NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 1

Hand shaking lemma - Corollary

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Let us go back to our high school mathematics, 1 + 3 is 4
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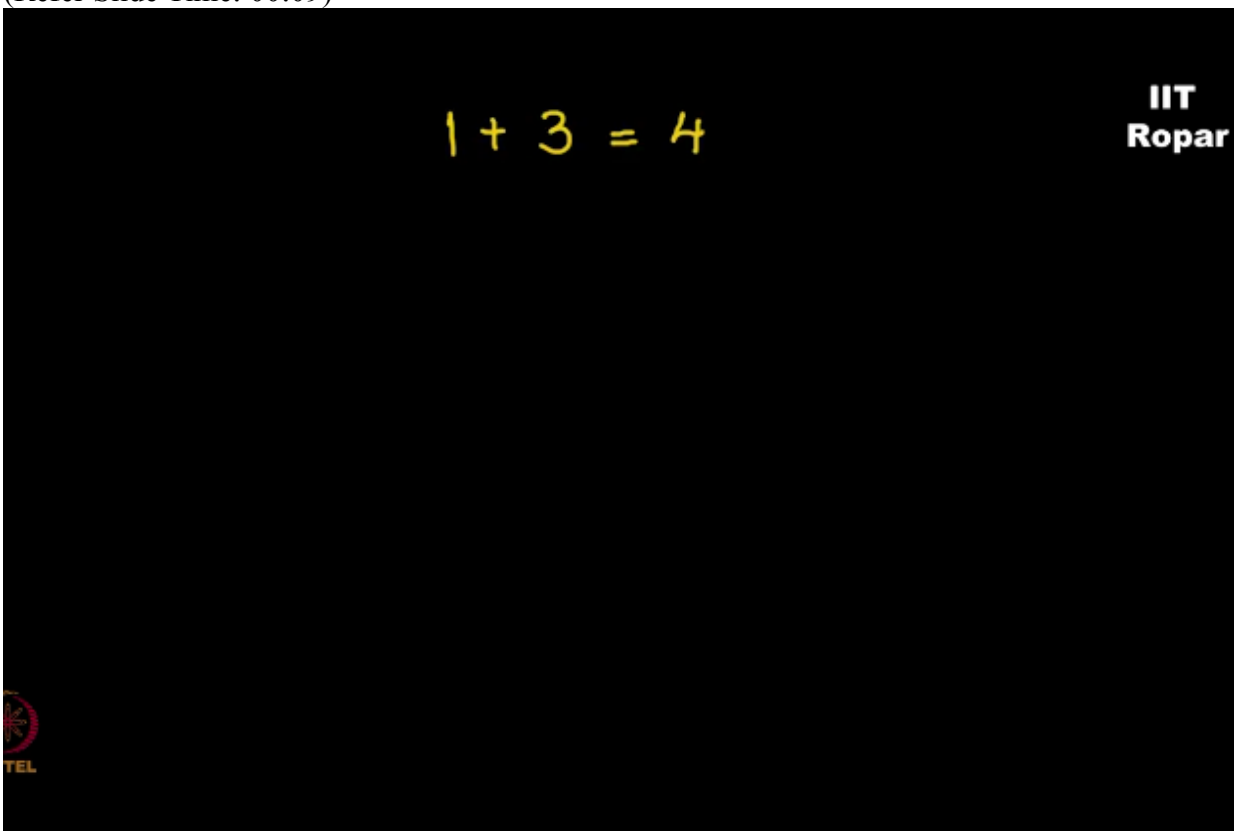

you see 4 is an even number, I'll add one more odd number here, 1 and 3 are already odd numbers, I'm going to add one more odd number, what do I get? 1+3+ let's say 5 is 9,
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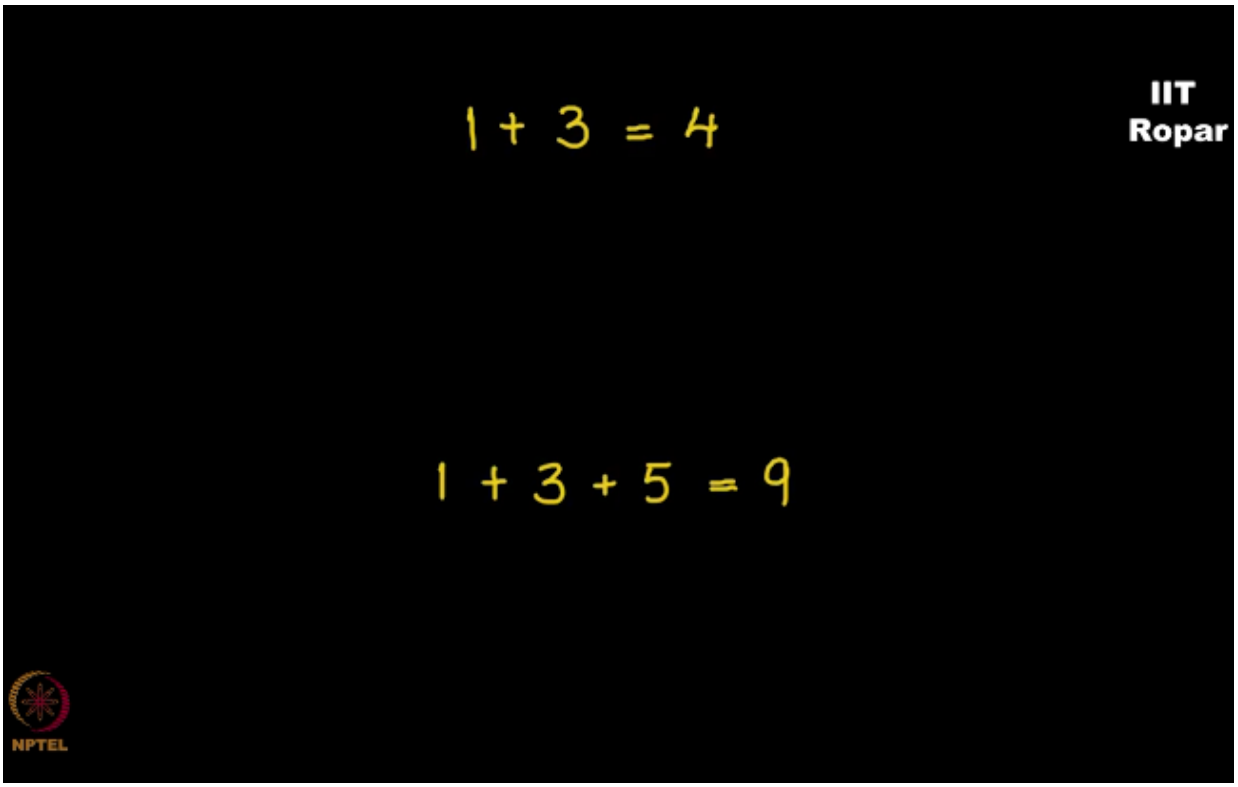

9 is an odd number, with these two expressions can we make or observe something?

When you have odd number of odd numbers, the sum is always odd,
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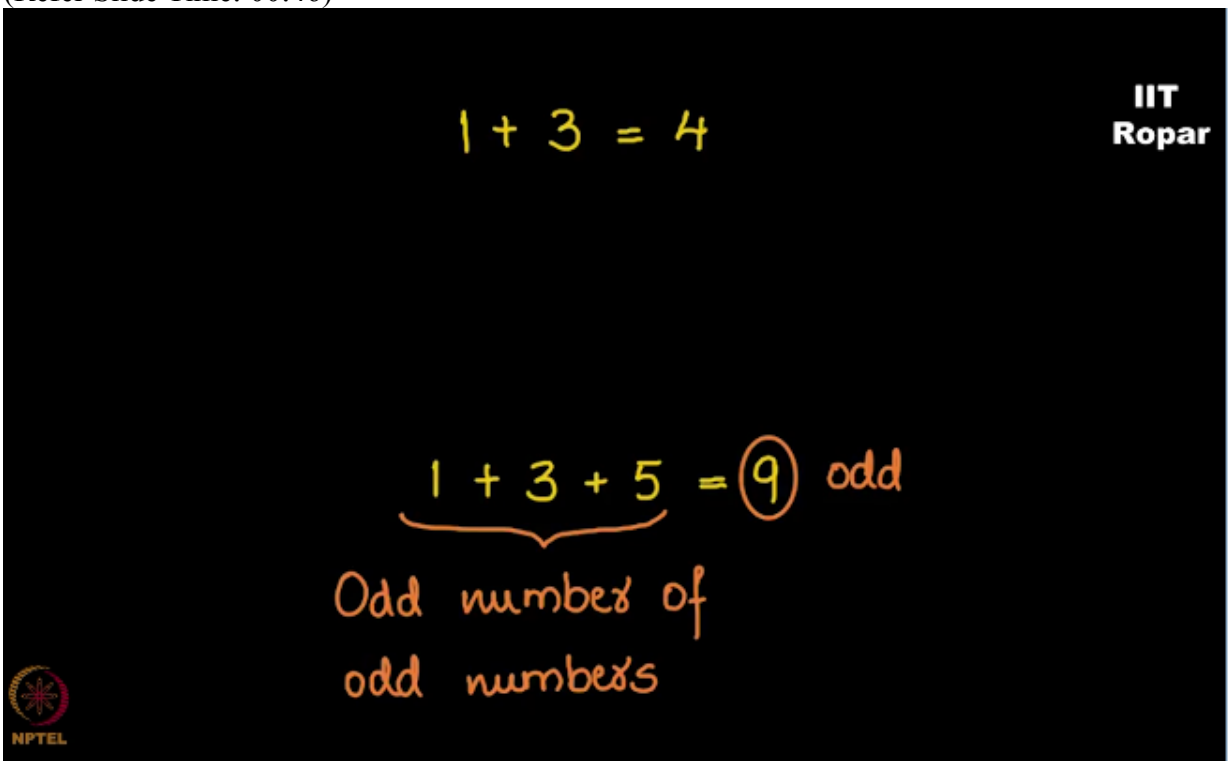

when you have even number of odd numbers the sum is an even number,

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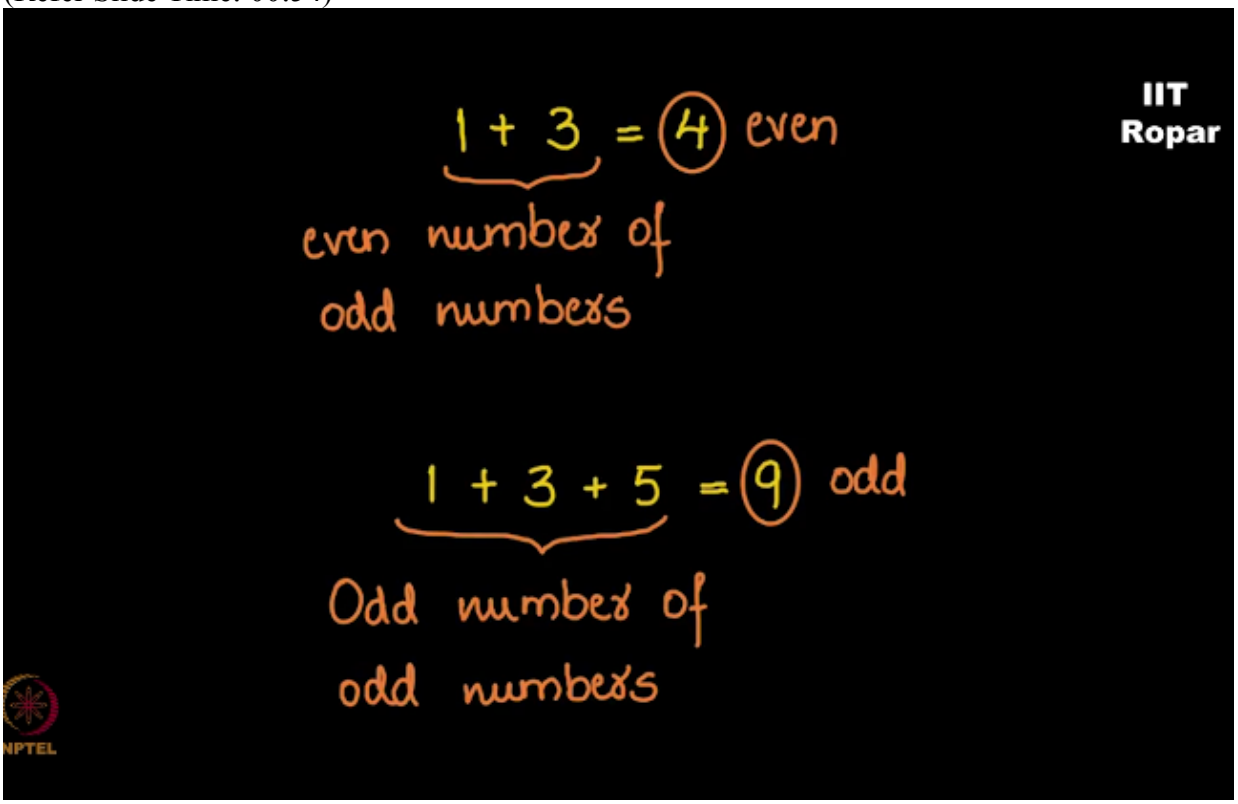

did you see this first case? It is an even number, why? We had even number of odd numbers and the second case it is an odd number, because we had odd number of odd numbers, we have 3 odd numbers here 1, 3, and 5, right,

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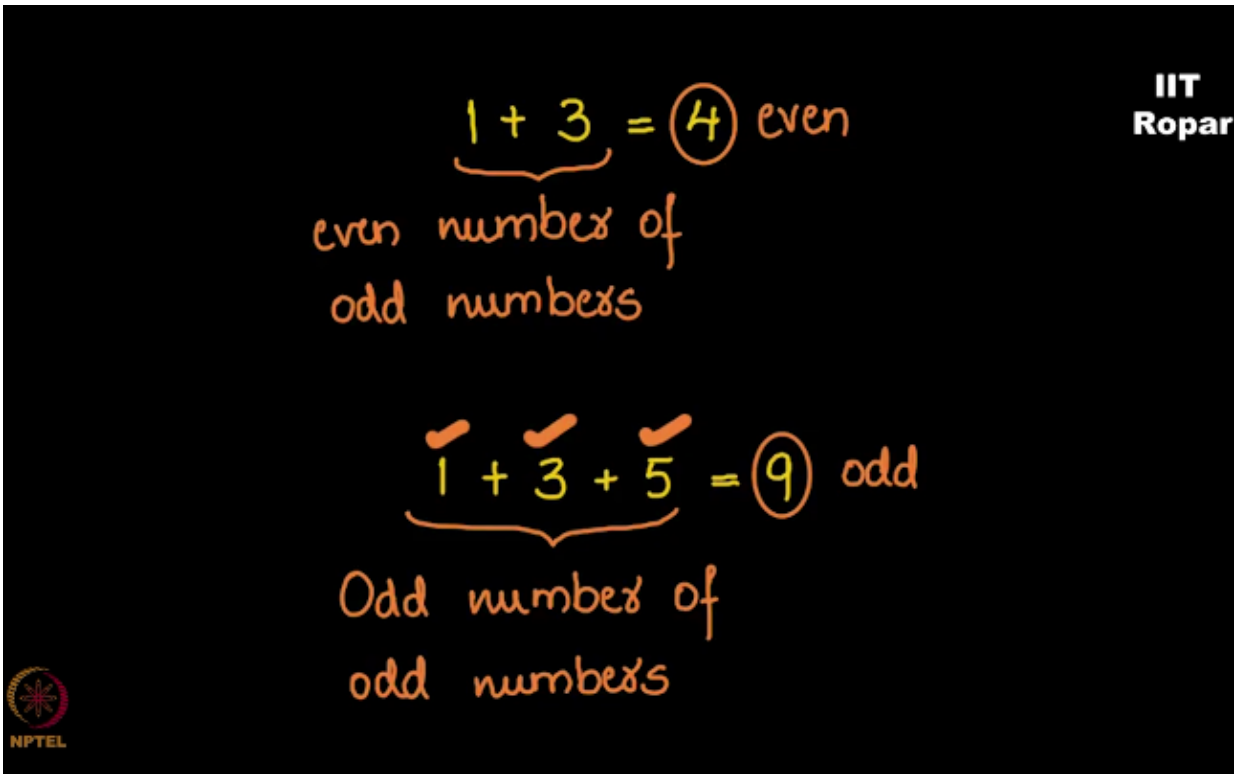

keep this two results in your mind, I'm going to prove something really very cute now, do you have already seen that some of the degrees is twice the number of edges, we have been talking of this.

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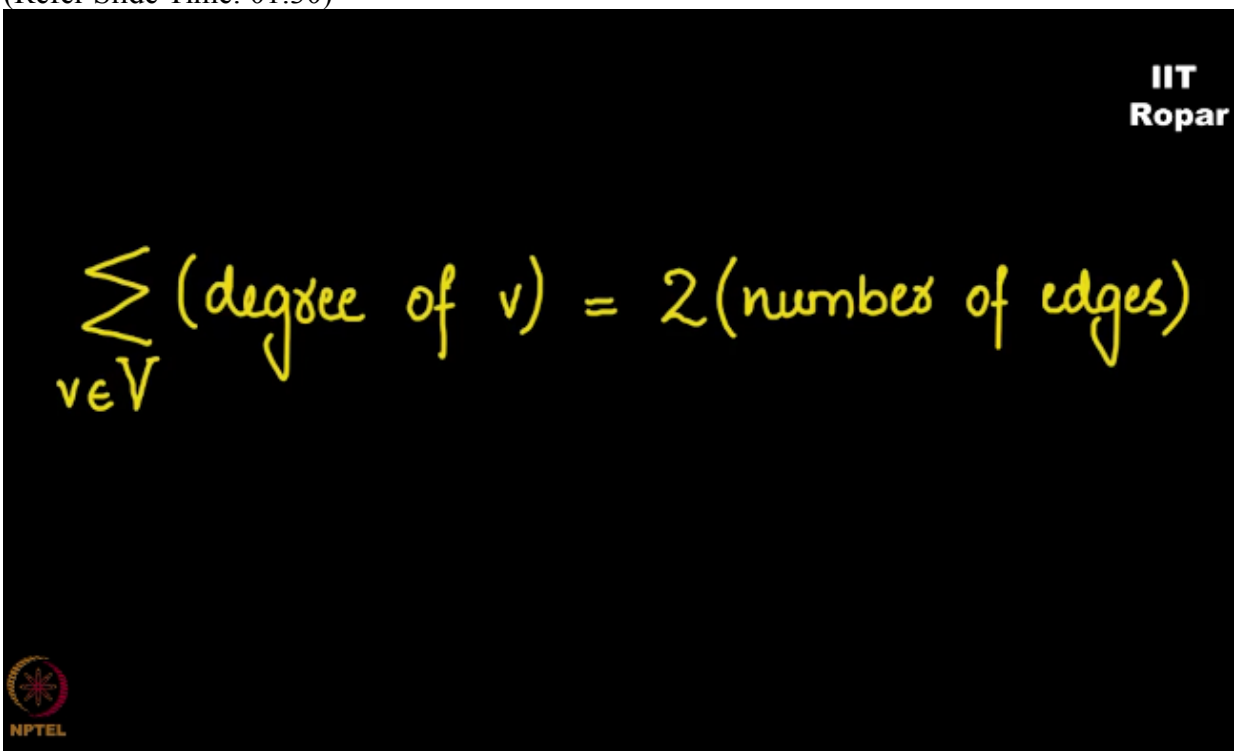

Now twice the number of edges this is an even number, right, 2 into say some M is even, multiple of 2 it is, what does that mean?
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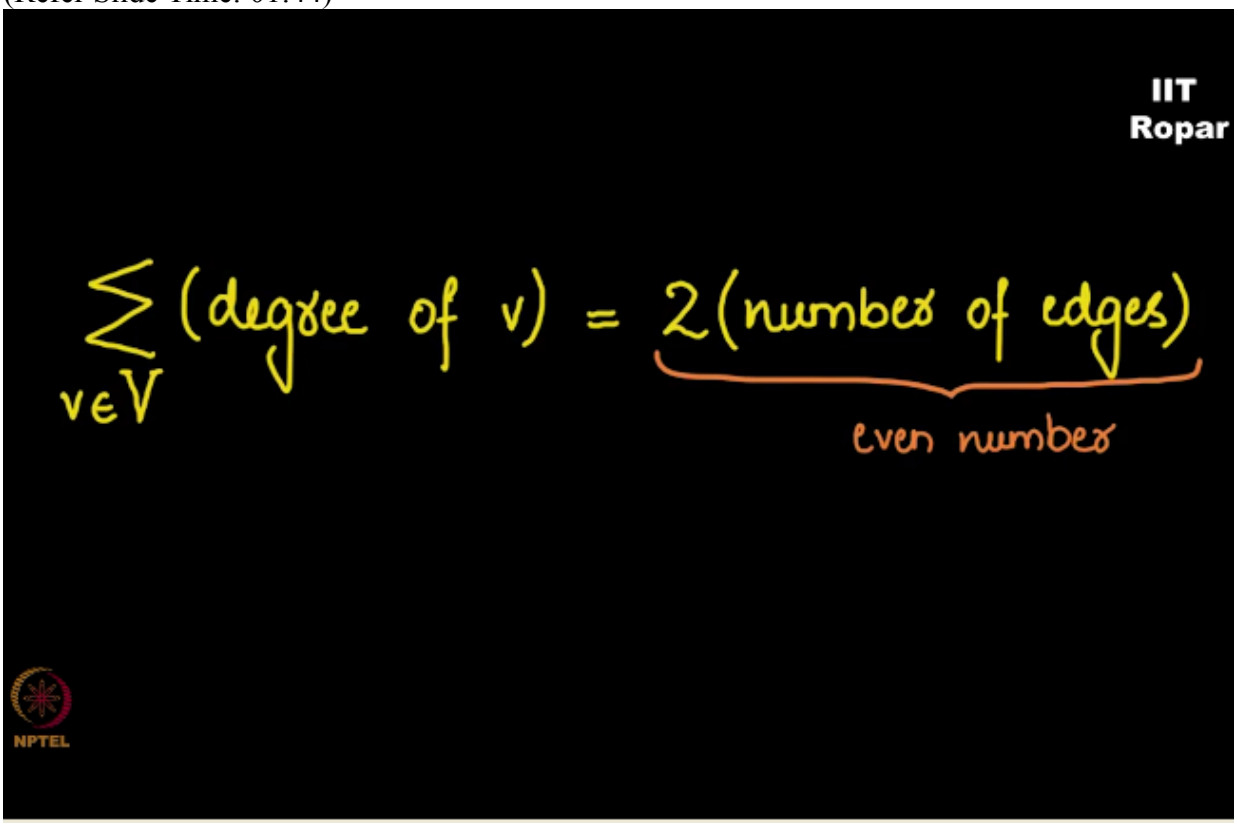

Sum of degrees this entire sum of degrees this is an even number, right, let me just write it as D1 + D2 and so on up to DN where what does this mean? N is the number of nodes in my graph G,
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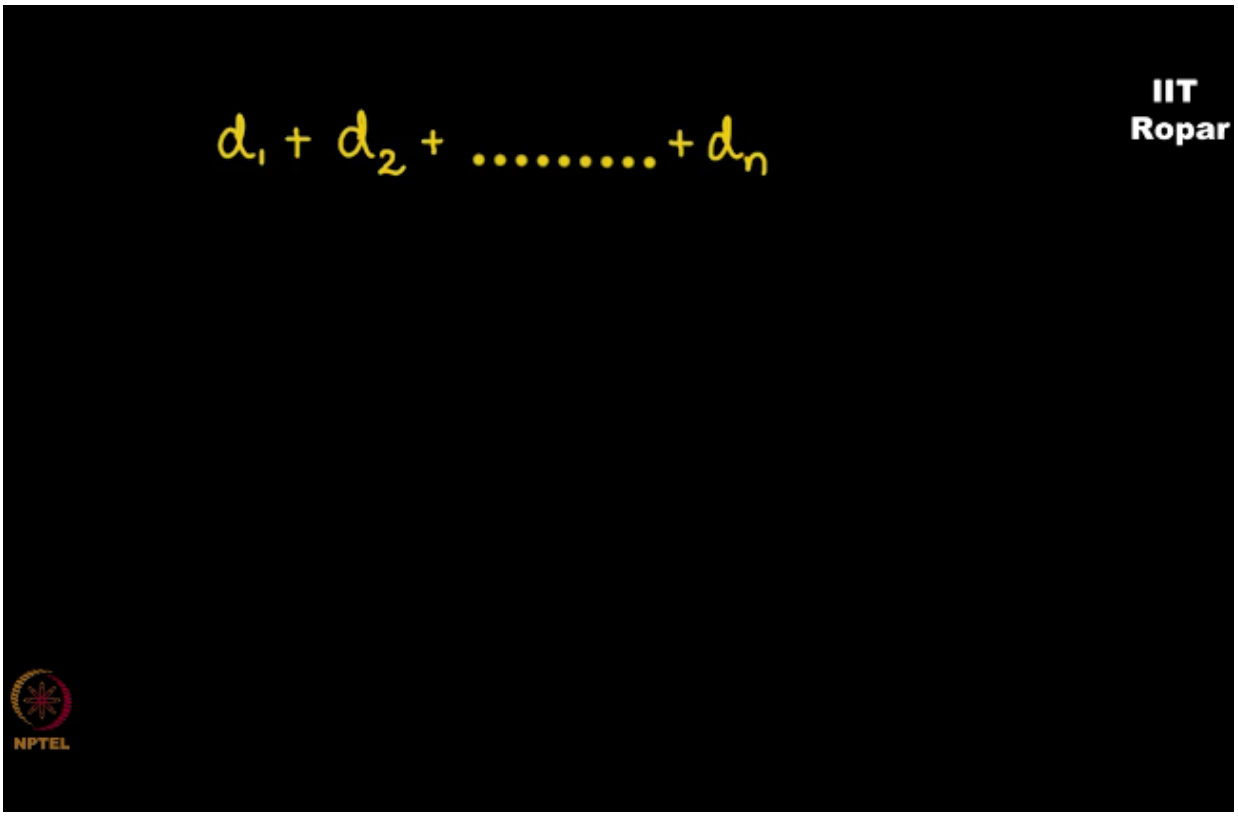

and some of their degrees is D1 + D2 + D3 and so on up to DN, now this is an even number, because it is equal to 2M, right.

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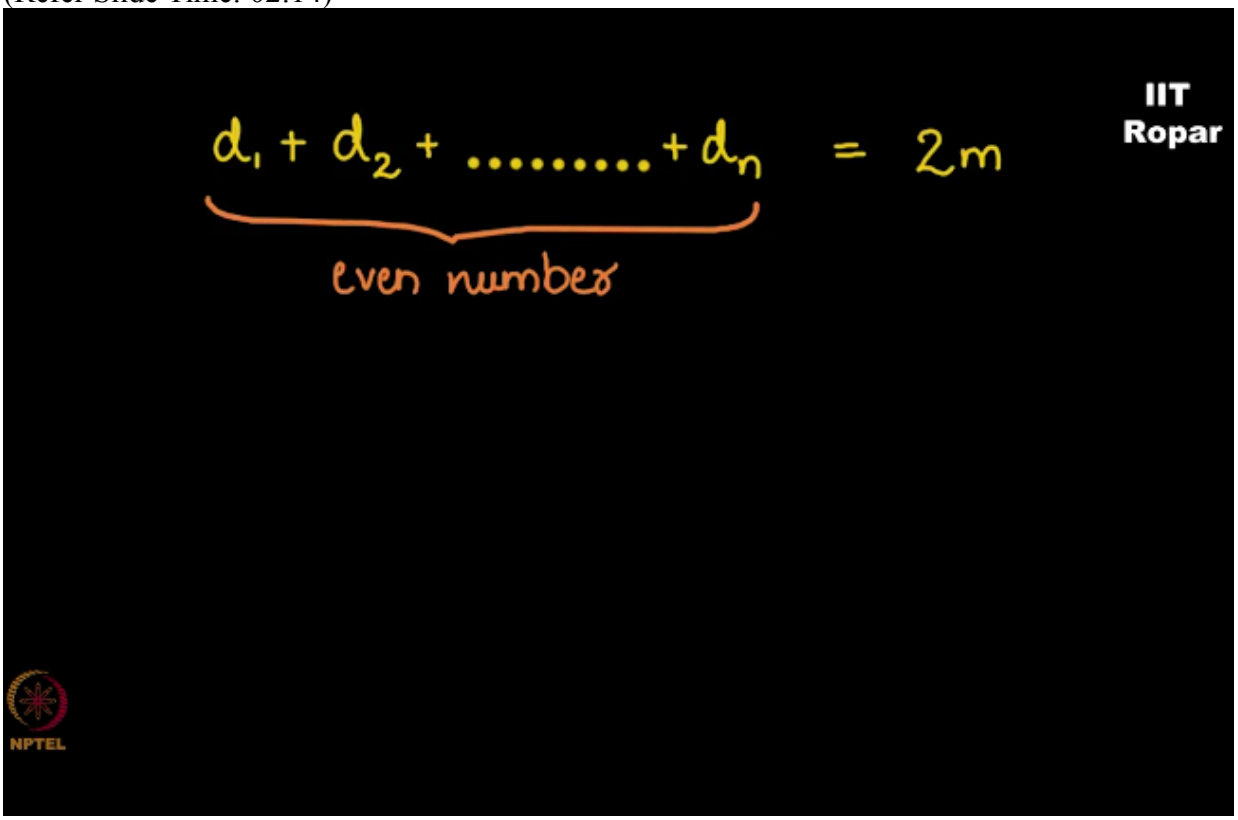

Now if this is an even number using the previous observations what can I conclude? I can conclude that either this number, what can I conclude about D1, D2, D3 and so on up to DN? We can conclude that all are even numbers, all that is D1, D2, DN all are even numbers or there are even number of odd numbers,

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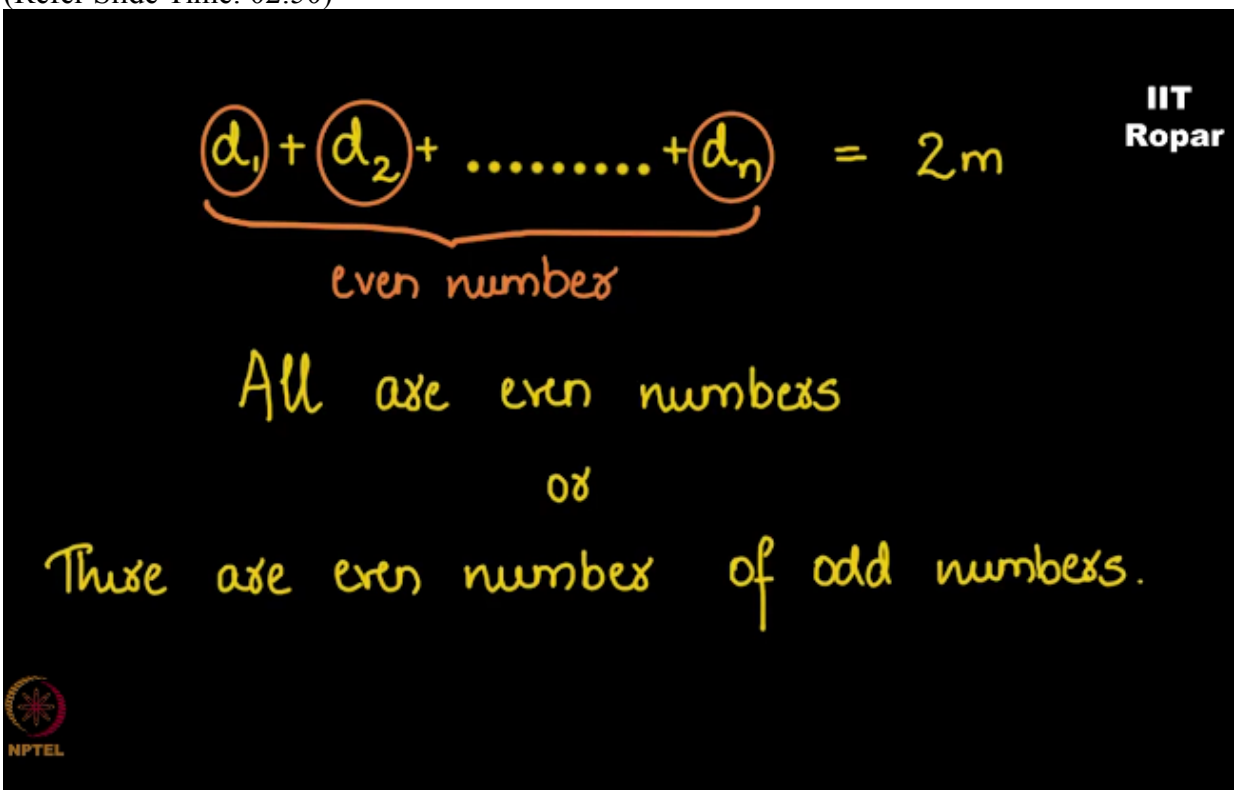

what do I mean by even number of odd numbers here, what does it signify? It signifies that there are even number of odd degree vertices in the graph,

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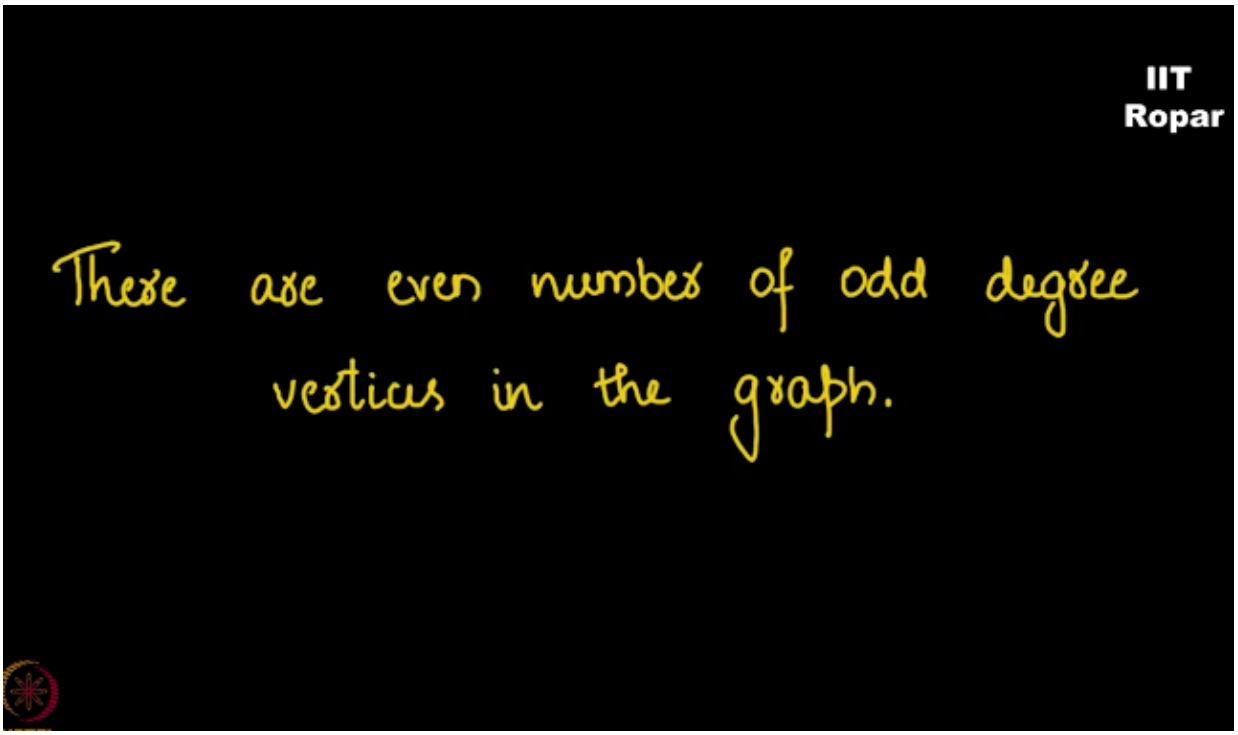

so either there are all even degrees or there are even number of odd degrees, that's how you're getting the sum to be even.

So let me state this explicitly now, every graph has even number of odd degree vertices,
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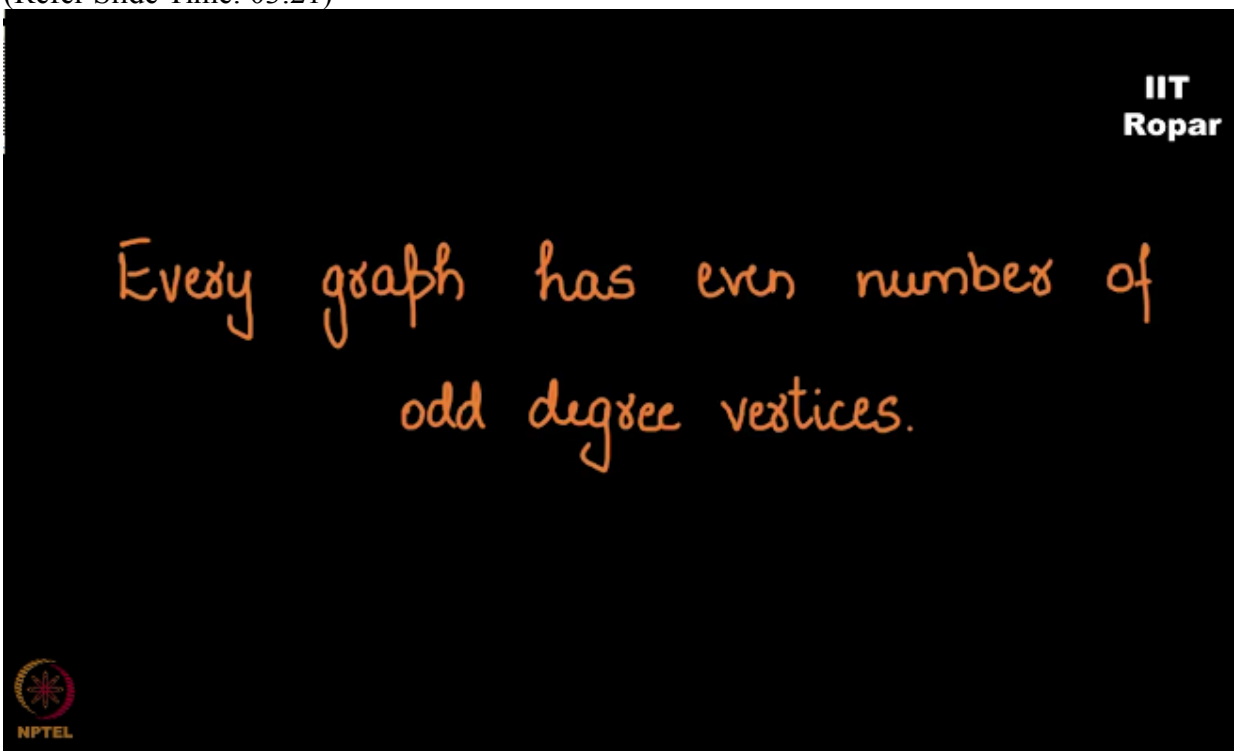

that is the number of vertices having odd degree are even, this is true in every graph, keep this in mind we'll use just to solve some problems.

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