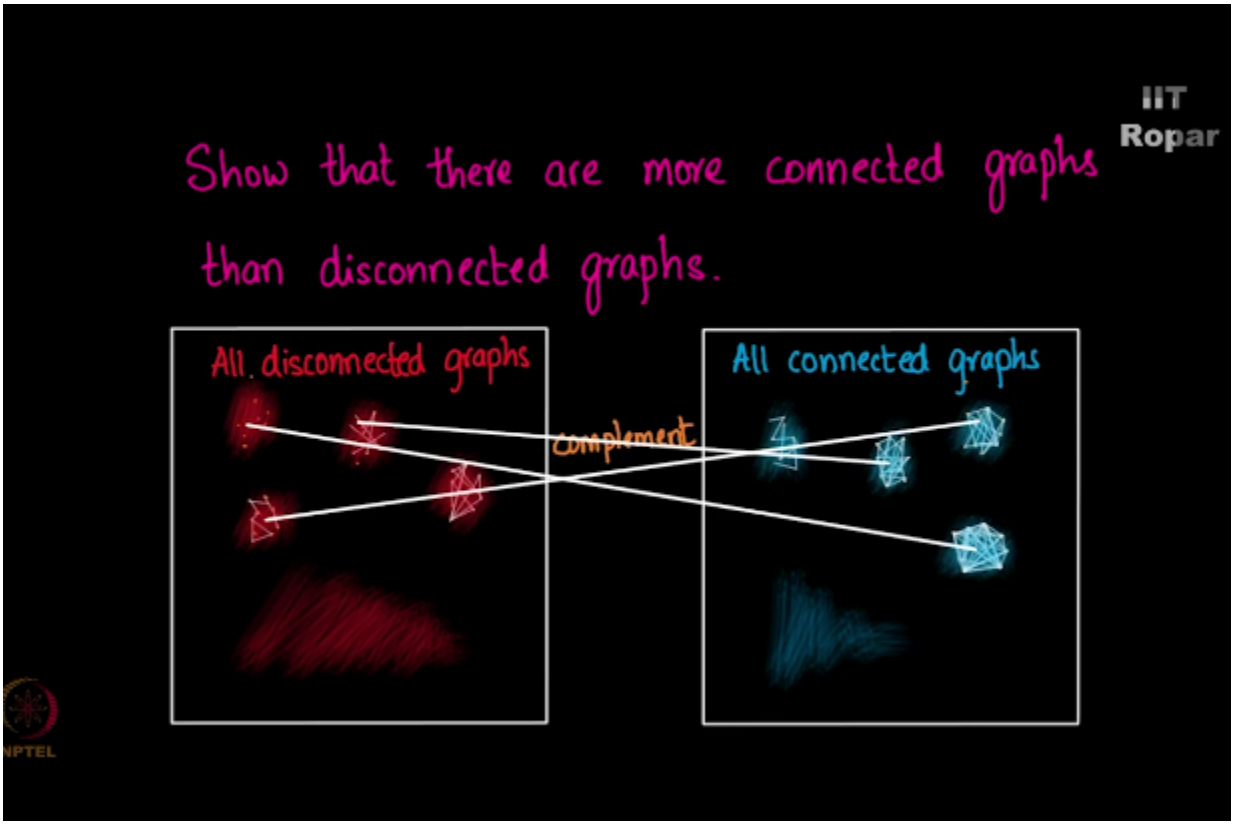

this disconnected graph is there in the left wing, look at disconnected graph it's a complement of this disconnected graph, you put an edge between this and this, right.

Now do you think two different disconnected graphs can go to the same connected graph? Not really, because complement of a disconnected graph is always what? Connected, we've seen that, right, so all you can say is the number of connected graphs on the right wing is at least as many as the disconnected ones, observe the proof carefully, I tried giving the proof in easy pieces, but then if you want to rigorously show it you may have to argue properly in the language of 1-1 functions, how is that? Think about it.
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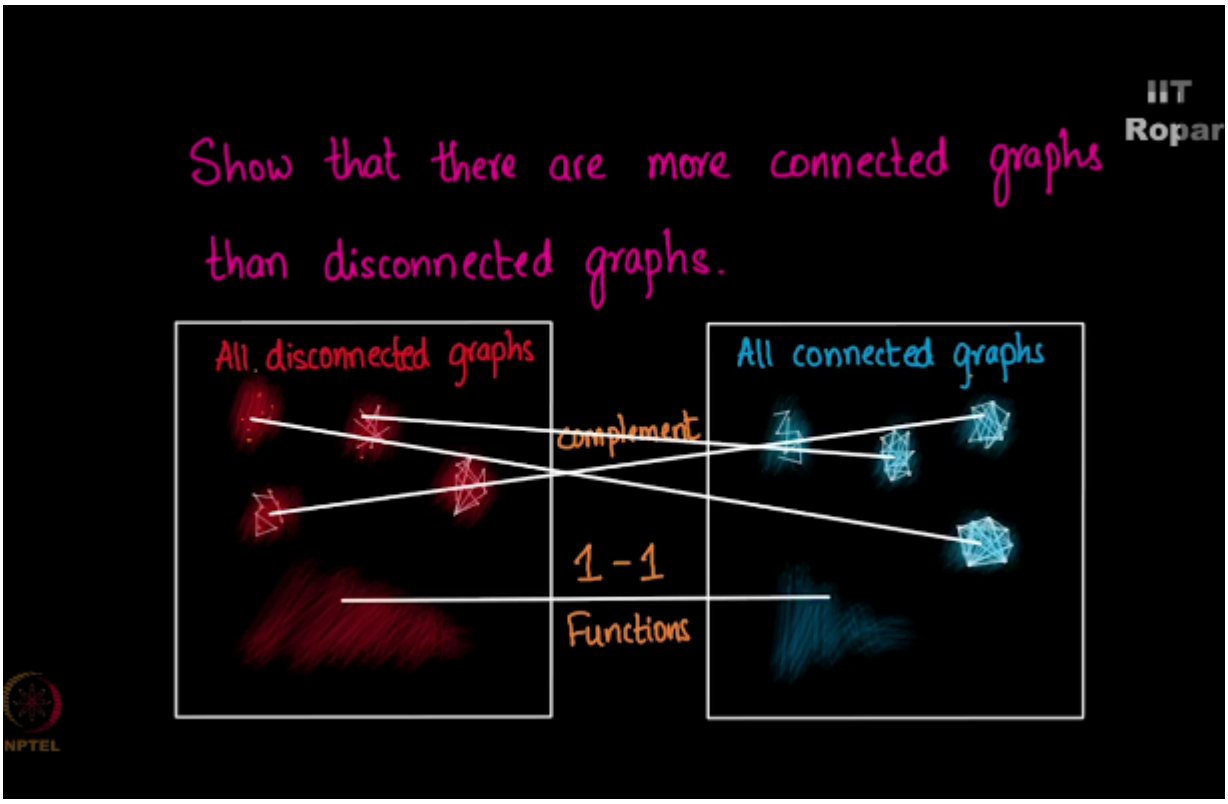

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