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NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics  
Graph Theory - 2

Self complement

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With a illustrations I hope the complement for graph was cleared to all of you, let us make a simple observation, if  $G$  is a simple undirected graph, then for any vertex  $V$  the following is true, degree of  $V$  in  $G$ , so I'm writing  $G$  as a subscript here, what does it mean? I'm considering the degree of  $V$  with respect to the graph  $G$  + degree of the vertex  $V$  in  $G$  complement, the sum of this is  $n-1$ .

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If  $G$  is a simple undirected graph, then  
for any vertex  $v$ ,  $\deg_G v + \deg_{\bar{G}} v = n-1$

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Well, you must be able to prove it yourself and the proof is just a one line proof, let me just tell you, you see if  $V$  has degree say  $K$  in the graph  $G$ , what does it mean? There are  $K$  edges adjacent to  $V$ , and in  $G$  complement all those  $K$  edges will not be there, so the sum must be  $n-1$ , either the edges must be in  $G$  or in  $G$  complement, and this is true only for simple undirected graph not otherwise.

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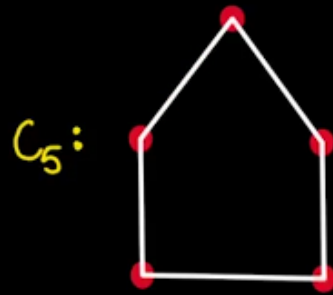
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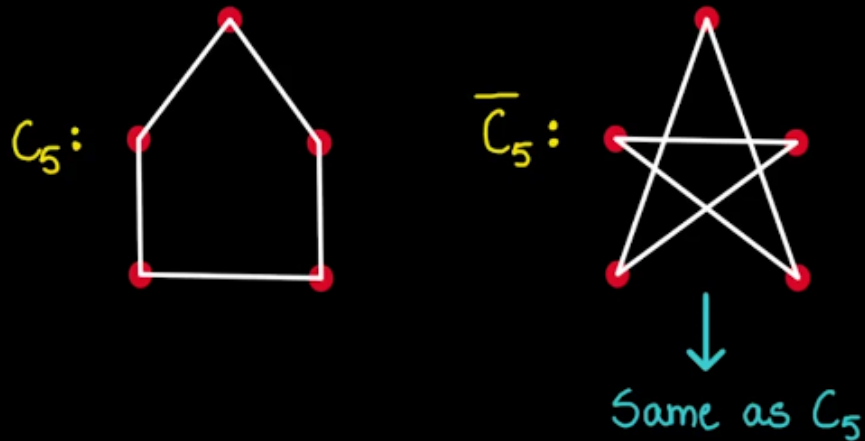
If  $\deg_G v = k$ , in  $\bar{G}$ , the  $k$  edges are not there.  
 $\therefore \deg_G v + \deg_{\bar{G}} v = n-1$

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Now look at this graph,  
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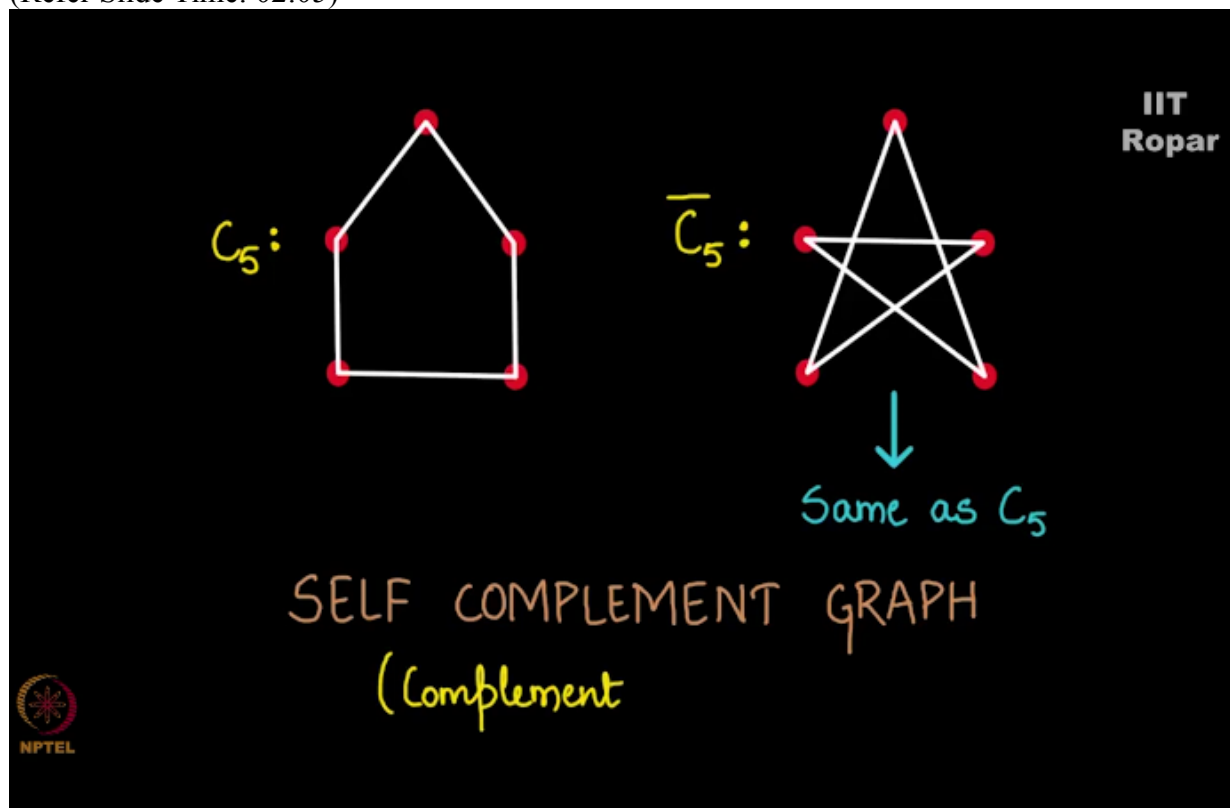


what is the complement of  $C_5$ ? This is a very special graph, what is the complement? The complement looks like this, well you must be very sharp to tell that these two were actually isomorphic we had already seen that,  
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now if I just rearrange this graph properly, if I untangle it you can see that it is same as  $G$ , you have obtained  $G$  itself, now do you see that  $G$  and  $G$  complement are the same? Such graphs are called as self-complement graphs, it is the complement of itself.

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