NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 2

Isomorphic graphs - An illustration

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Do you see these two graphs here
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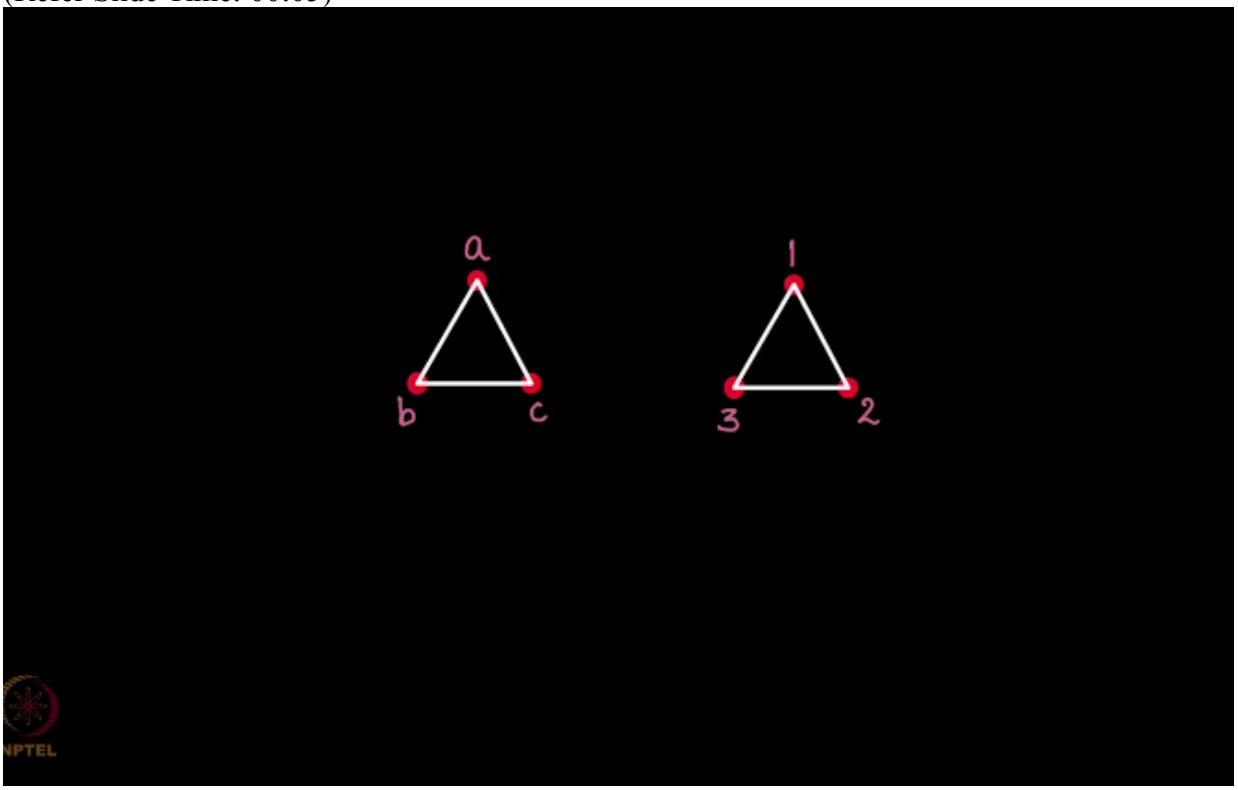

they are isomorphic, and this is the symbol for isomorphism,
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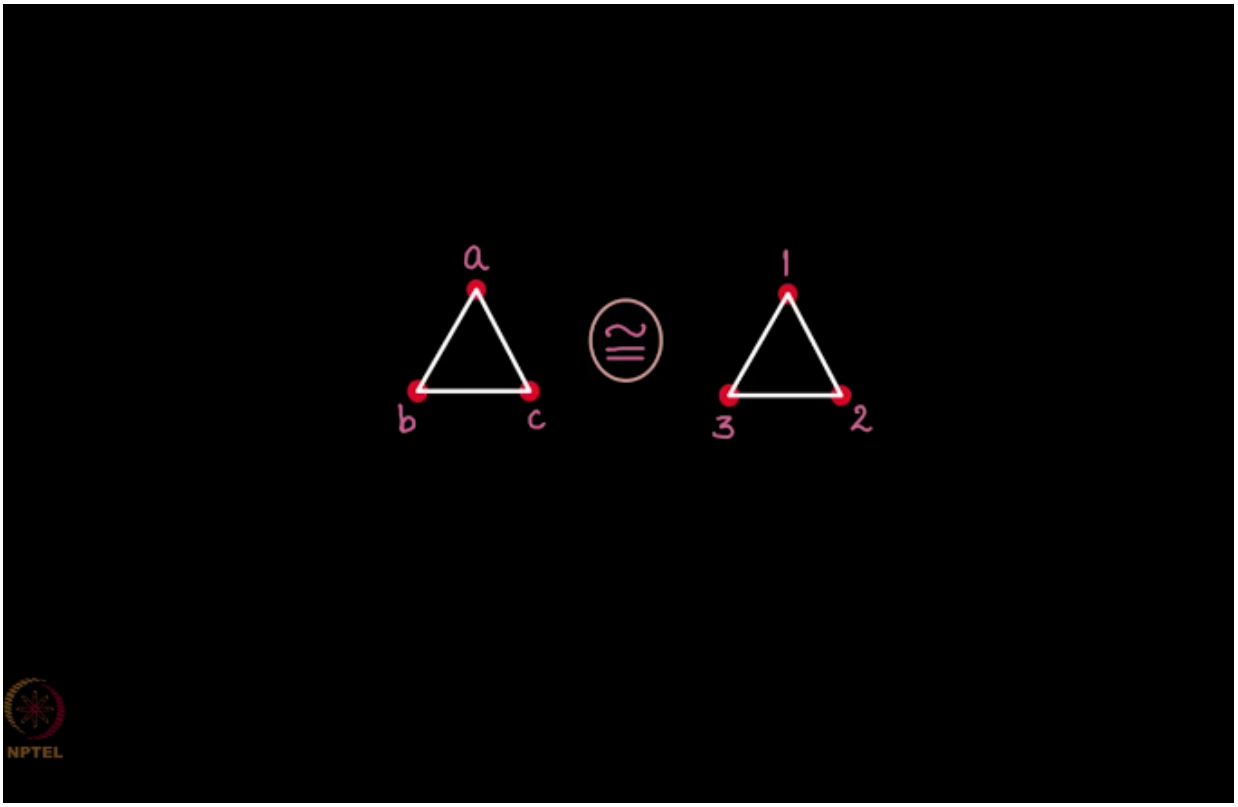

do you see that only labels are different but the graphs are the same, they are isomorphic.

Do you see this graph here and this graph, they are also isomorphic, how?
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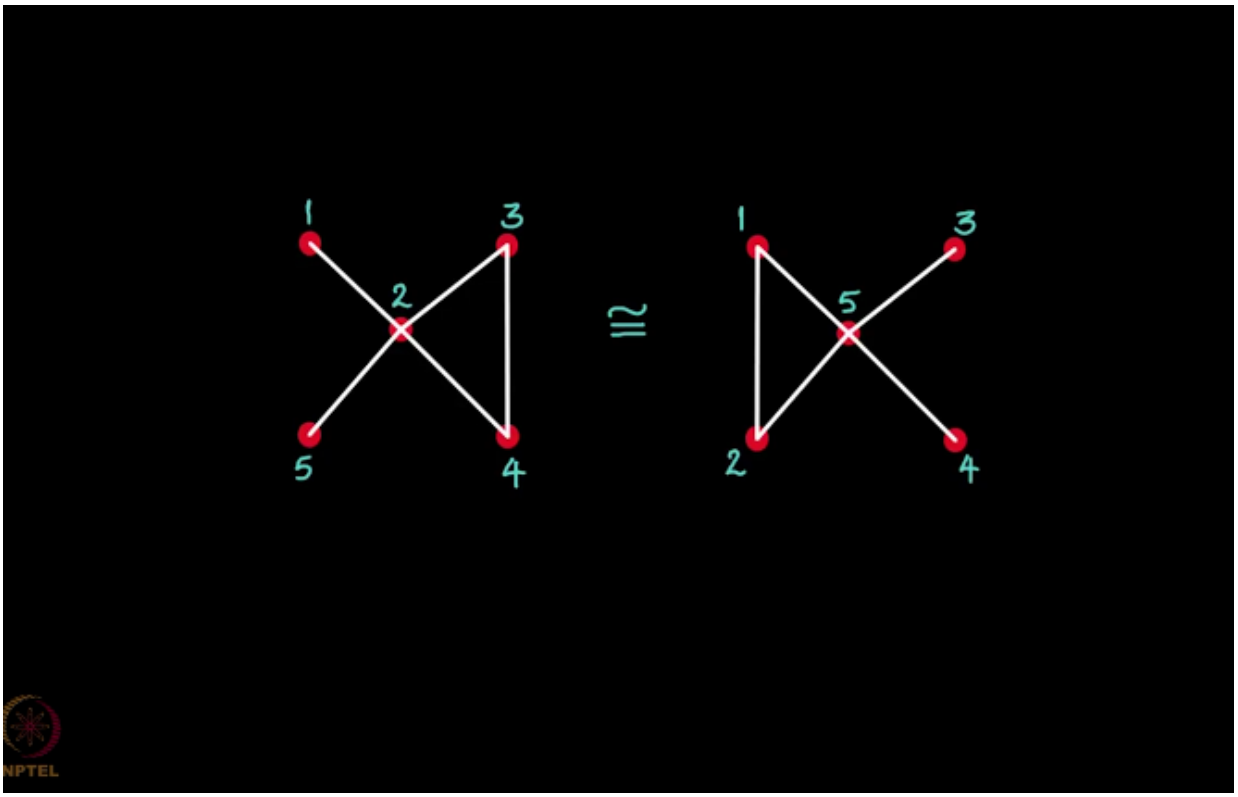

It is just that the labels are rearranged if you keenly observe, if I consider this to be a group of friends and this two a group of friends, here you see that 2 is the leader among them,

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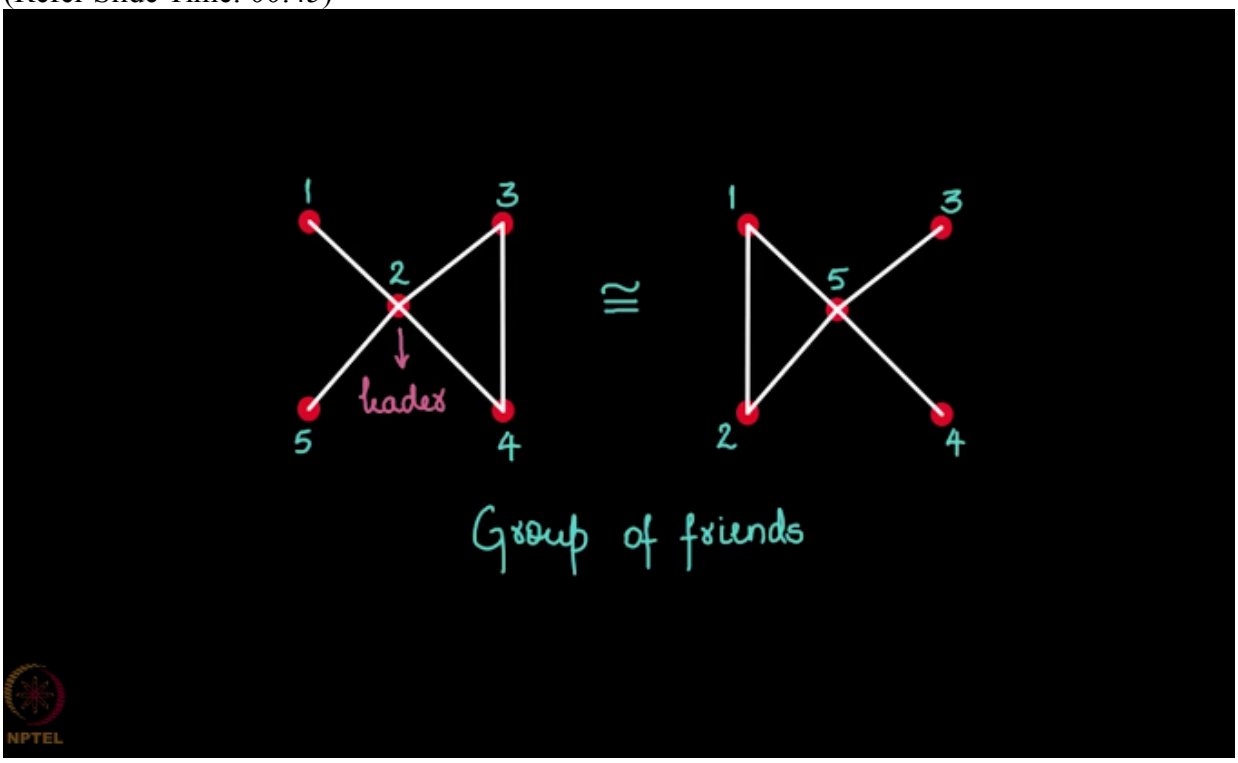

many people are connected to 2, but here 5 is the leader,

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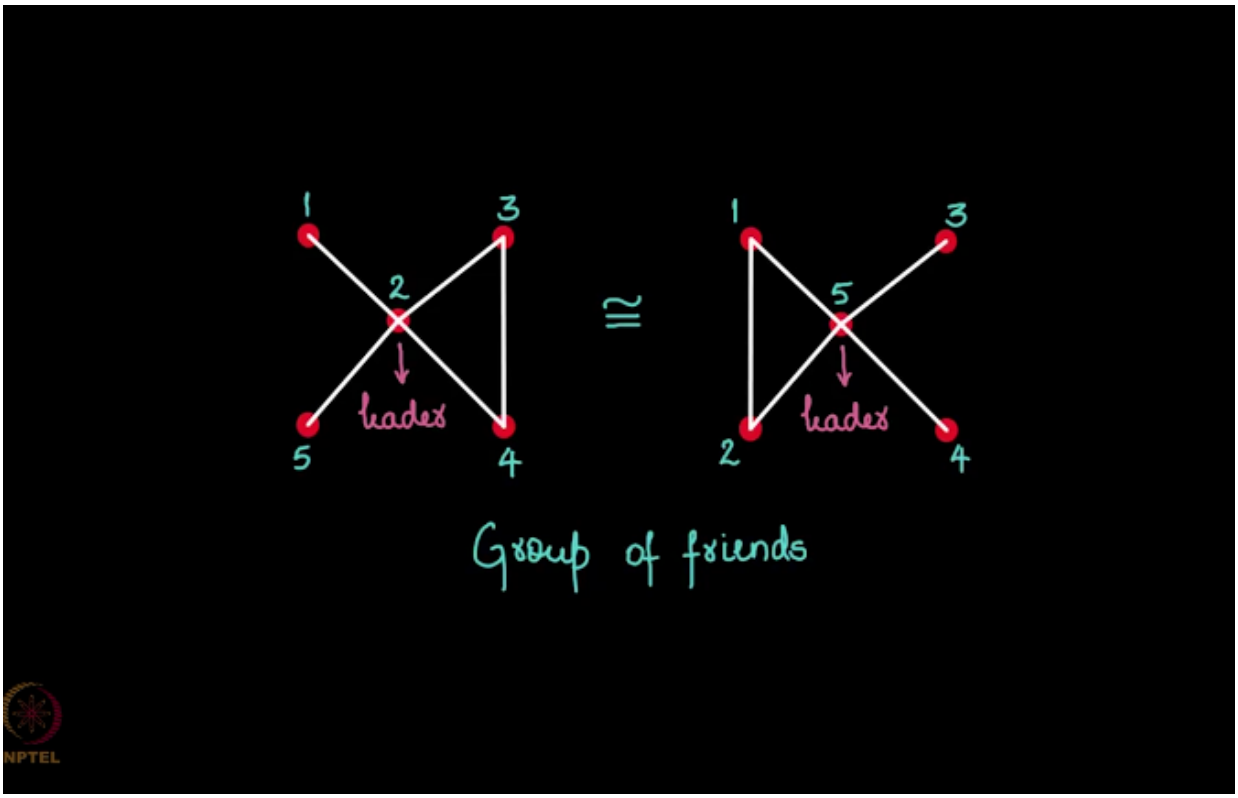

only the labels are rearranged whereas the structure of the graph is the same, and hence these are isomorphic.

We have a nice observation to make about graphs which are isomorphic, if G and H are isomorphic graphs, then you see we must or we can conclude that the number of vertices are same in both the graphs, formally speaking cardinality of VG = cardinality of VH,
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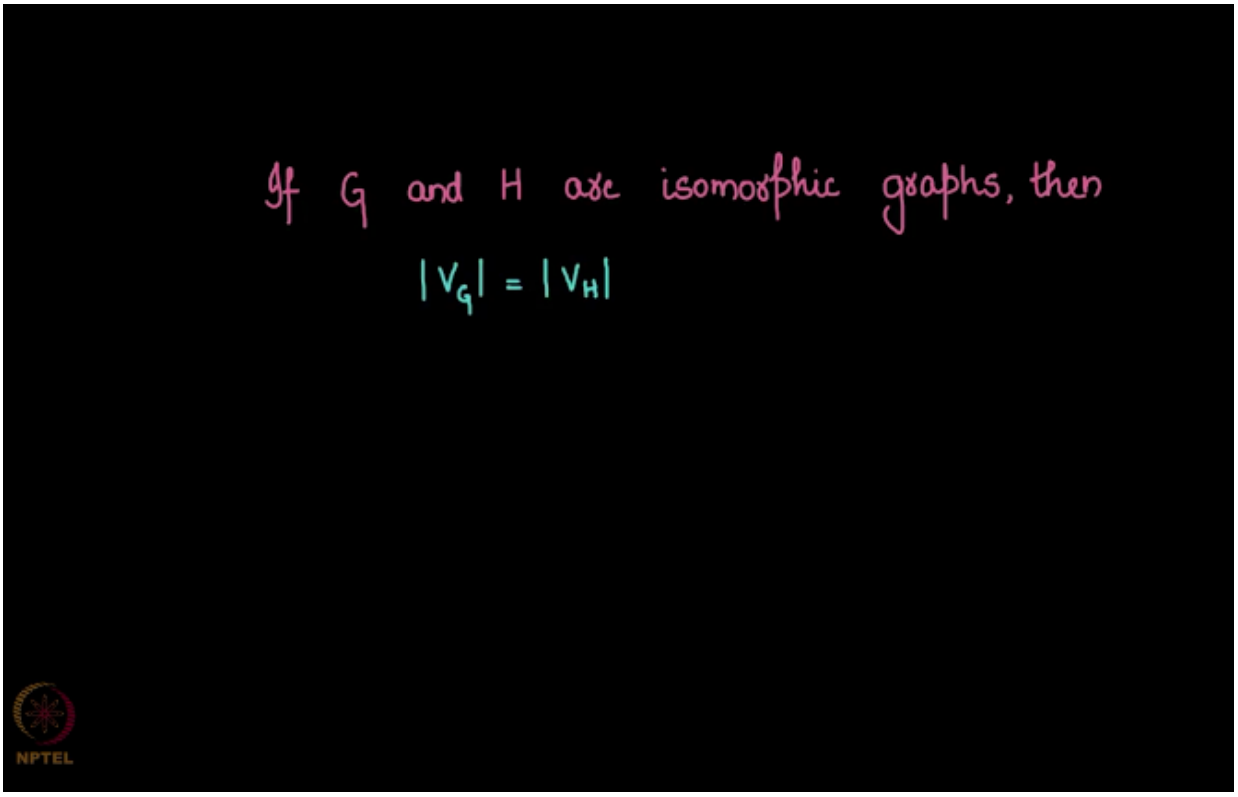

and the same goes to for the edge set as well, EG = EH the cardinalities,

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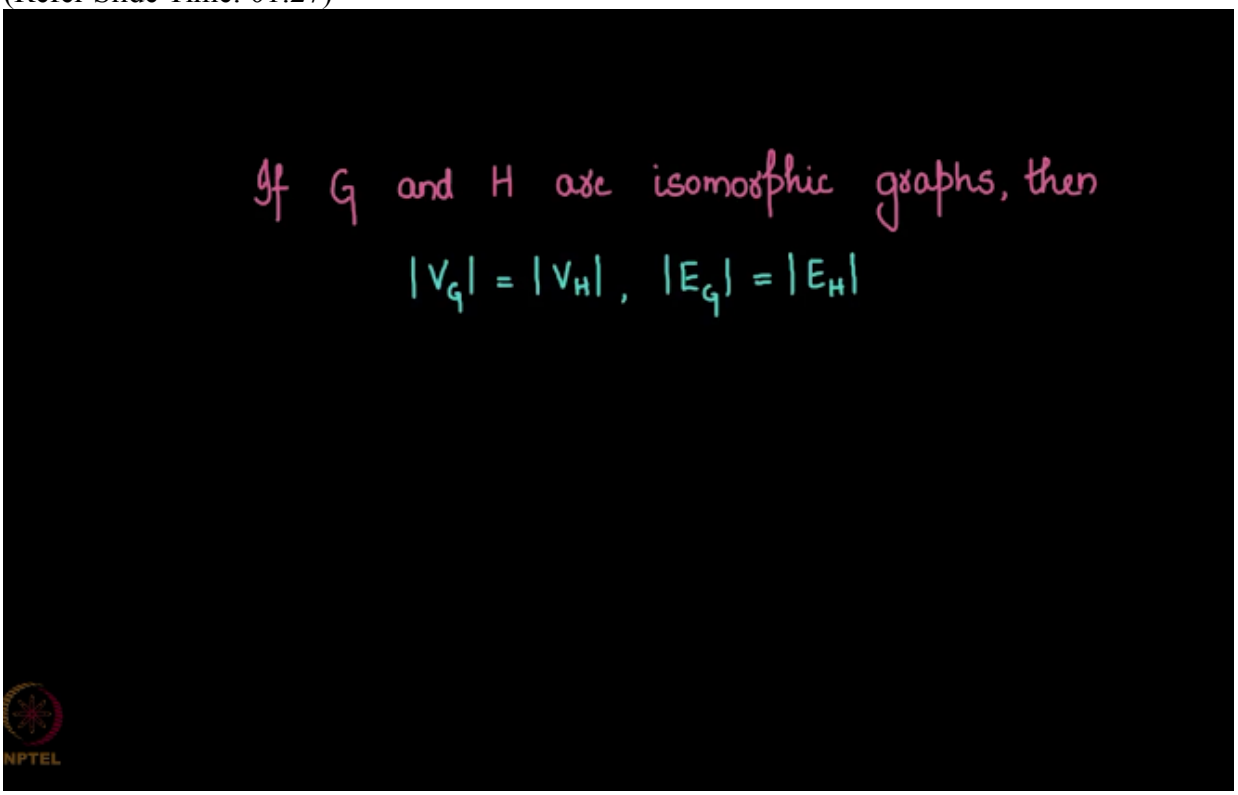

one more thing the degree sequence of G = the degree sequence of H,

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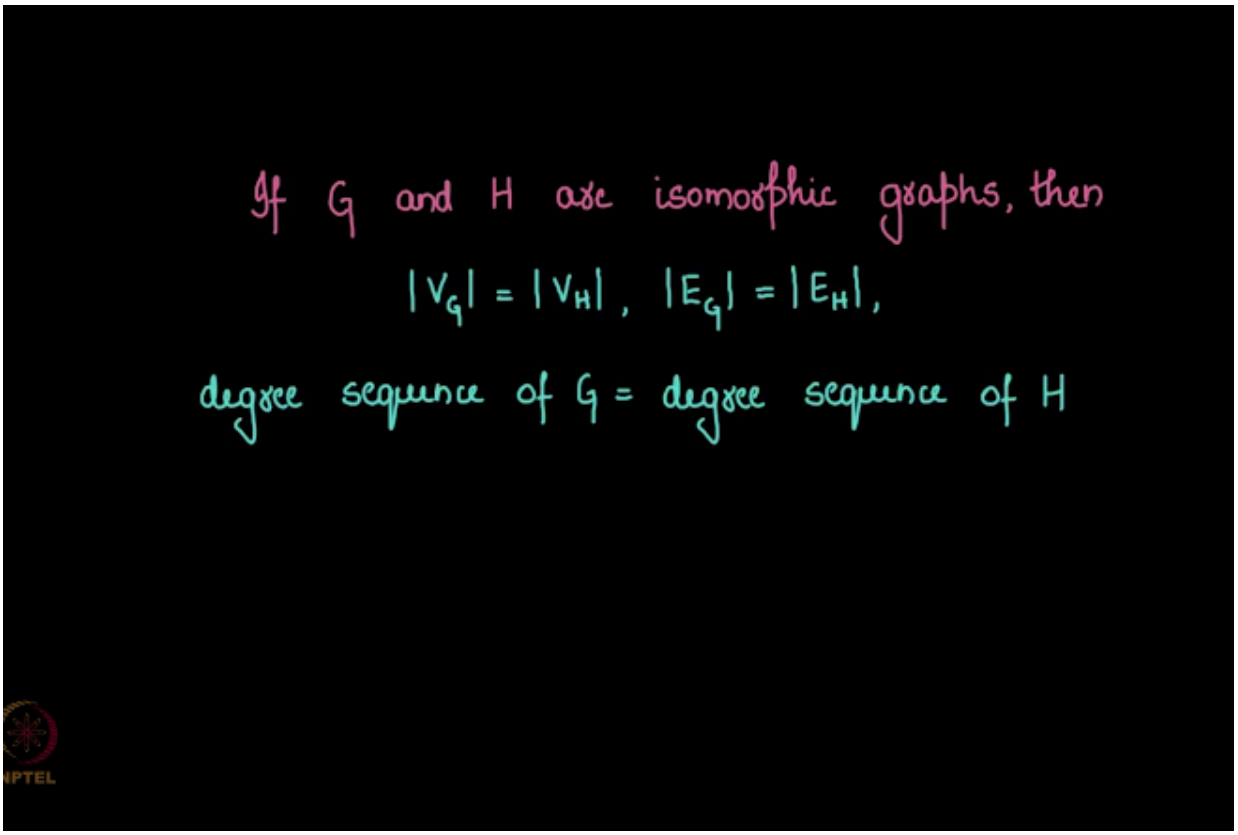

so if G and H are isomorphic graphs, then all these three hold true.

Now what does this mean? Supposing these, one of these is not true that is if P implies Q we know that naught Q implies naught P, so what can we tell? If one of these three is not true then G and H cannot be isomorphic.

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If G and H are isomorphic graphs, then

$|V_G| = |V_H|$, $|E_G| = |E_H|$,

degree sequence of G = degree sequence of H

If one of the three conditions is not true,
G and H are not isomorphic.

Let us take an example to verify that, consider this graph on 7 vertices and there are 8 edges here, so cardinality of V is 7, and cardinality of E is 8,

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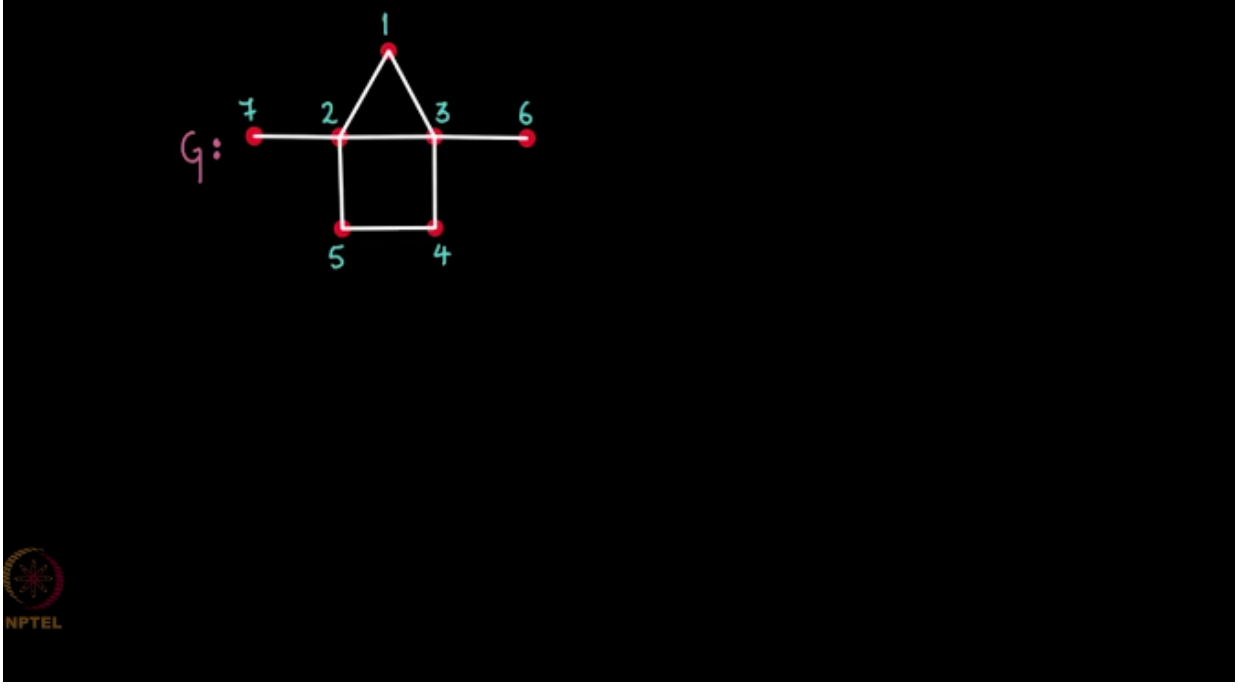

consider this graph, here again cardinality of V is 7 and we have 8 edges,
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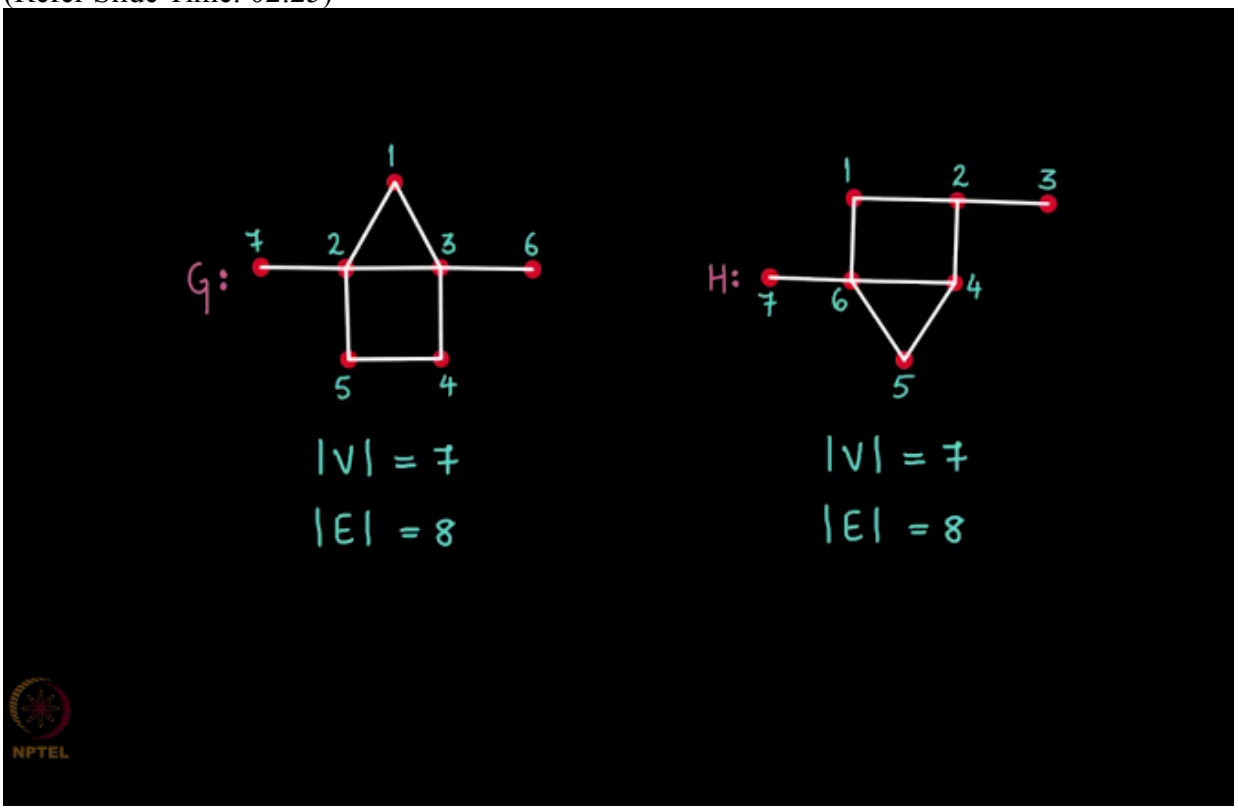

but look at the labeling, only the labeling is different whereas as the structure is the same, but let us write the degree sequence for G and H, for G is 4, 4, 2, 2, 2, 2, 1, 1 this is the degree sequence for G, but for H it is 4, 3, 3, and 2, 2, 1, 1,
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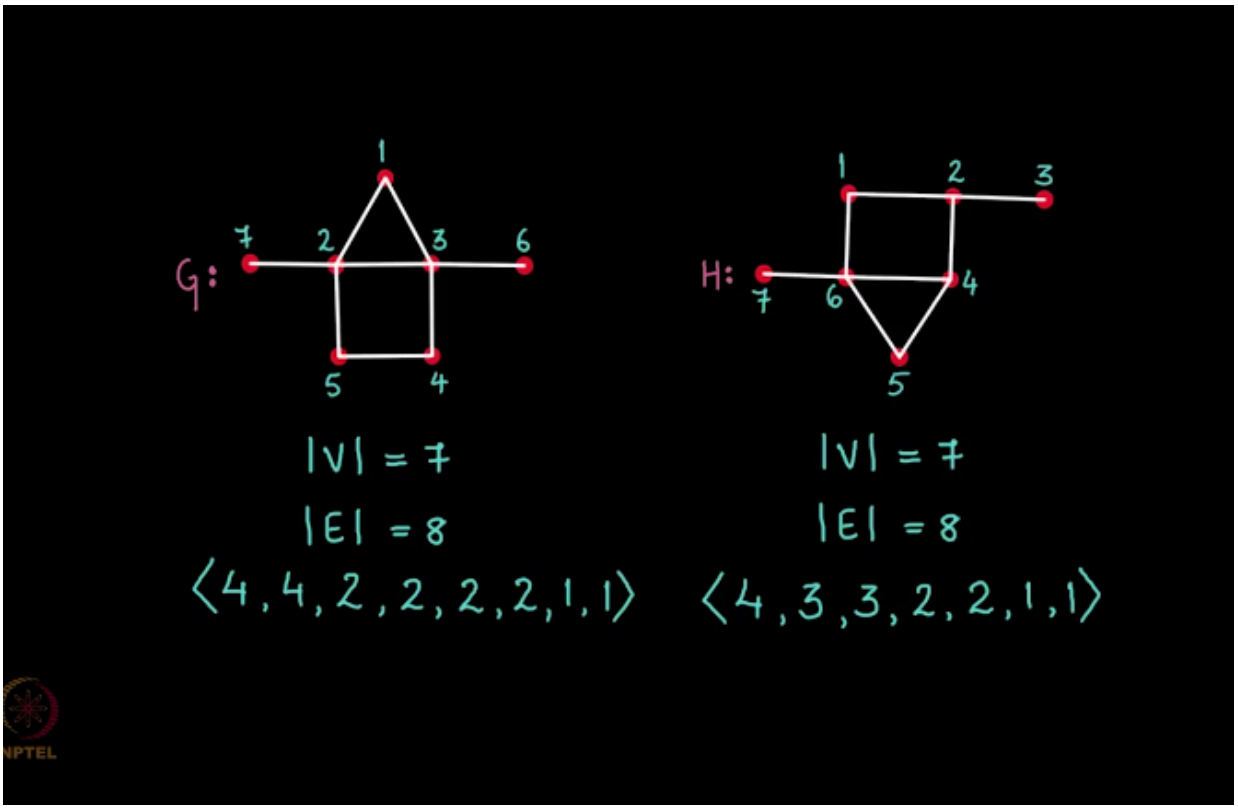

well do you see that the degree sequences are different, and hence the two graphs are not isomorphic,

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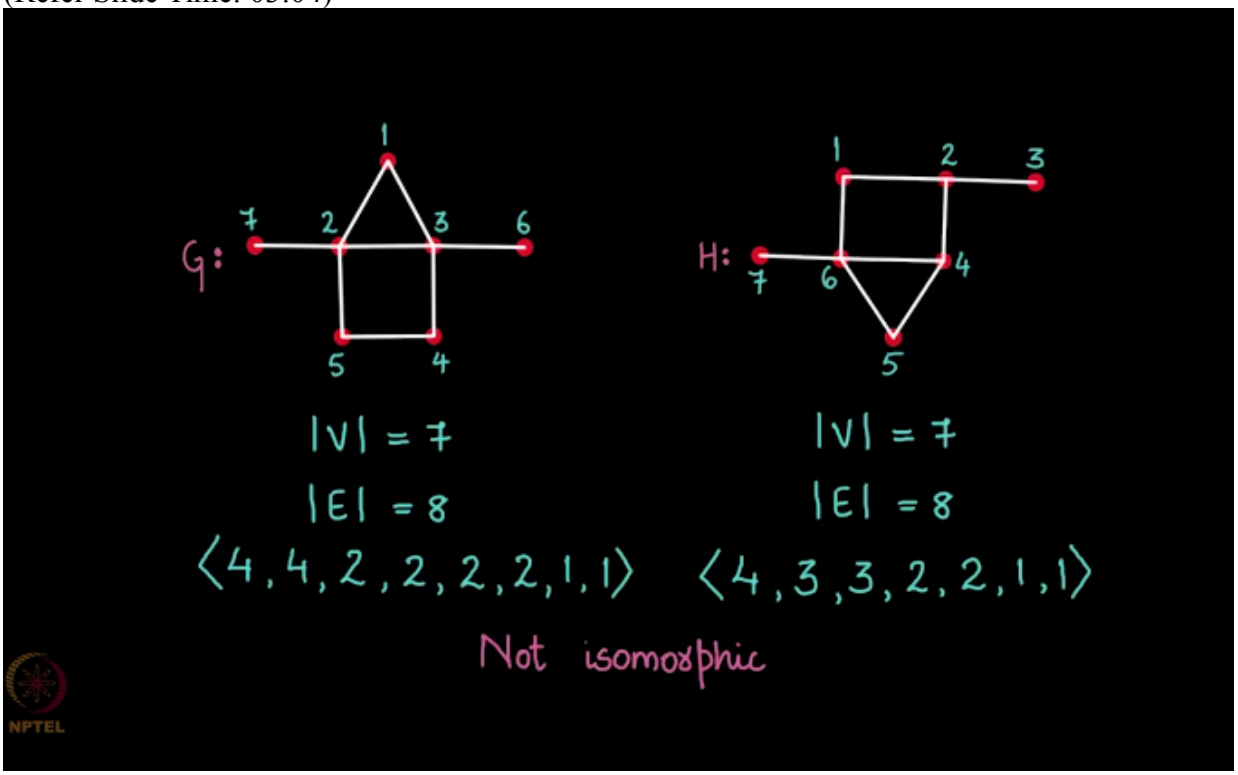

even if one of them gets violated G and H cannot be isomorphic, then what is this graph isomorphic to, let us see, it is isomorphic to this graph
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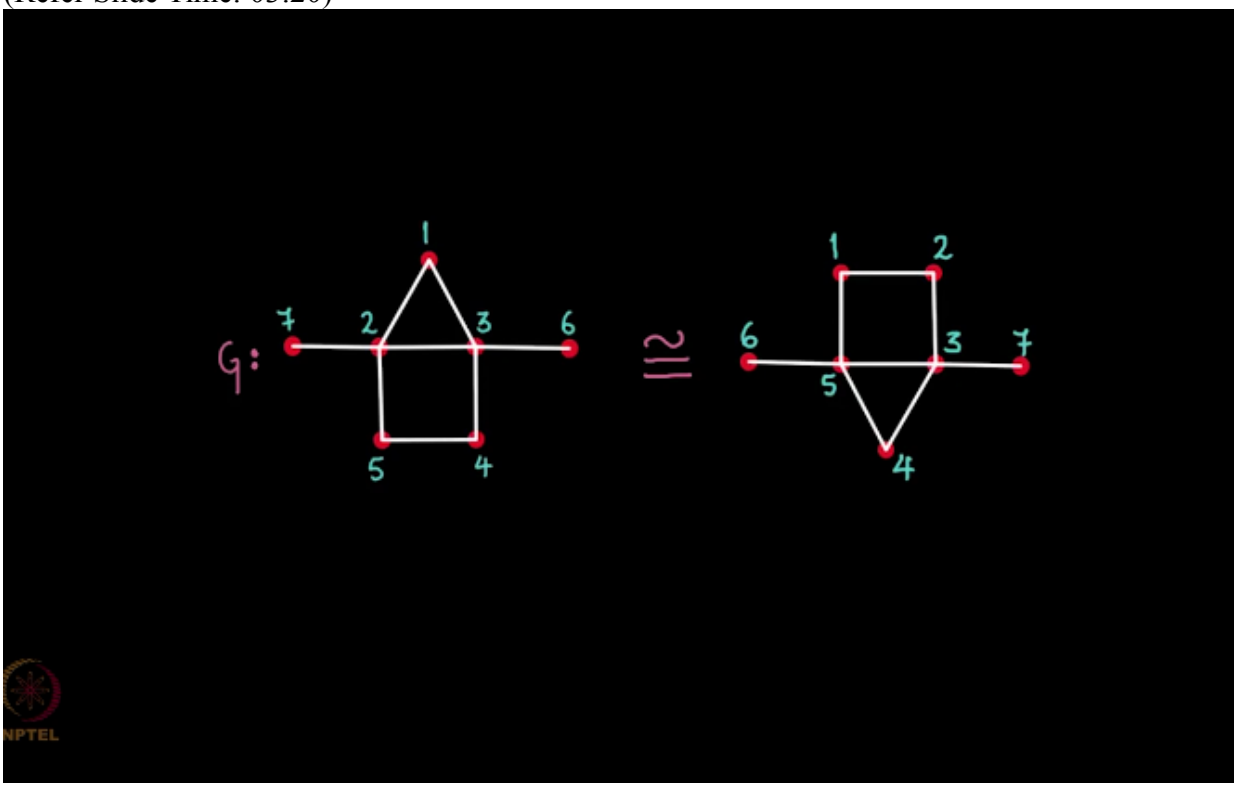

with this labeling, please check the degree sequence, it is a challenge for you all.

Look at these two graphs,
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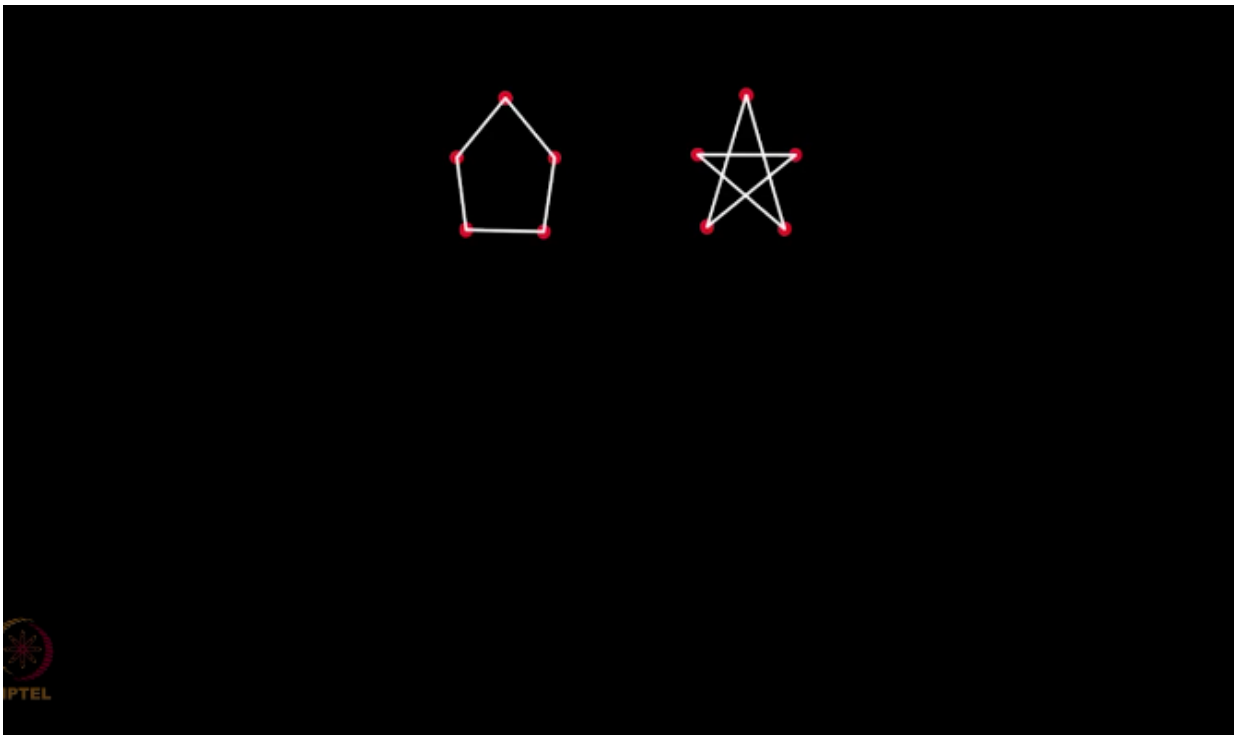

I claim that these two graphs are isomorphic, how? Let us see, here the labeling is A, B, C, D, E, and here it is 1, 2, 3, 4, 5,

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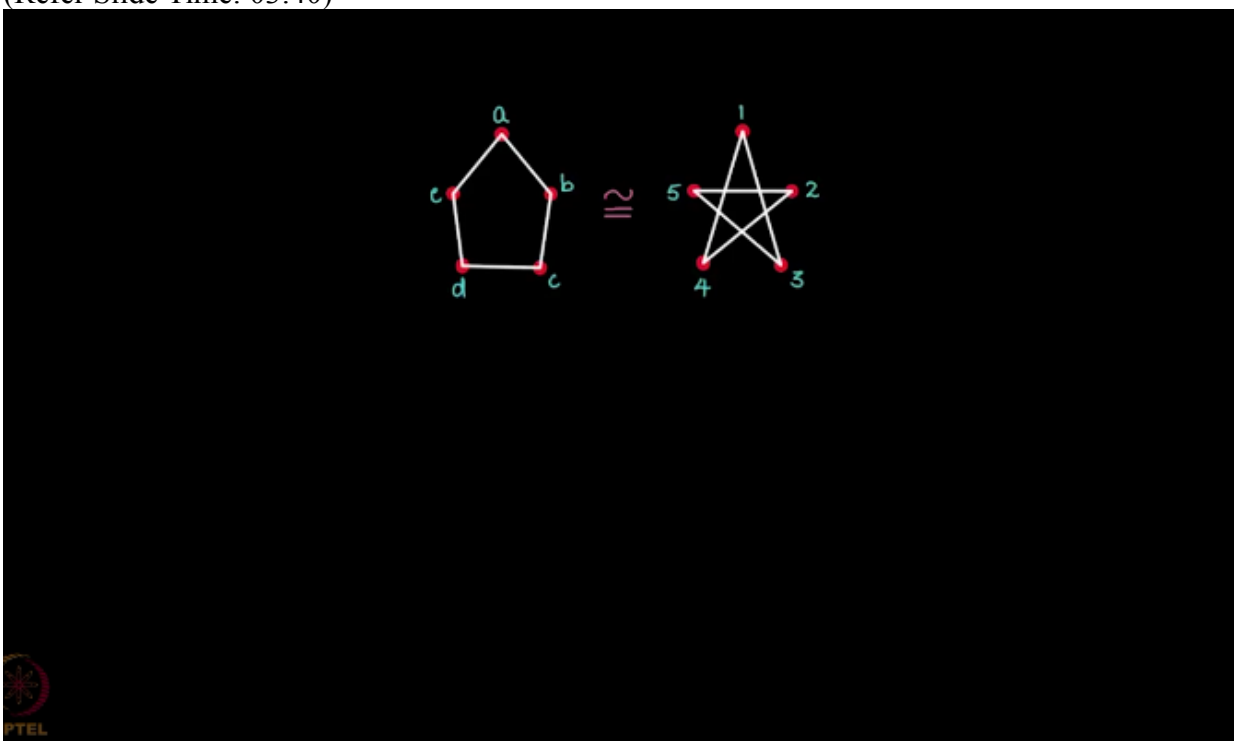

now I write it, I write this graph like this, are just opened up one tangling you see like this, and I just flip this 3 and 1 again, flipping in the sense

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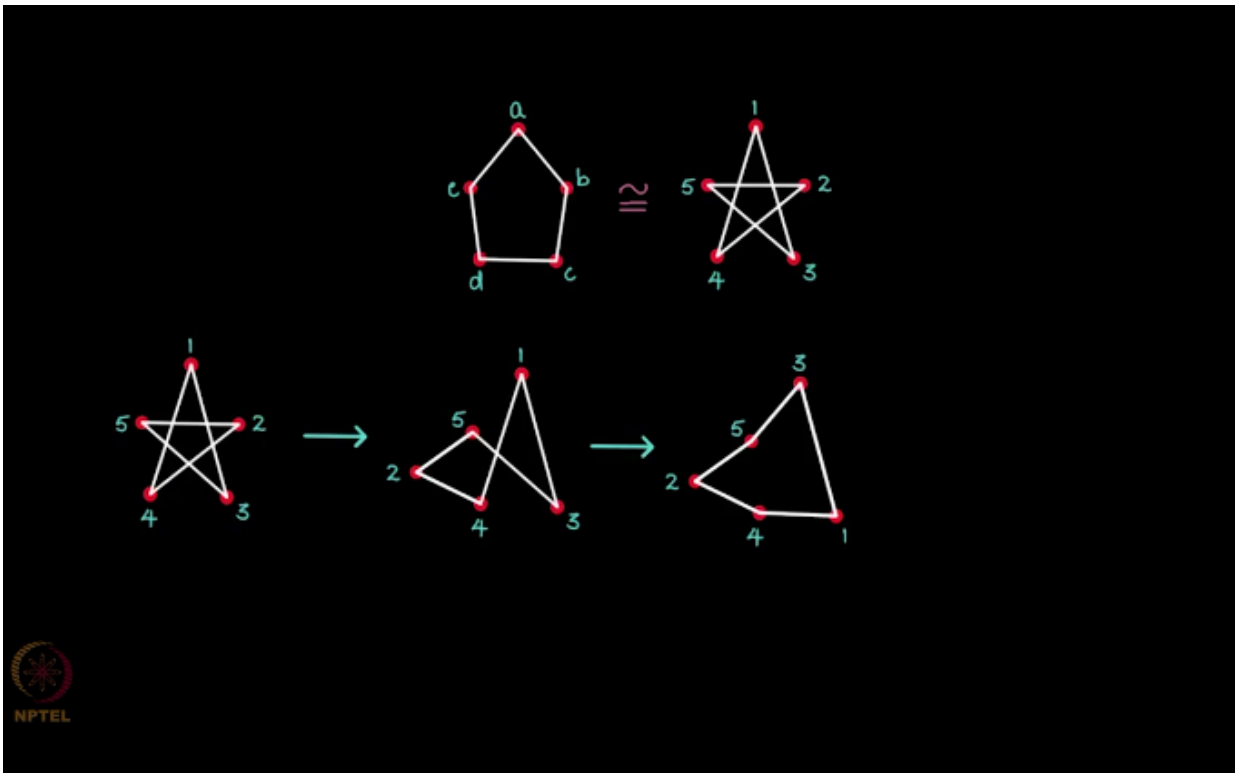

I'm just moving if you take a thread and arrange it in the form of a star like this you can probably try this out as an experiment, you can move this way, and now this way and then you see that with some rearrangement you obtained a C5 with this labeling,

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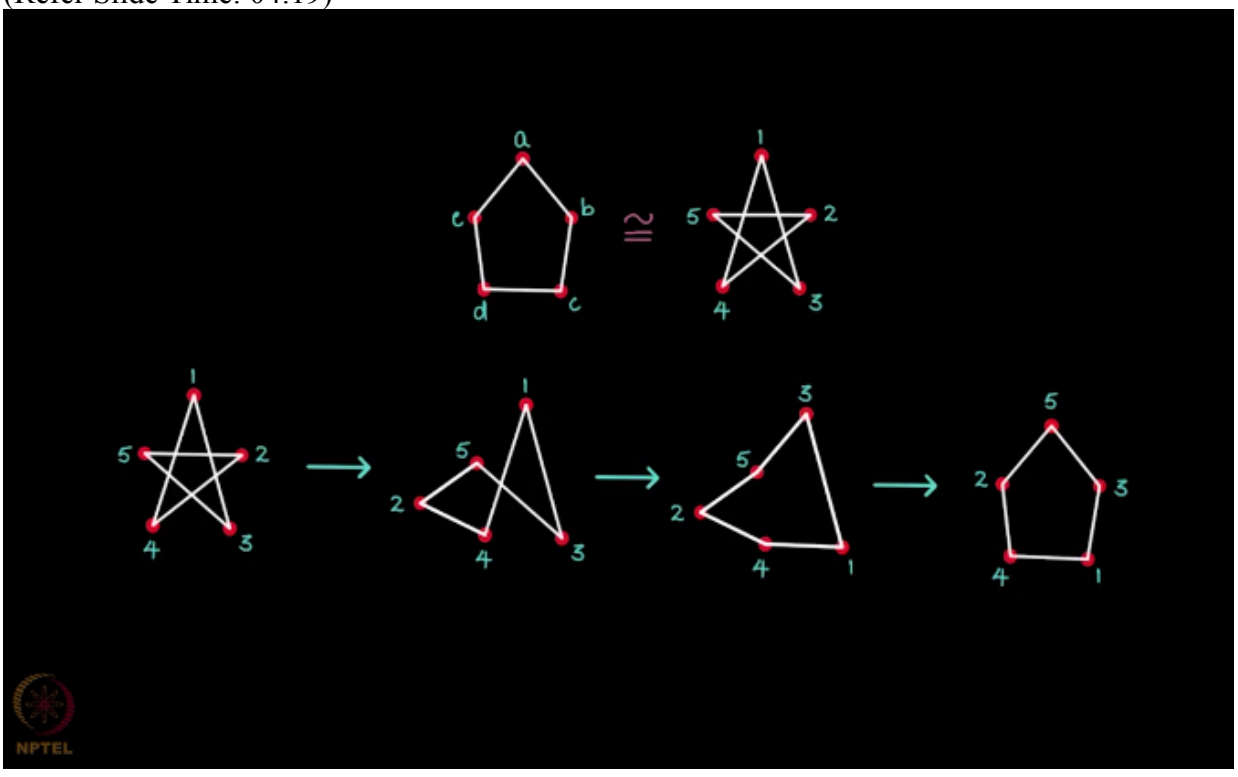

whatever are the labels we have obtained C5, so these two graphs though they look different, one is a cycle and other looks like a star, but they are isomorphic, so this is an example for graphs which look different but they are the same.

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