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NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 1

Havel Hakimi theorem - Part 5

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I can now stretch the entire story narrated so far improper basis and I'm now going to use the previous definition and state a very nice result. A sequence S, a degree sequence D1 greater than or equal to D2, greater than or equal to D3 and so on, greater than or equal to DN with D1 less than or equal to N-1
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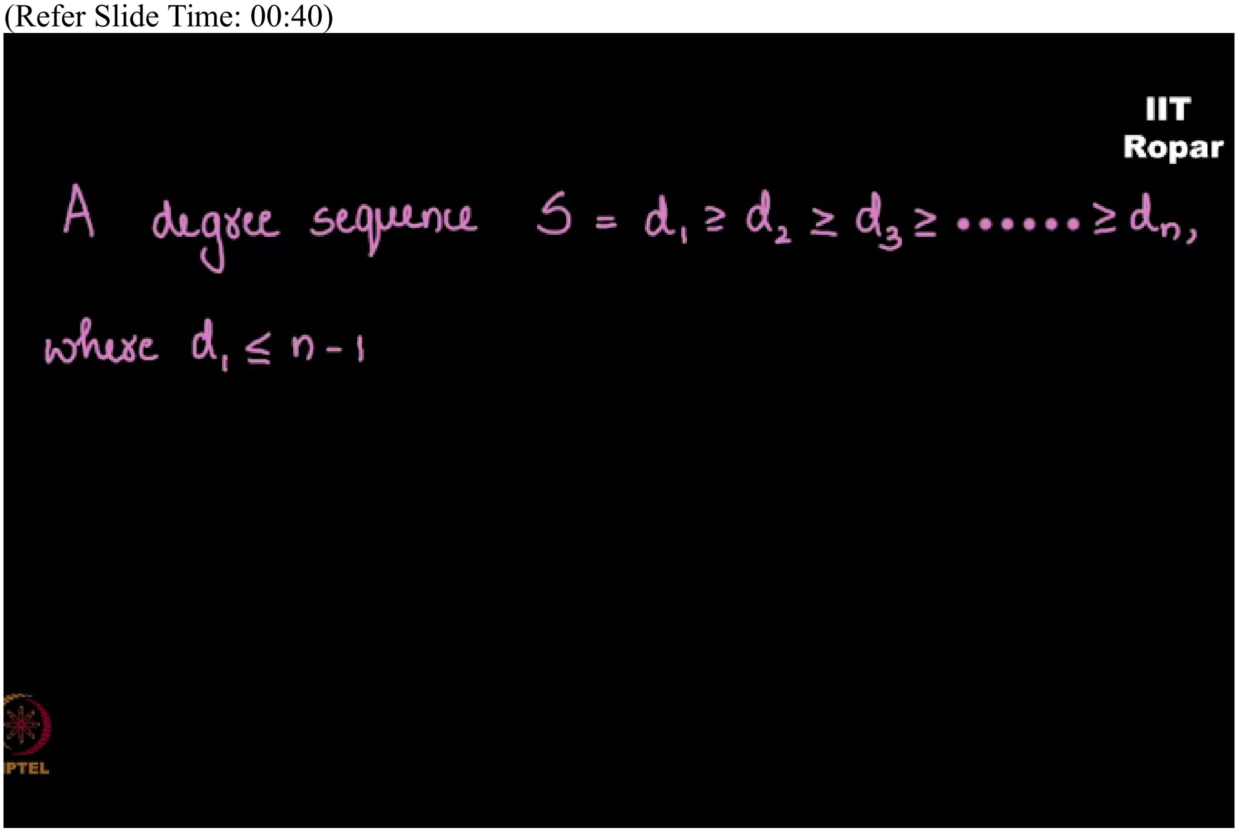

and this what is the number of vertices, this sequence is graphic if and only if the reduced sequence,
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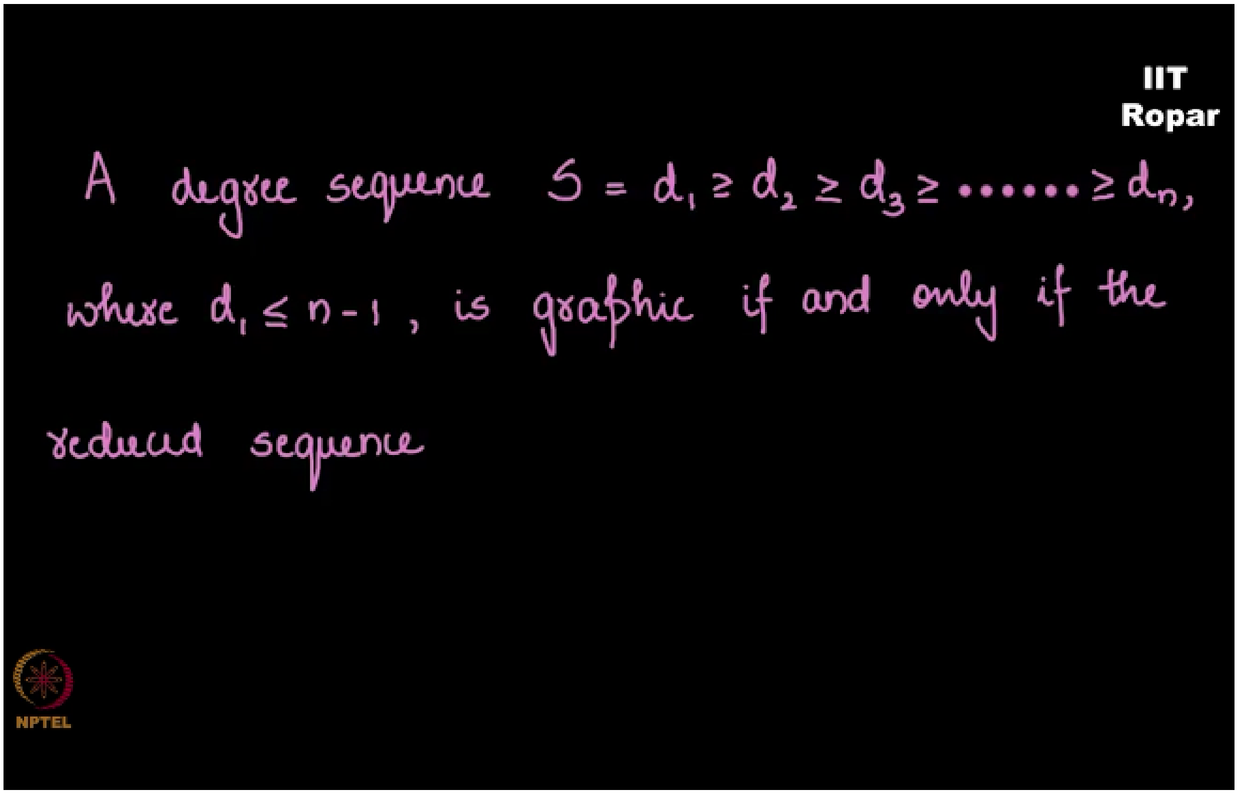

now don't worry much about the Jordon's or the complicated words used, since we've already solved a problem I see how to draw the graphs with the sequence you must be able to visualize it parallelly as I've stated it.

Now it is graphic if and only if the reduced sequence S dash which is star, D2-1, D3-1 so on up to DN,

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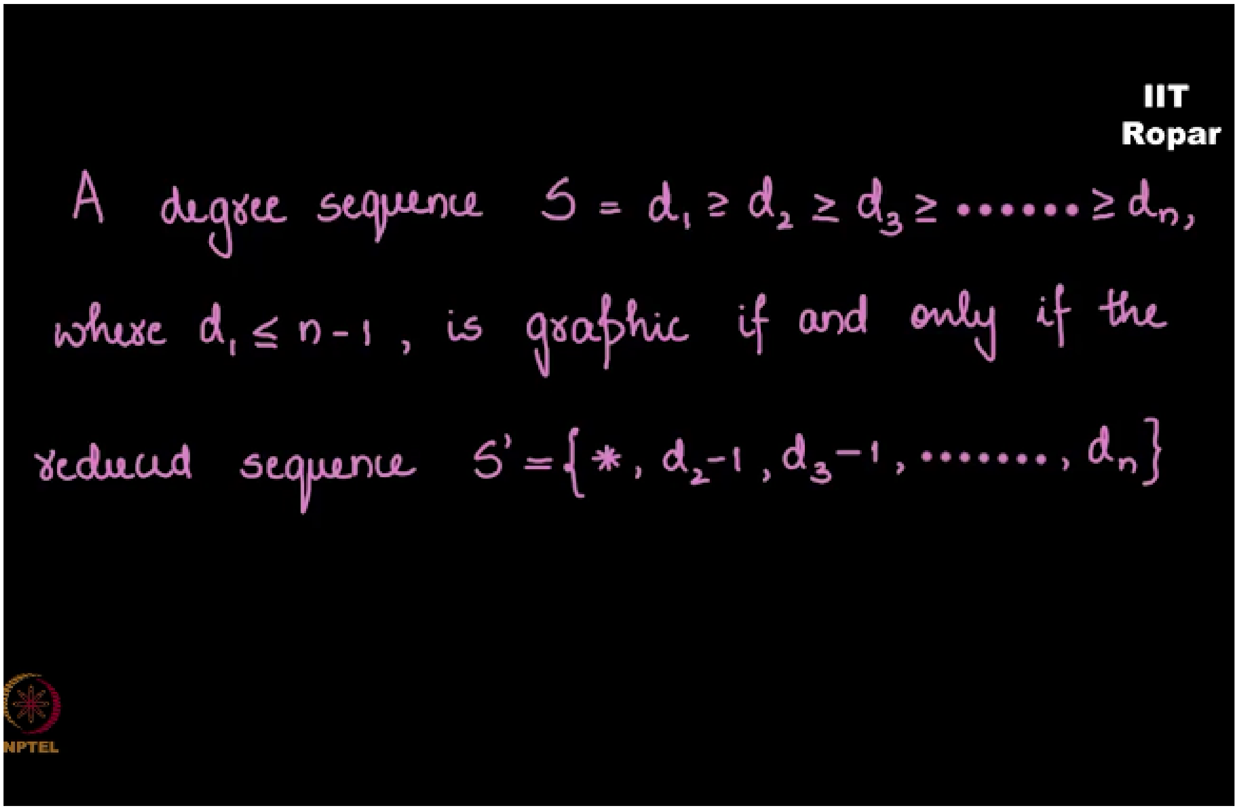

why is this the reduced sequence? Each time and modified previous sequence we reduce the degree by 1.

Now this sequence S dash is graphic,
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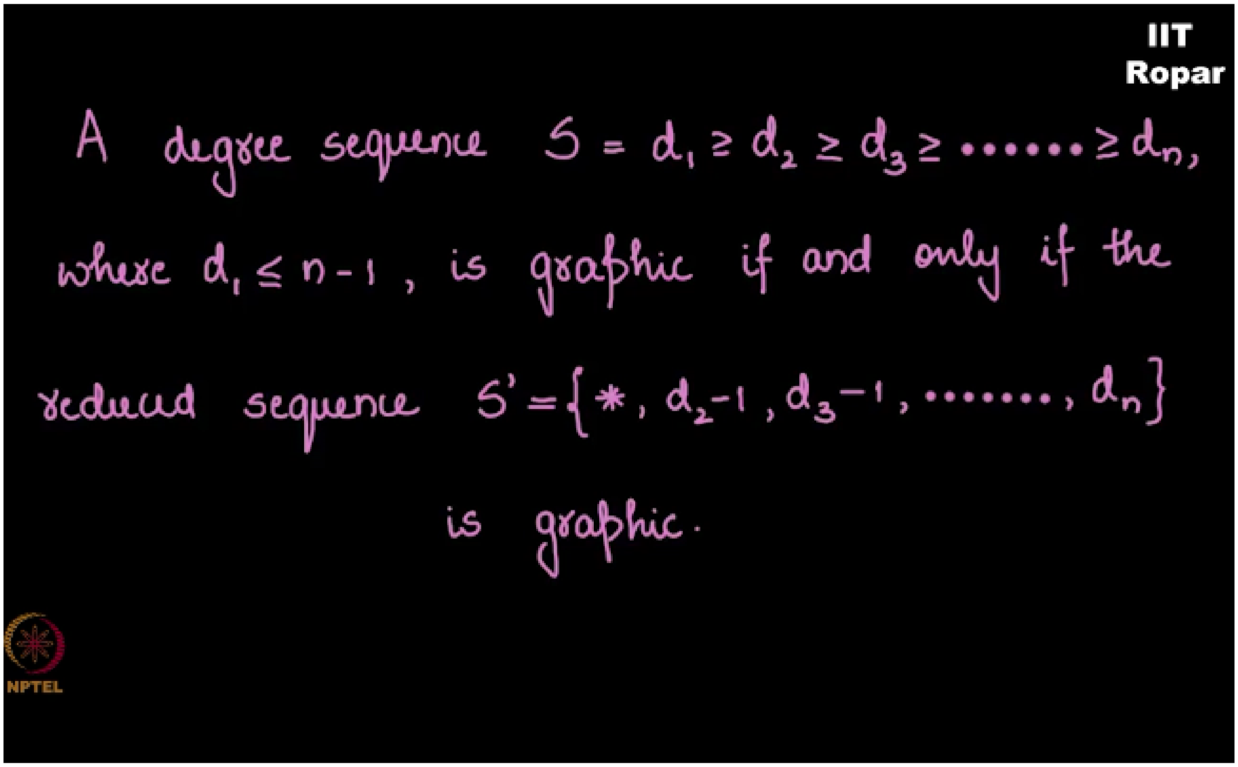

so a sequence S is graphic if and only if the reduced sequence is graphic, please note this word if and only if is very important, why?

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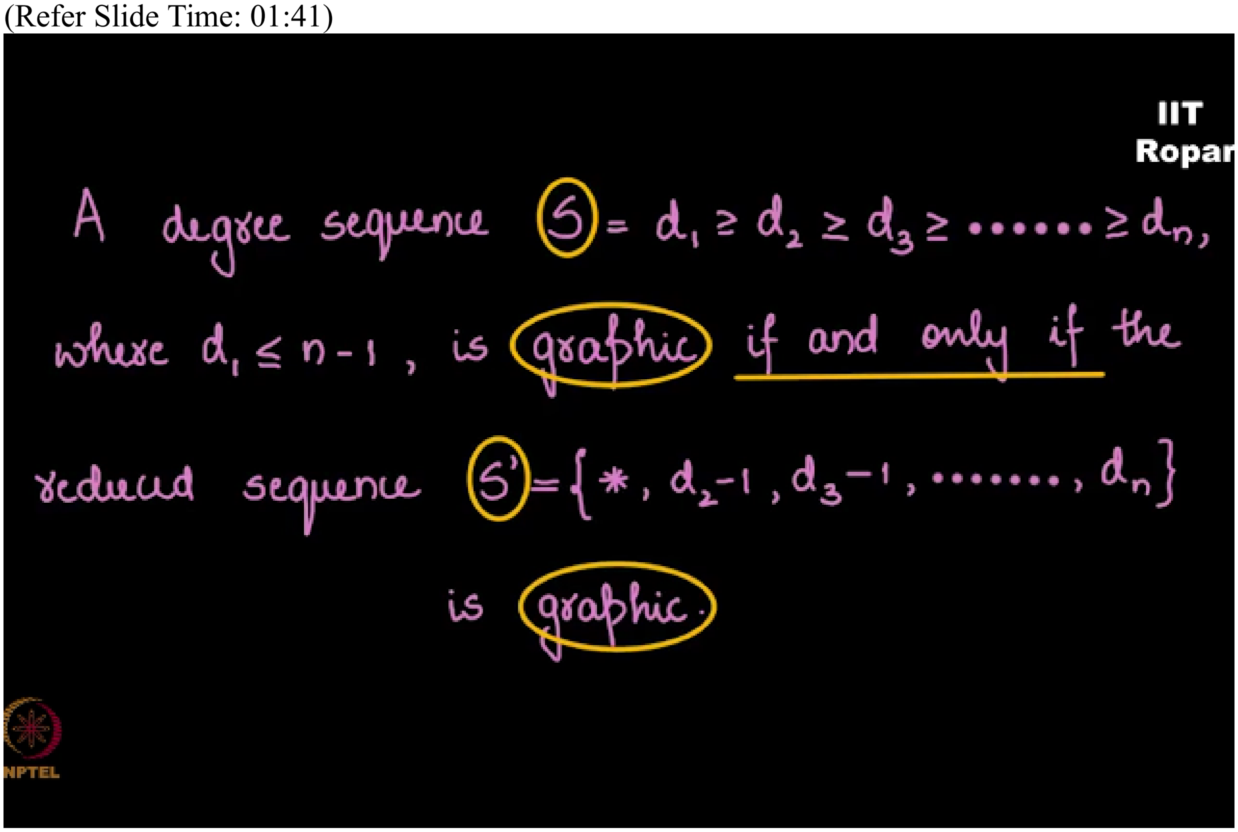

in case the reduced sequences graphic then the given sequence is graphic.

Now the most important part is the following, if the last sequence contains all 0s or stars, then it is graphic,
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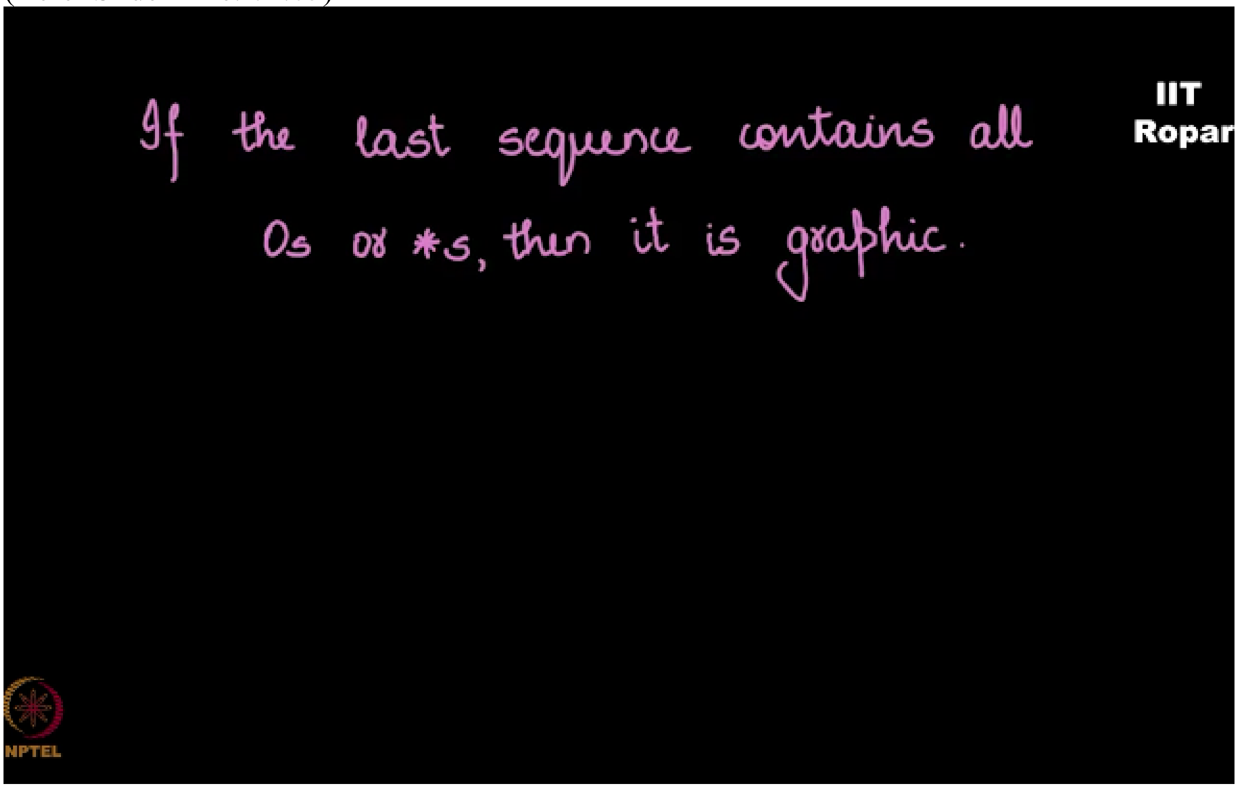

this is very important, why? You need not even draw the graph at each step like how we did earlier, right, you have a sequence you reduce it, again reduce it you go to the next sequence and so on,
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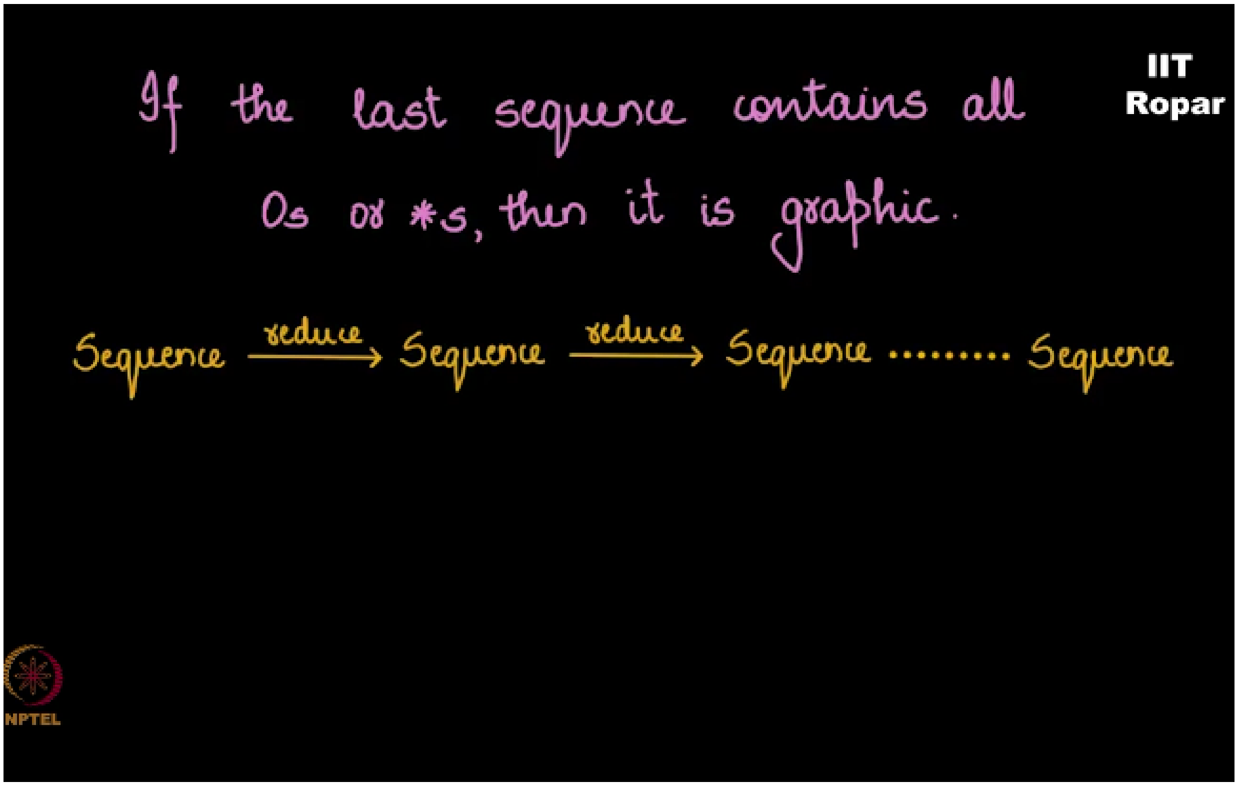

once you finish the process the last sequence which you get if it contains all 0s then the given sequence is set to be graphic.

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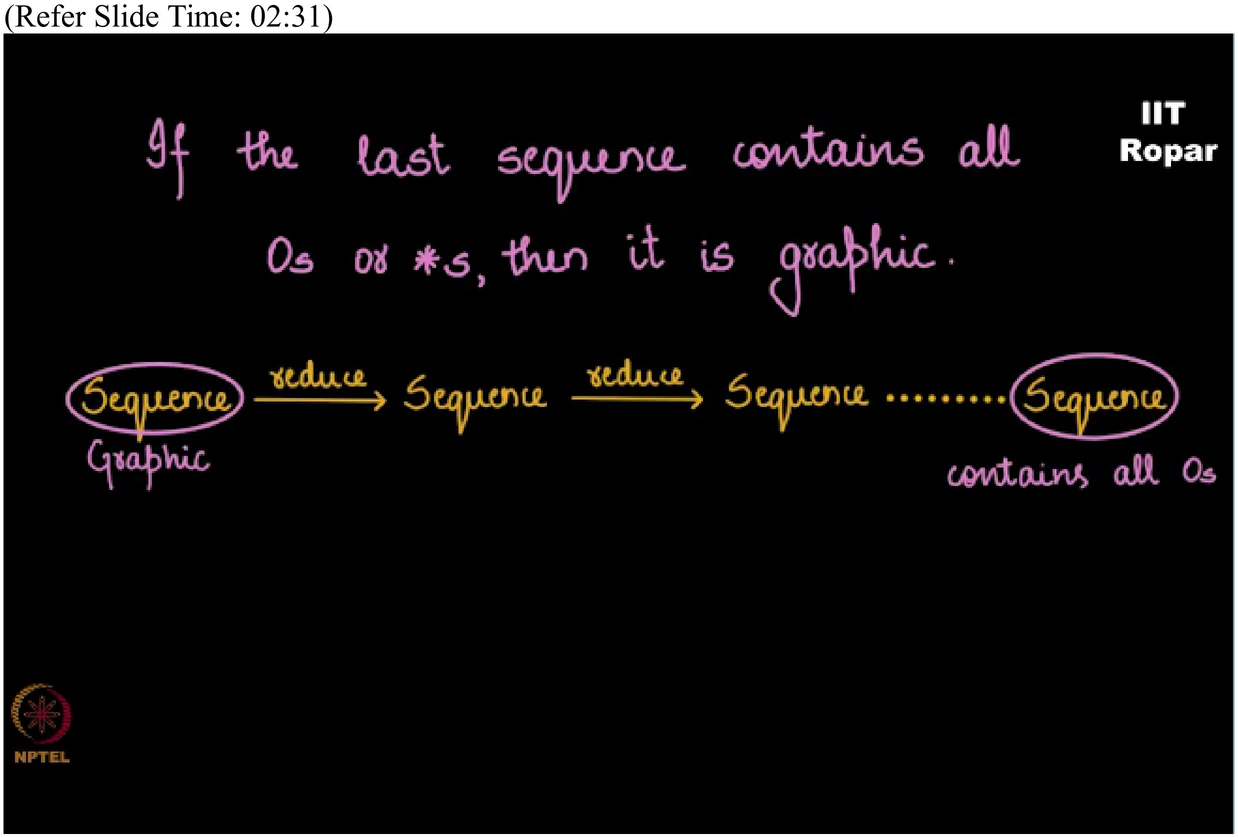

Now this theorem is called the celebrated Havel Hakimi theorem,

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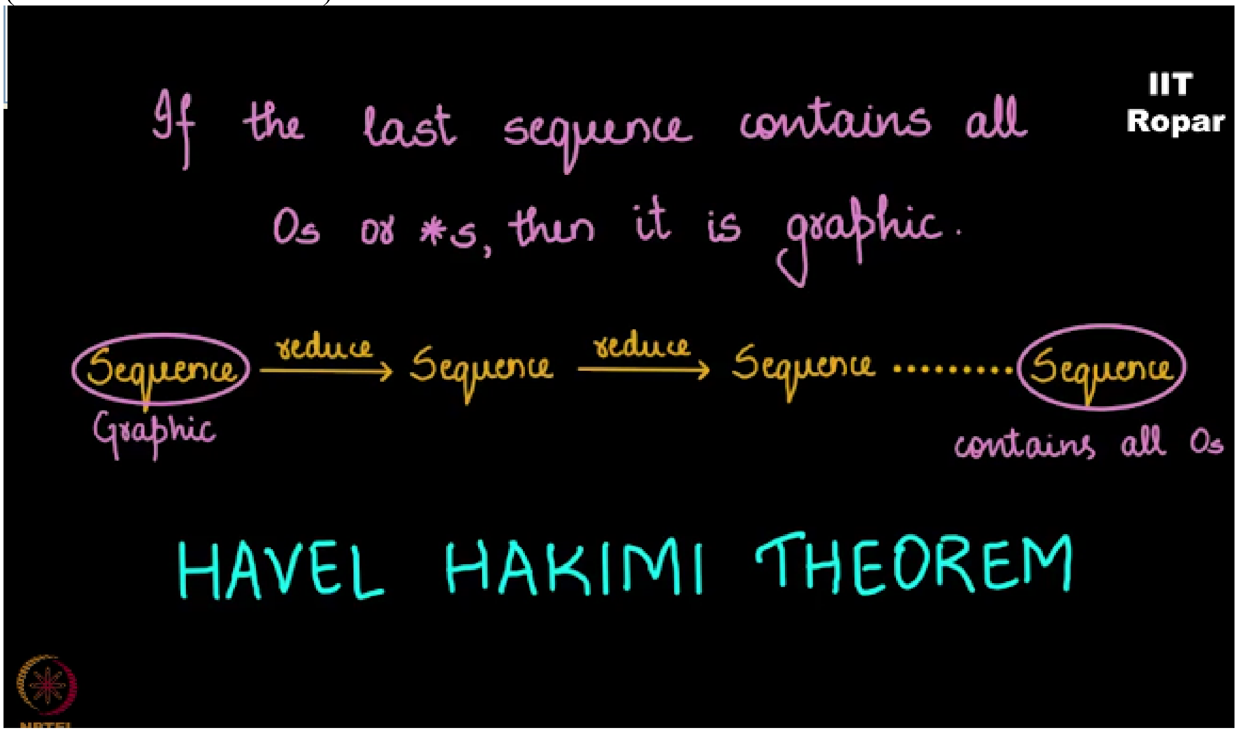

they are not going to prove this theorem as it involves a lot of technicalities, we did two previous problem that is 5, 5, 3, 3, 2, 2, 2 in order to motivate you to clean the wound this theorem.

Now go back to that video, look at how the graph was drawn and you will now realize that the graph drawing is actually not required, that entire process is not required, why? just reducing the sequence and writing the sequences is enough for you to judge if the sequence is graphic or not.

Now this is the right time for you to try out the sequence which was given in the previous video 5, 5, 5, 5, 2, 2, 2, try if this sequence is graphic

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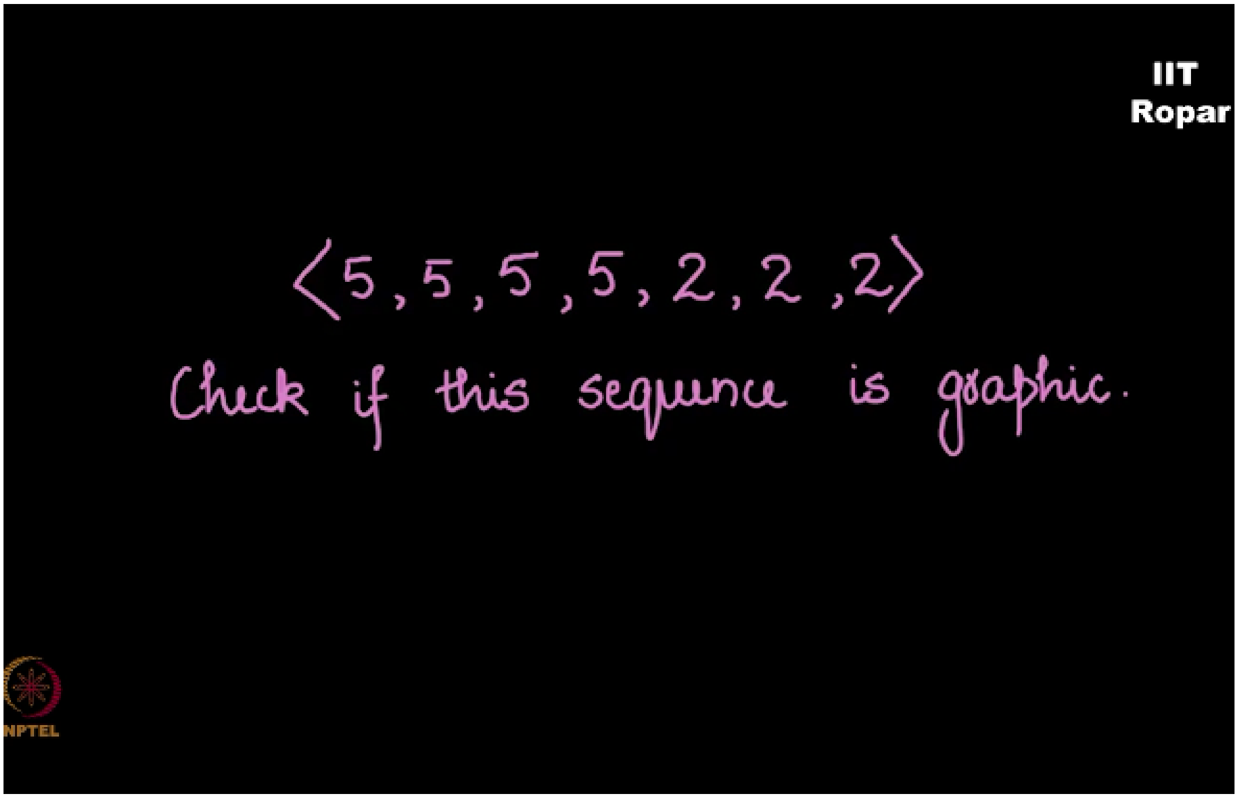

this is the right stage for you to try out and check if it is graphic.
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I'm now going to show you the solution for the question is the sequence 5, 5, 5, 5, 2, 2, 2, graphic, when we are not going to give you an explanation here, but you can watch the video and understand whether the sequence is graphic or not, observe the solution.
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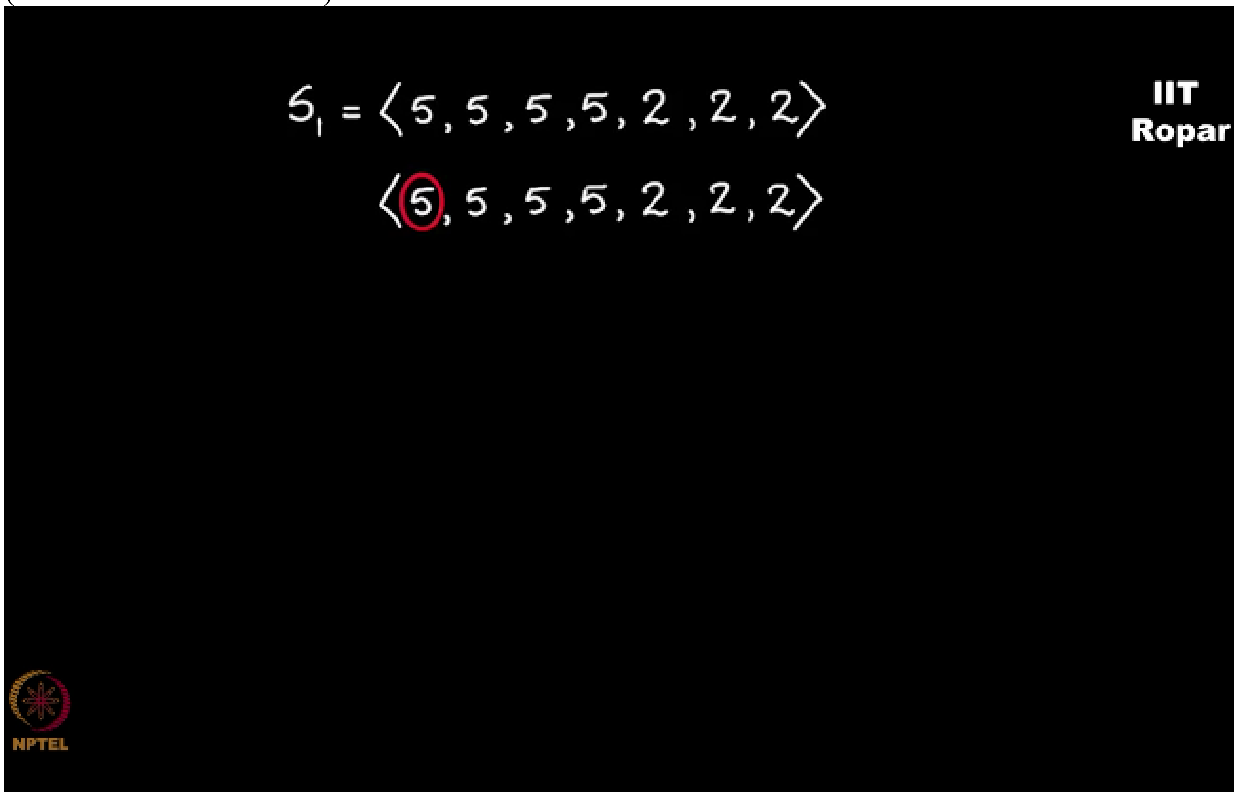

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$$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$$

$$\langle 5, 4, 4, 4, 1, 2, 2 \rangle$$

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$$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$$

$$S_2 = \langle *, 4, 4, 4, 1, 1, 2 \rangle$$

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$$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$$

$$S_2 = \langle *, 4, 4, 4, 1, 1, 2 \rangle$$

$$S_2' = \langle *, 4, 4, 4, 2, 1, 1 \rangle$$

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$$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$$

$$S_2 = \langle *, 4, 4, 4, 1, 1, 2 \rangle$$

$$S_2' = \langle *, 4, 4, 4, 2, 1, 1 \rangle$$

$$\langle *, 4, 4, 4, 2, 1, 1 \rangle$$

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$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$

$S_2 = \langle *, 4, 4, 4, 1, 1, 2 \rangle$

$S_2' = \langle *, 4, 4, 4, 2, 1, 1 \rangle$

$\langle *, *, 3, 3, 1, 0, 1 \rangle$

$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$

$S_2 = \langle *, 4, 4, 4, 1, 1, 2 \rangle$

$S_2' = \langle *, 4, 4, 4, 2, 1, 1 \rangle$

$S_3 = \langle *, *, 3, 3, 1, 0, 1 \rangle$

IIT Ropar

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IIT Ropar

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$$S_3 = \langle *, *, 3, 3, 1, 0, 1 \rangle$$

$$S_3' = \langle *, *, 3, 3, 1, 1, 0 \rangle$$

$$S_4 = \langle *, *, *, 2, 0, 0, 0 \rangle$$

IIT Ropar

$$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$$

$$S_2 = \langle *, 4, 4, 4, 1, 1, 2 \rangle$$

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$$S_3 = \langle *, *, 3, 3, 1, 0, 1 \rangle$$

$$S_3' = \langle *, *, 3, 3, 1, 1, 0 \rangle$$

$$S_4 = \langle *, *, *, 2, 0, 0, 0 \rangle$$

$$S_5 = \langle *, *, *, *, -1, -1, 0 \rangle$$

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$$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$$

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$$S_3 = \langle *, *, 3, 3, 1, 0, 1 \rangle$$

$$S_3' = \langle *, *, 3, 3, 1, 1, 0 \rangle$$

$$S_4 = \langle *, *, *, 2, 0, 0, 0 \rangle$$

$$S_5 = \langle *, *, *, *, -1, -1, 0 \rangle$$ Degree cannot be negative.

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$$S_1 = \langle 5,5,5,5,2,2,2 \rangle$$

$$S_2 = \langle *,4,4,4,1,1,2 \rangle$$

$$S_2' = \langle *,4,4,4,2,1,1 \rangle$$

$$S_3 = \langle *,*,3,3,1,0,1 \rangle$$

$$S_3' = \langle *,*,3,3,1,1,0 \rangle$$

$$S_4 = \langle *,*,*,2,0,0,0 \rangle$$

Not graphic $S_5 = \langle *,*,*,*,-1,-1,0 \rangle$ Degree cannot be negative.

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$$S_3 = \langle *,*,3,3,1,0,1 \rangle$$

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Not graphic $S_4 = \langle *,*,*,2,0,0,0 \rangle$

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$S_1 = \langle 5, 5, 5, 5, 2, 2, 2 \rangle$

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$S_2' = \langle *, 4, 4, 4, 2, 1, 1 \rangle$

Not graphic $S_3 = \langle *, *, 3, 3, 1, 0, 1 \rangle$

$S_3' = \langle *, *, 3, 3, 1, 1, 0 \rangle$

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Not graphic $S_2 = \langle *, 4, 4, 4, 1, 1, 2 \rangle$

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