NPTEL

NPTEL ONLINE COURSE

Discrete Mathematics

Let Us Count

Multinomial theorem

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The sports club of a school has 36 girls and they wish to form four volleyball teams of nine girls each. In how many ways can they do this? So there is a sports club of 36 girls and four volleyball teams have to be formed let me name it as T1, T2, T3 and T4 and each team will have 9 girls. So to form team one, 9 girls will be chosen from 36 girls. In how many ways and 36C9 ways. Let me keep it aside. Now for team two, again 9 girls will be chosen but for team one 9 girls have already been taken and we have 36 minus 9 which is 27. So for team two we have 27 girls remaining out of them 9 girls have to be chosen. Again this can be done in 27C9 ways.

Now for team three, what will be the possibility? We have 9 plus 9, 18 girls already out out of the 36 girls. So we have 18 girls remaining out of them 9 girls have to be chosen and this can be done in 18C9 ways and the fourth team will have just 9 girls remaining and they can be chosen in 9C9 ways. So for team one we have 36C9, team two we have 27C9. Team 3 we have 18C9 and for team 4 for we have 9C9 and we want all these to happen so by the rule of product we multiply all these and get 36C9 into 27C9 into 18C9 into 9C9. So in these many number of ways four teams can be formed. Ley me calculate so this becomes 36 factorial by 9 factorial into 27 factorial into 27 factorial by 9 factorial into 18 factorial into 18 factorial by 9 factorial into 9 factorial into 9 factorial by 9 factorial into 0 factorial. After simplification what remains is 36 factorial by 9 factorial into 9 factorial into 9 factorial into 9 factorial. So in these many number of ways 4 teams can be formed with 9 girls in each team.

So this was an example now let me just write in terms of notations. The number of ways to choose n1 objects from a given set of n objects and n2 objects from the remaining that is n minus n1 objects. n3 objects from the remaining that is n minus n1 minus n2 object please note I am again and again choosing from the remaining objects and this goes on and choosing nK minus 1 objects from n minus n1 minus n2 and so on minus nK minus 1. So in how many ways can we do this that will be given by nCn1 into n minus n1Cn2 into n minus n1 minus n 2Cn3 product goes on into n minus n1 minus n2 this goes on up to nK minus 1CnK. Let me repeat it if it's not clear. You can see it this way. We're choosing n1 objects from n, n2 objects from what remains here n3 objects from what remains here and so on up to nK. Now by simple calculation this comes out to be n factorial by n1 factorial into n2 factorial into so on and the product goes on up to nK factorial.

Now this can be written as it can be denoted as nCn1, n2, n3 and so on up to nK. What did I do just now?

I have chosen a particular number of objects from a given fixed number of objects, n was fixed and I kept on choosing a particular number that is n1, n2, nK from this set n. So what we are arriving at is called the famous multinomial theorem. So what I stated as n factorial by n1 factorial into n2 factorial up to nK factorial is called the coefficient of the term x1 power n1 into x2 power n2 into x3 power n3 so on up to xK power nK in the expansion of x1 plus x2 plus so on up to xK whole power n. You have already seen binomial; bi means 2 so when K is 2 this turns out to be binomial theorem. So this is the generalization of binomial theorem. So what we have to keep in mind is that this term n factorial by n1 factorial into n2 factorial up to nK factorial will be the coefficient of x1 power n1 into x2 power n2 up to xK power nK. So what does this actually mean? This means from this set of n objects x1 is chosen n1 times, x2 is chosen from the set of n minus n1 objects n2 times, x3 is chosen from the set of n minus n1 minus n2 n3 times and so on xK is chosen from the set of n minus n1 minus n2 up to minus nK, how many times, nK times. And this choosing can be done in n factorial by n1 factorial into n2 factorial so on up to nK factorial in these many ways and this is called the multinomial theorem.

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