Discrete Mathematics

Let Us Count

Combination with Repetition - Problems

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So here is a question. In how many ways can we write 100 as sum of four numbers? That is A+B+C+D should be equal to 100. Now you see 30+30+20+20 becomes 100. This is one possibility. Please note, 30+20+30+20 is another possibility. You see change of ordering leads to another possibility. That is how we are counting here. So you probably are wondering why should we consider these two are different possibilities that is a rule led by us, right. I mean that's how we are going to count. So in how many ways can we write 100 as A+B+C+D and please note we say non-negative integer which means 0 is also allowed. So 100+0+0+0 is one possibility. 0+100+0+0 is another possibility. 0+0+0+100+0 is another. 0+0+0+0+100 is another. So let us answer this question.

Now that you are very familiar with the formula, you already see that this is nothing else but 100 slots plus three separators. So it's going to be 100+3 slots and in this 103 in how many ways can you place three placeholders which will be 103, 103C3. N plus r minus 1 C r minus 1. And here is 100, r here is four so it's going to be 100 plus 4 minus 1 C 4 minus 1 which is 103C3. Assume I have this jar which can contain 100 candies and these candies come in seven different colors, let's say seven different flavors and seven different colors. And let these colors be VIBGYOR which is violet, Indigo, Blue, Green, Yellow, Orange, Red. The question now is in how many ways can I fill this jar which can actually hold 100 candies with candies which come in seven colors. Corrrect? Now isn't this question the same as in how many ways can you write 100 as a1 plus a2 plus a3 plus a4 plus a5 plus a6 plus a7. Think about it. Correct.

You see you pouring 100 violet candies is one possibility that translates to 100+0+0+0+0+0 correct and so on. So what's the answer? Pretty clear. So 100 equals 7 slots which is n plus r minus 1 C r minus 1. I have a advice for you all and the advice goes like this, do not see the formula always. Try to solve it without the formula and try to get used to the formula.

So what will be the answer? 106 slots and 6 placeholders. Six vertical sticks in how many ways can you put six vertical sticks on 106 slots. That would be n equals 100 r equals 7, n plus r minus 1 turns out to be 106C6 and there we are. That's the answer.