NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 1

Relation between walk and path
-An induction proof

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Let us not take a look at formal proof of what the professor has given an induction proof, I want to prove it using the tool induction, so my statement goes like this, yes there exists a UV walk, that yes a walk starting from U and ending at V and G then there exists a UV path in G, this is the statement.
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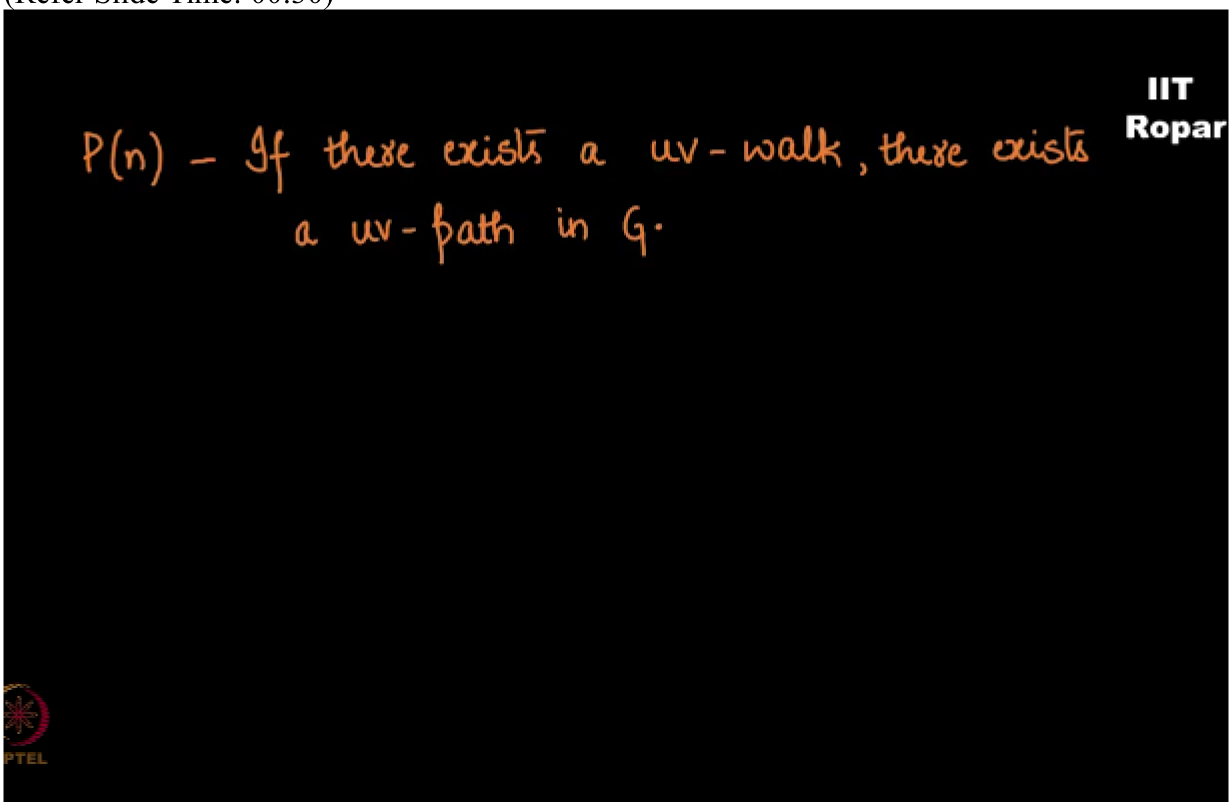

Now what are we going to induct on? We are going to induct on the length of the walk, so what will be the basic step? Basic step will be length as 1 that is the length of the walk is one unit,
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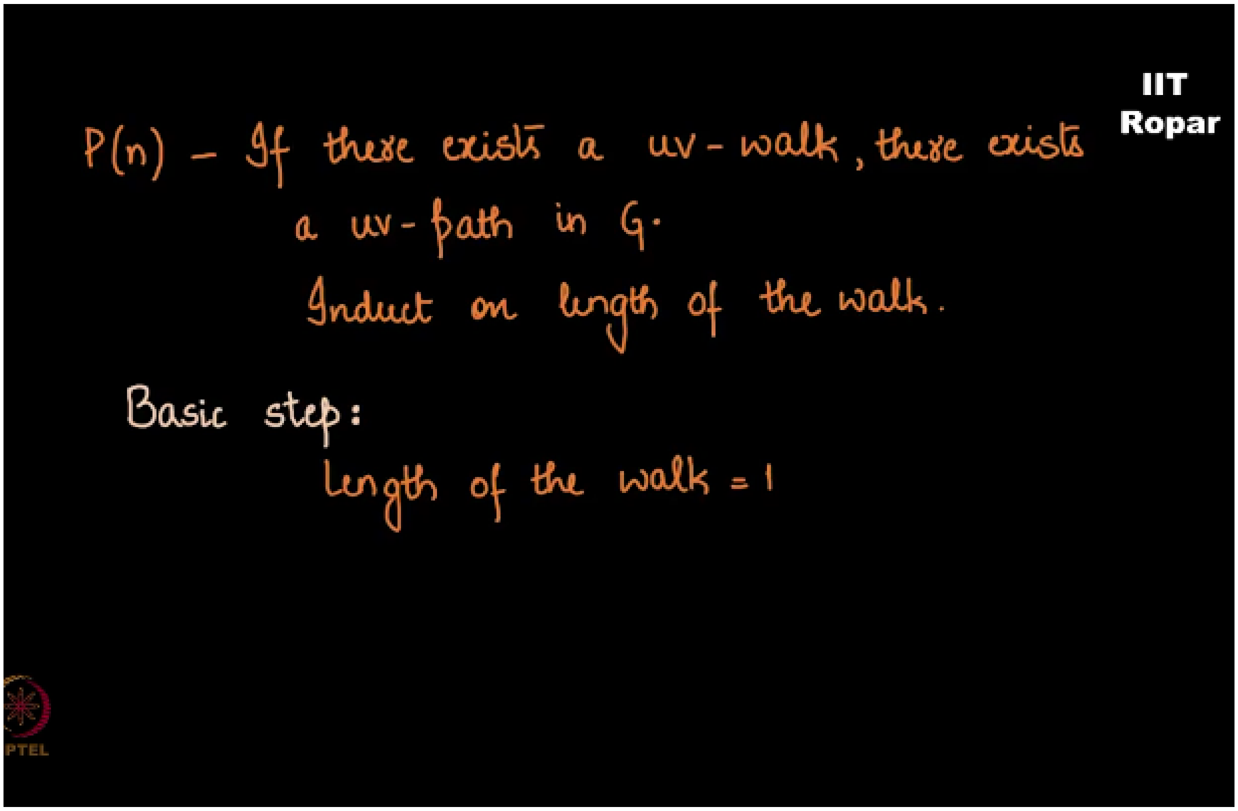

what does it mean if length is 1? It means that there is 1 edge, only 1 edge in the walk so it goes like this, so starting from U one edge V, now this is a path, this is a walk, and as well a path, this is the basic step and hence basic step if you see that this is true,

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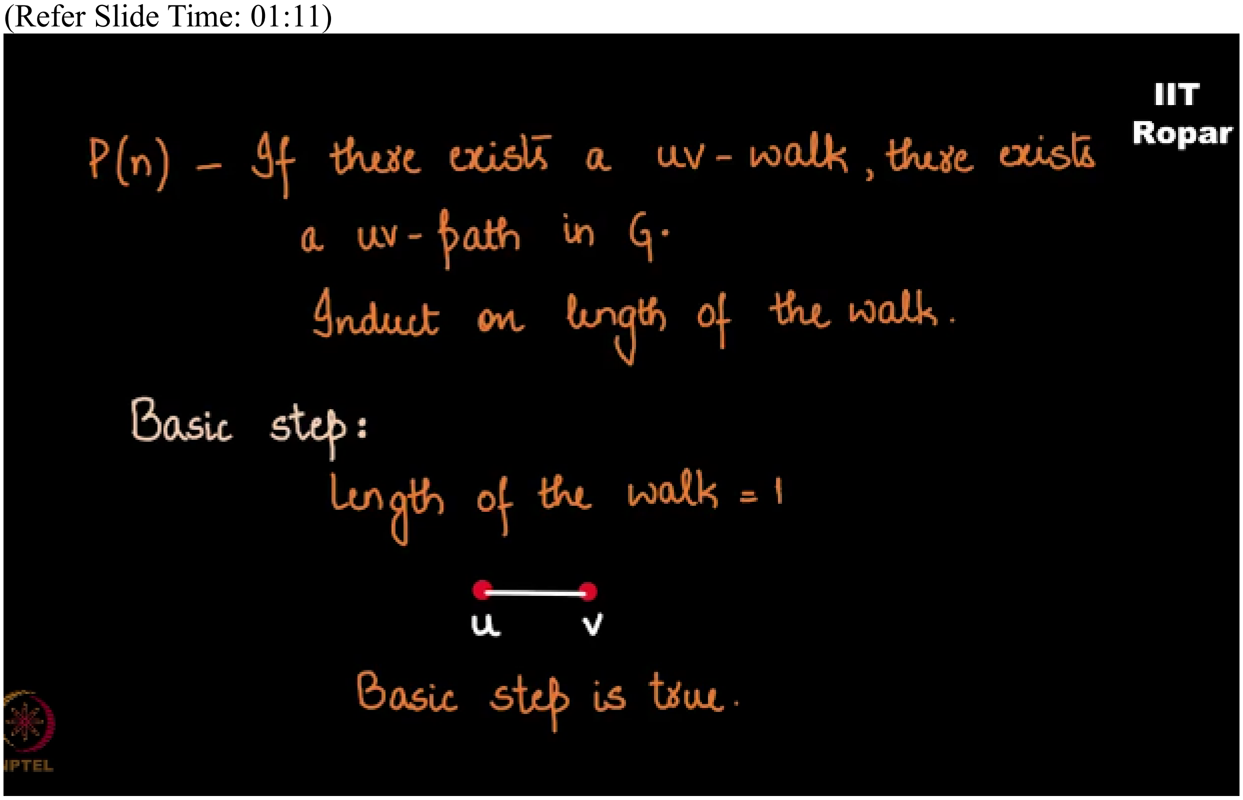

going to the induction hypothesis what does it mean, I assume that the statement holds true, if the walk, the length of the walk is less than or equal to K, so if there exists a UV walk in G, here is a UV path in G of length and less than or equal to K, it was UV this to be true.

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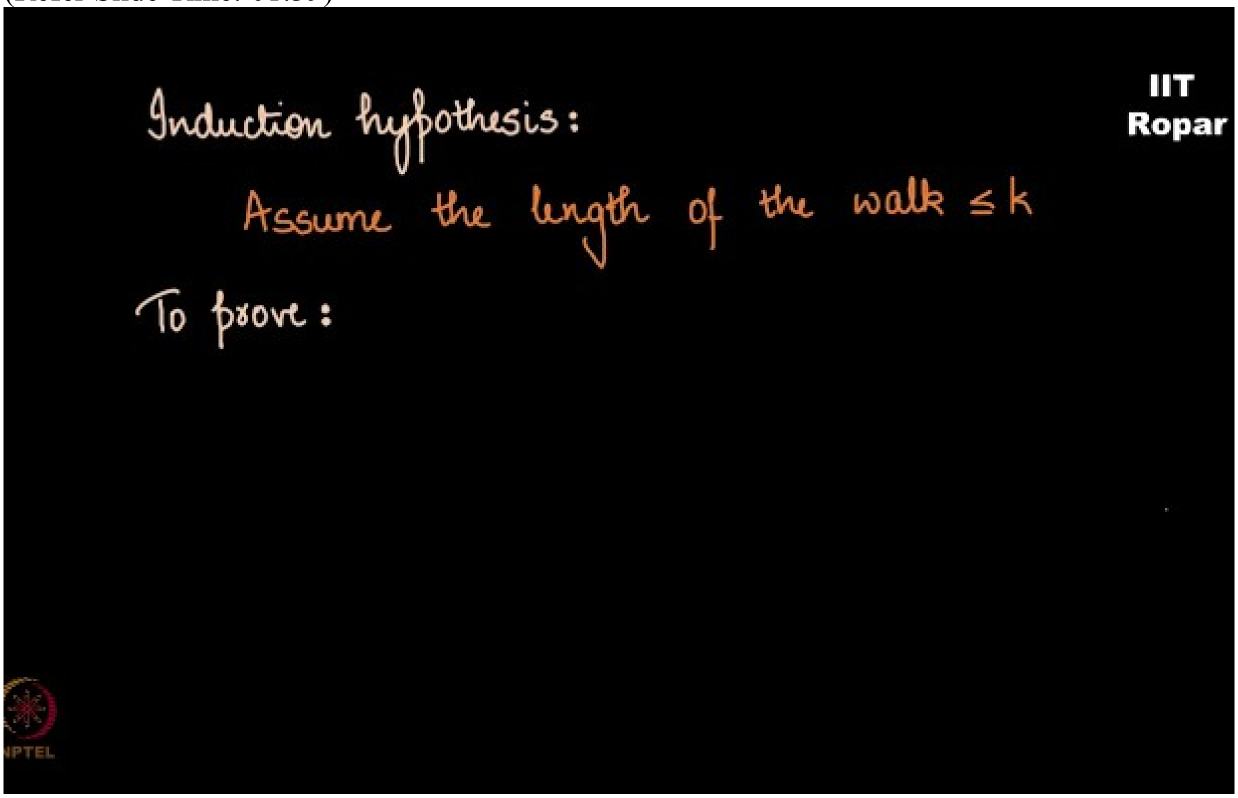

Now what do we have to prove, we have to prove that if you assume that there exists a path of length less than or equal to K then the theorem holds true for length K+1,

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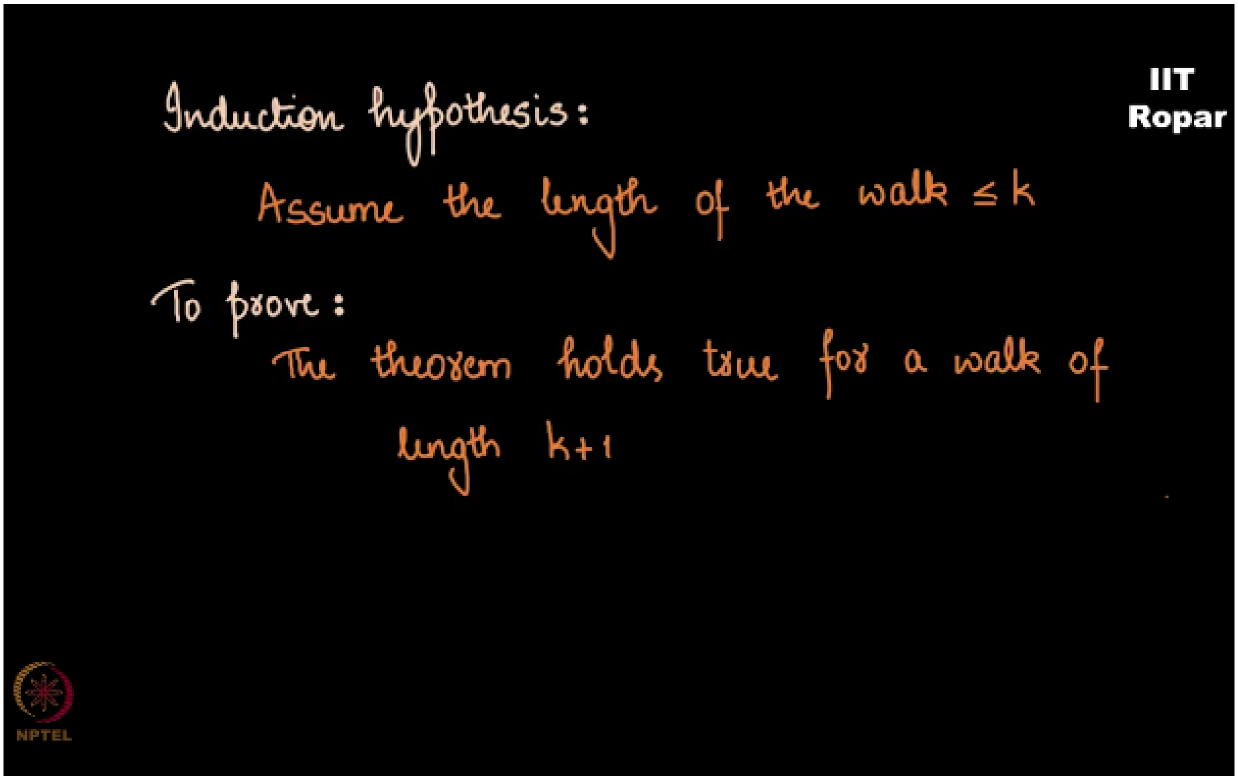

so for a length, walk of length K+1 there exists a UV path in G, so assuming that W of UV is a walk of length K+1, if there is no vertex which is getting repeated then we are done, it's a path by definition.
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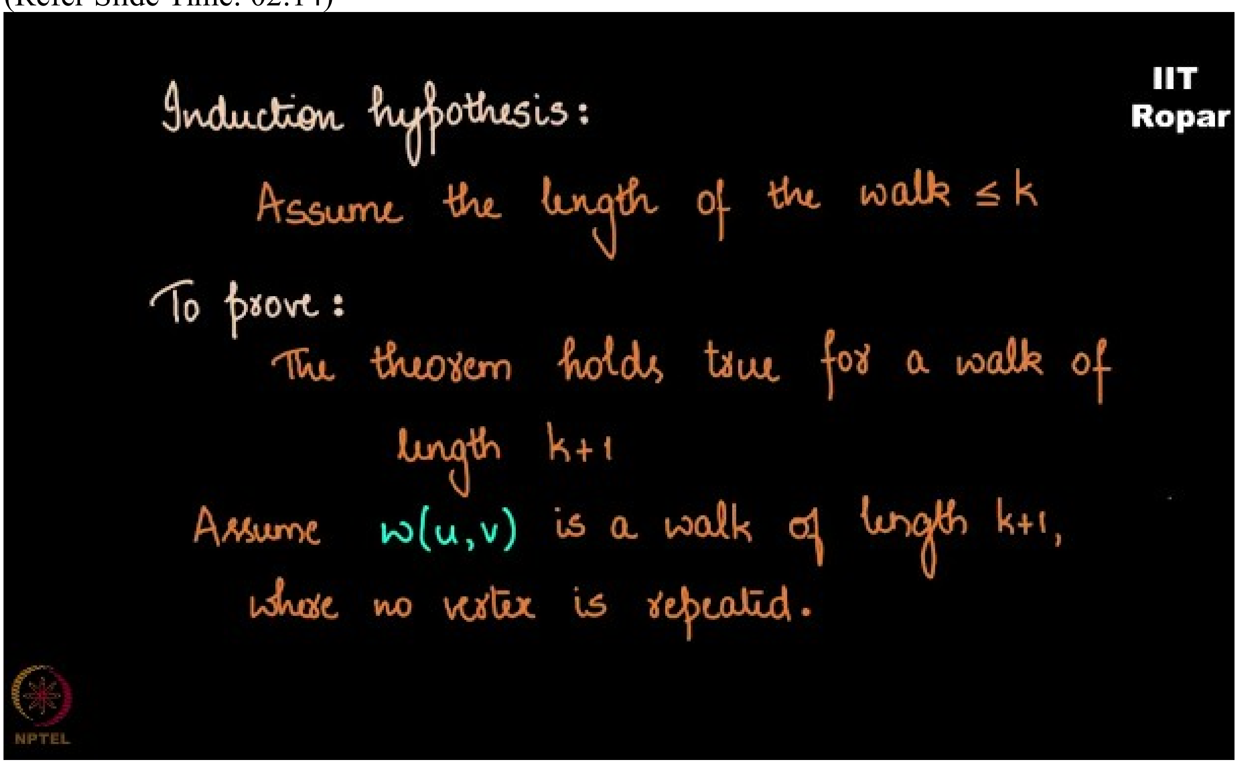

Now let us assume that a vertex gets repeated, what are we going to do? We can write it as, let the walk, W, U, V, be U starting from U, edge E, Y, another vertex say A, H, E to so on, I am going to go to the edge say EI, then X, X is the vertex, now this vertex is going to repeat, that is what we have

assumed, after X there is some edge so on and again you encounter X then over edge see EJ so on up to the last vertex, where do you reach? V, so this is the walk,
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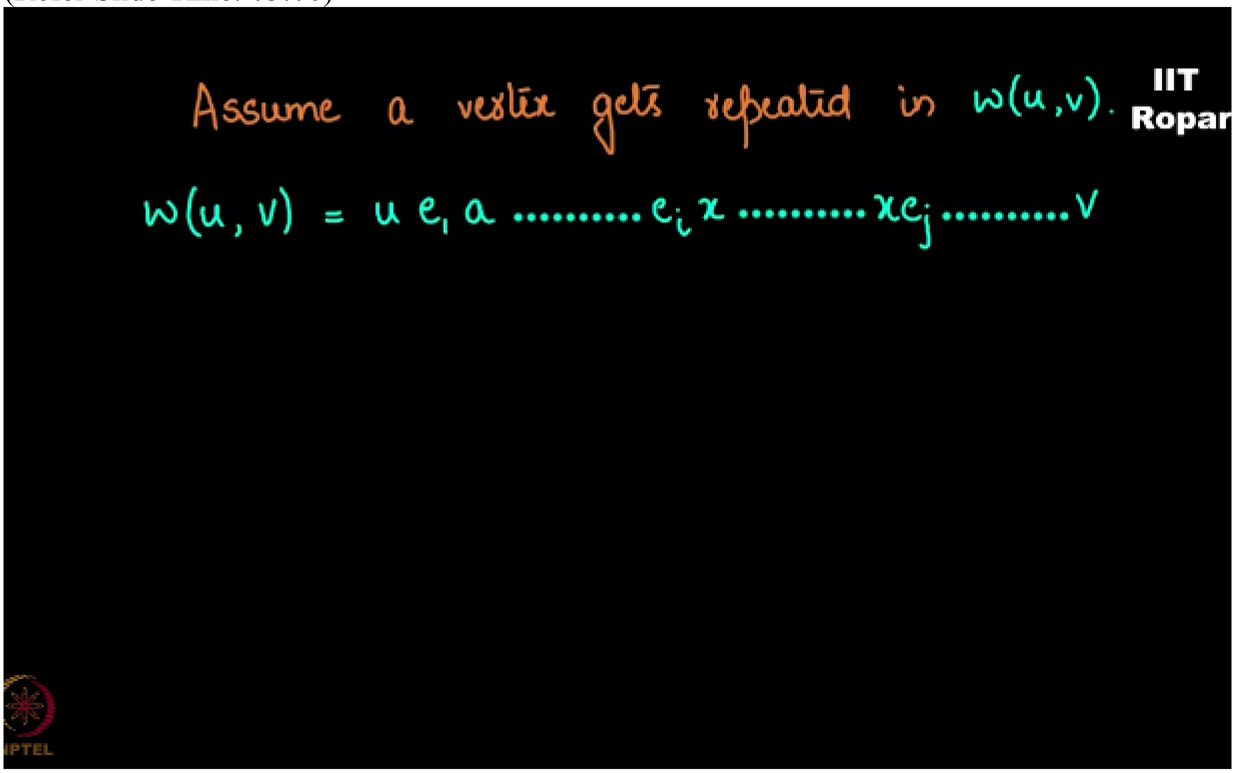

take a moment to observe the walk.

Now what am I going to do is I'm going to chop off all the vertices and edges like this,
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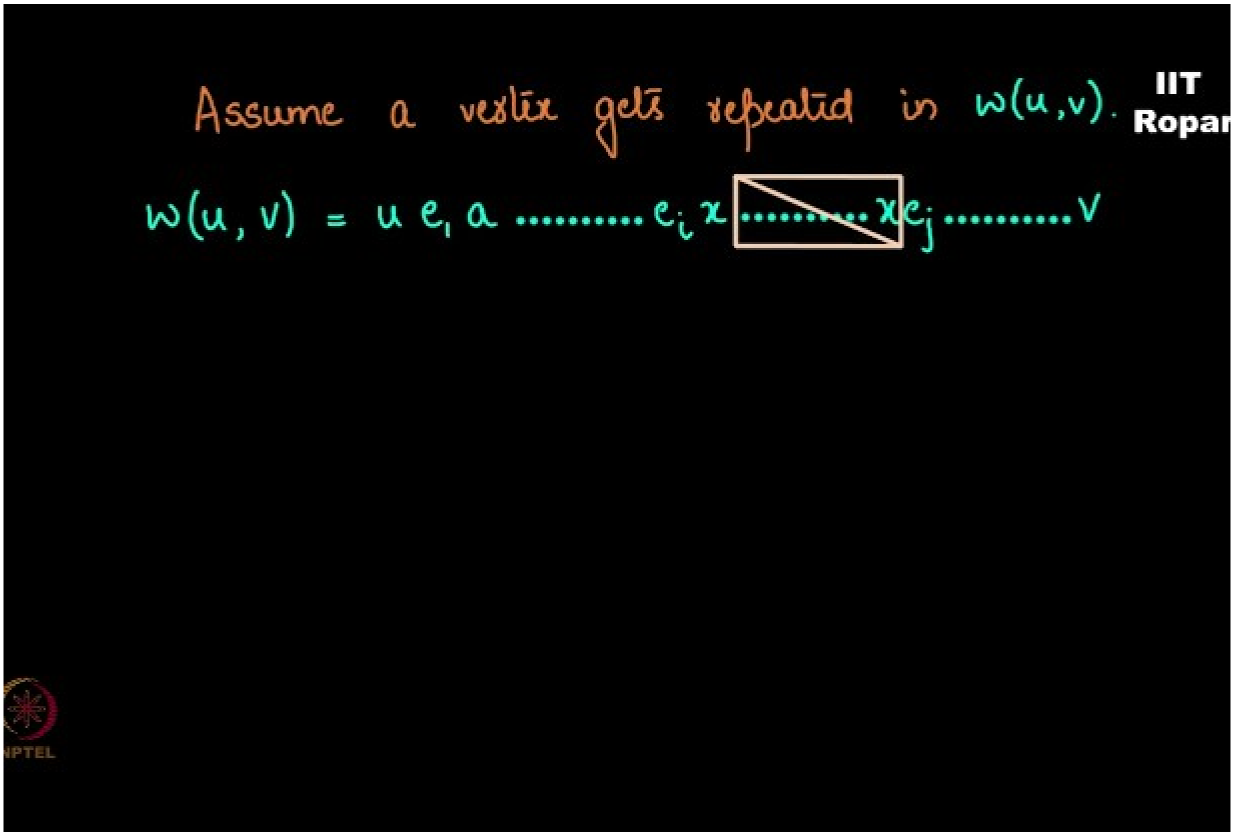

so what am I going to do, there are a few edges at least one in between the two X, so I am going to remove those vertices, edges, and this repeated X, so what am I going to get? If W dash is a walk from UV, I am going to write it as U, E1, say A, E2, so on EI, X, EJ so on up to V, did you see that this chopped off part is not there now, and we have reached EJ here,

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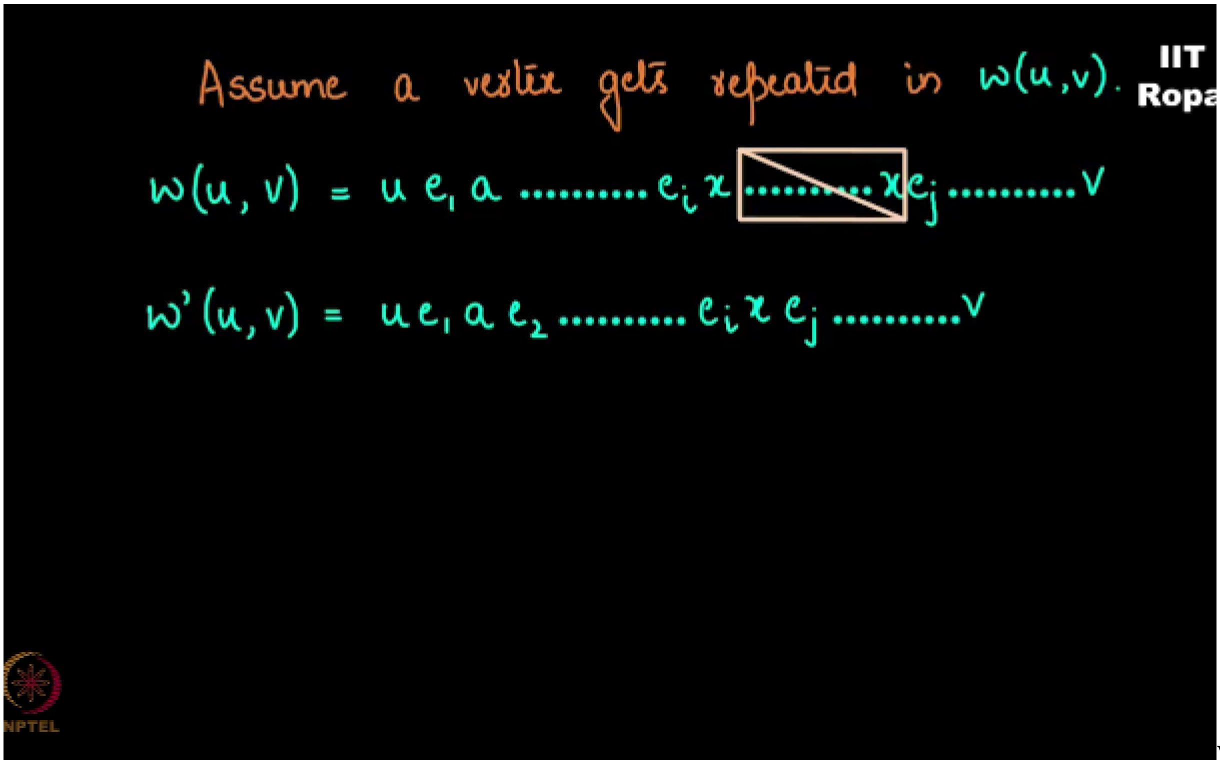

what do we see? We see that this is a walk of length less than or equal to K, because I have removed at least one edge so it is a walk of length less than or equal to K,

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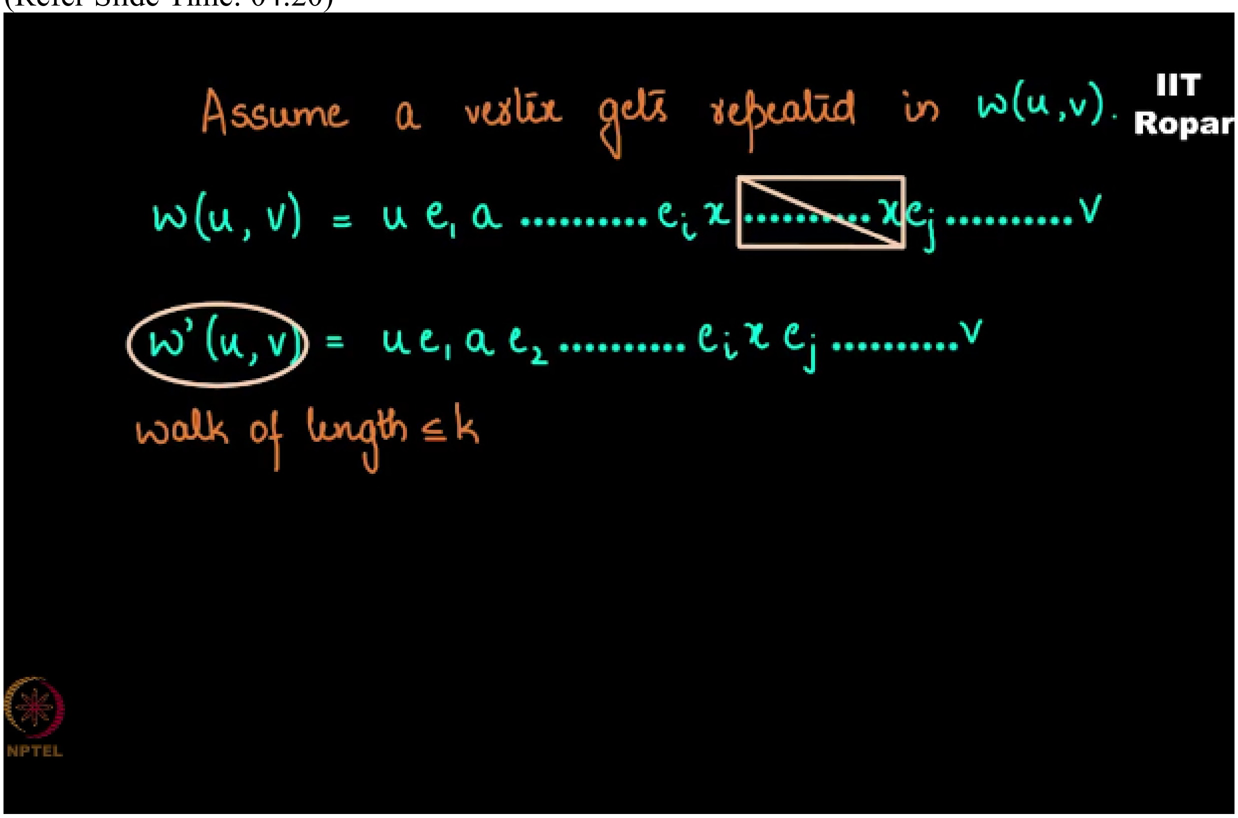

take a moment to observe the transition from the previous step to the step.

Now what did we conclude from the induction hypothesis? We can conclude that there exists a UV path, because this is a walk of length less than or equal to K, hence there must exist a path
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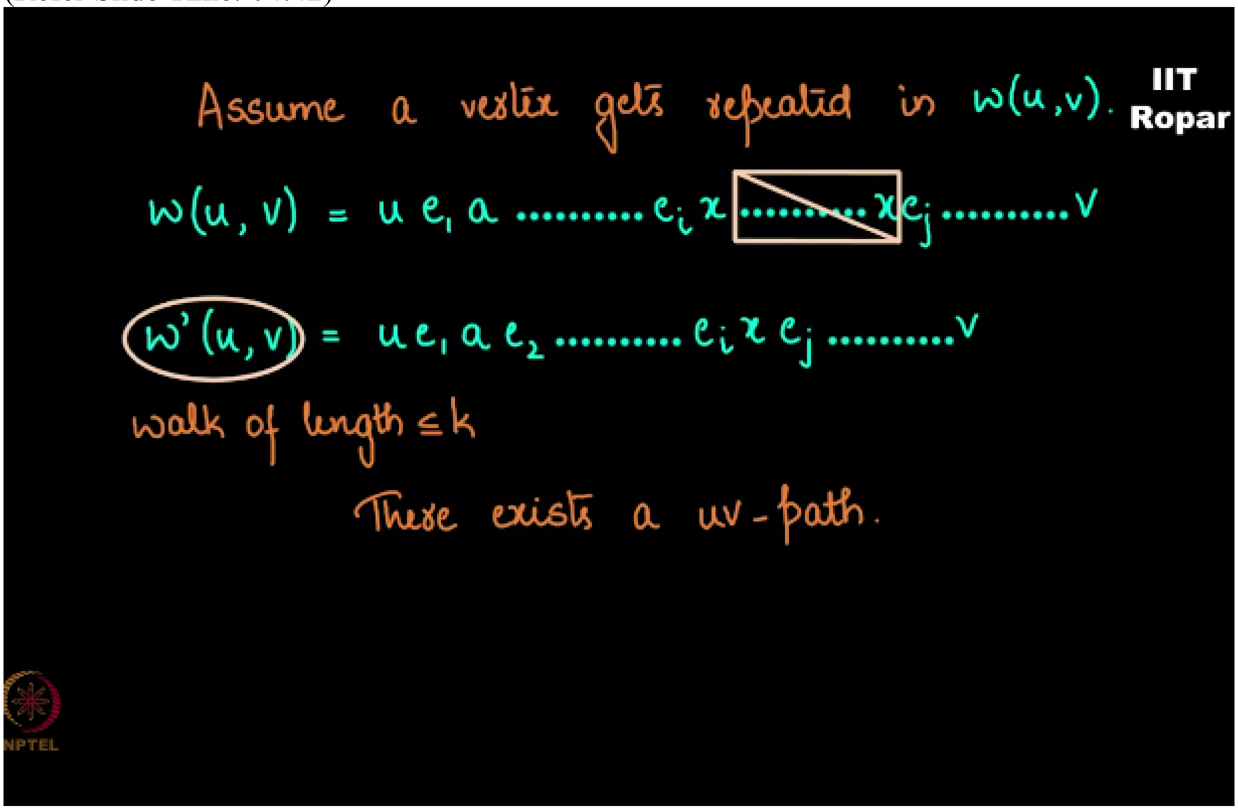

so we have proved that if there is a walk in the graph then there is a path in the graph.
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