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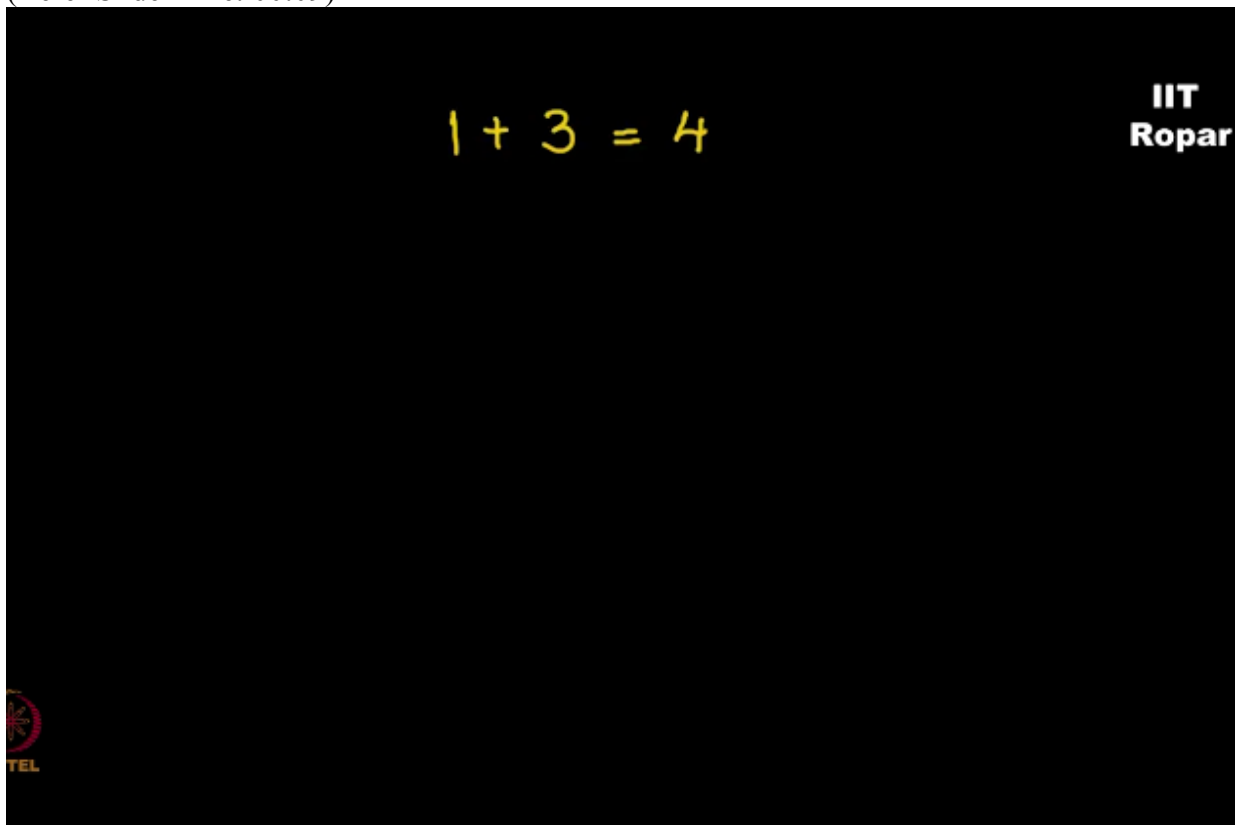
NPTEL ONLINE CERTIFICATION COURSE

**Discrete Mathematics
Graph Theory - 1**

Hand shaking lemma - Corollary

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Let us go back to our high school mathematics, $1 + 3$ is 4
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you see 4 is an even number, I'll add one more odd number here, 1 and 3 are already odd numbers, I'm going to add one more odd number, what do I get? $1+3+$ let's say 5 is 9,
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$$1 + 3 = 4$$

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$$1 + 3 + 5 = 9$$



9 is an odd number, with these two expressions can we make or observe something?

When you have odd number of odd numbers, the sum is always odd,
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$$1 + 3 = 4$$

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$$1 + 3 + 5 = \textcircled{9} \text{ odd}$$

Odd number of
odd numbers



when you have even number of odd numbers the sum is an even number,

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The image shows a blackboard with handwritten text in orange and yellow. At the top right, it says "IIT Ropar". In the center, there are two examples. The first example shows the equation $1 + 3 = 4$ with a bracket under "1 + 3" and the number "4" circled, followed by the word "even". Below this is the text "even number of odd numbers". The second example shows the equation $1 + 3 + 5 = 9$ with a bracket under "1 + 3 + 5" and the number "9" circled, followed by the word "odd". Below this is the text "Odd number of odd numbers". In the bottom left corner, there is a small logo for NPTEL.

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$$1 + 3 = 4 \text{ even}$$

even number of odd numbers

$$1 + 3 + 5 = 9 \text{ odd}$$

Odd number of odd numbers

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did you see this first case? It is an even number, why? We had even number of odd numbers and the second case it is an odd number, because we had odd number of odd numbers, we have 3 odd numbers here 1, 3, and 5, right,

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$$\underbrace{1 + 3}_{\text{even number of odd numbers}} = 4 \text{ even}$$

$$\underbrace{\overset{\checkmark}{1} + \overset{\checkmark}{3} + \overset{\checkmark}{5}}_{\text{Odd number of odd numbers}} = 9 \text{ odd}$$

Odd number of
odd numbers





keep this two results in your mind, I'm going to prove something really very cute now, do you have already seen that some of the degrees is twice the number of edges, we have been talking of this.

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$$\sum_{v \in V} (\text{degree of } v) = 2(\text{number of edges})$$



Now twice the number of edges this is an even number, right, 2 into say some M is even, multiple of 2 it is, what does that mean?
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$$\sum_{v \in V} (\text{degree of } v) = \underbrace{2(\text{number of edges})}_{\text{even number}}$$


Sum of degrees this entire sum of degrees this is an even number, right, let me just write it as $D_1 + D_2$ and so on up to D_N where what does this mean? N is the number of nodes in my graph G ,
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$$d_1 + d_2 + \dots + d_n$$



and some of their degrees is $D_1 + D_2 + D_3$ and so on up to D_N , now this is an even number, because it is equal to $2M$, right.

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$$\underbrace{d_1 + d_2 + \dots + d_n}_{\text{even number}} = 2m$$



Now if this is an even number using the previous observations what can I conclude? I can conclude that either this number, what can I conclude about D_1, D_2, D_3 and so on up to D_n ? We can conclude that all are even numbers, all that is D_1, D_2, D_n all are even numbers or there are even number of odd numbers,
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$d_1 + d_2 + \dots + d_n = 2m$
even number

All are even numbers
or
There are even number of odd numbers.

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what do I mean by even number of odd numbers here, what does it signify? It signifies that there are even number of odd degree vertices in the graph,
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There are even number of odd degree vertices in the graph.

so either there are all even degrees or there are even number of odd degrees, that's how you're getting the sum to be even.

So let me state this explicitly now, every graph has even number of odd degree vertices,
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Every graph has even number of odd degree vertices.

that is the number of vertices having odd degree are even, this is true in every graph, keep this in mind we'll use just to solve some problems.

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