NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Discrete Mathematics
Graph Theory - 1

Degree and degree sequence

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Observe the graph that we just discussed, it had four nodes, S, R, C and D representing 4 friends, Sudarshan, Ravi, Chaya, and Deepak.
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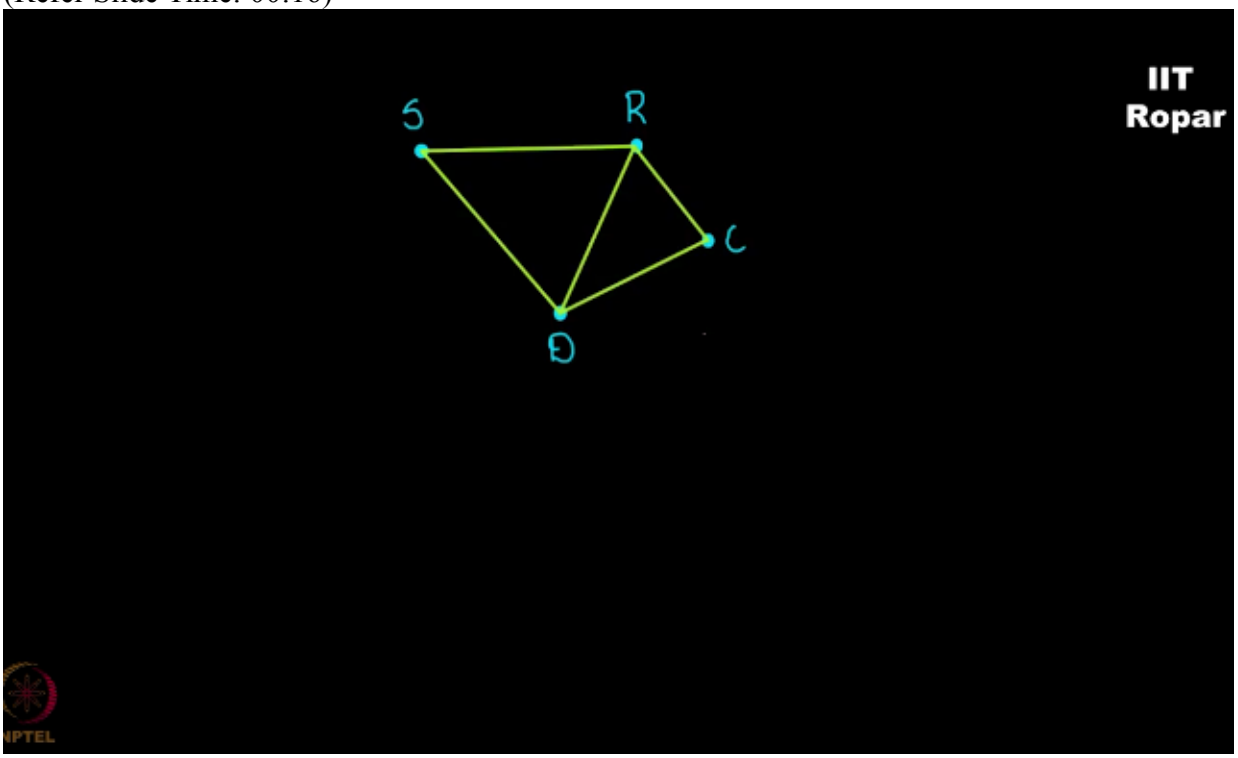

Now I'm going to write some numbers on it, you tell me what am I trying to say, Sudarshan has the number 2, Ravi has the number 3, Chaya has 2, Deepak has 3,
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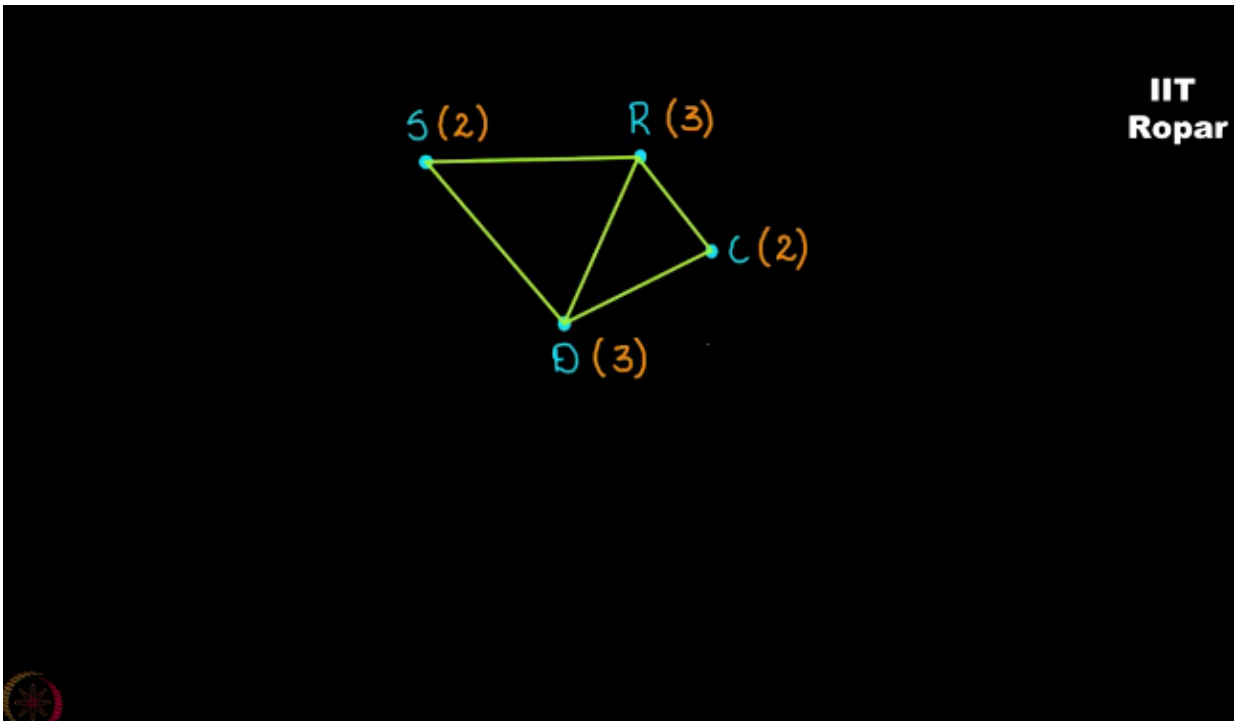

what is it this numbers are denoting here? Sudarshan 2 means, yeah I think you would have guessed, Sudarshan has 2 friends, Ravi has 3 friends, Chaya has 2 friends, Deepak has 3 friends, right,

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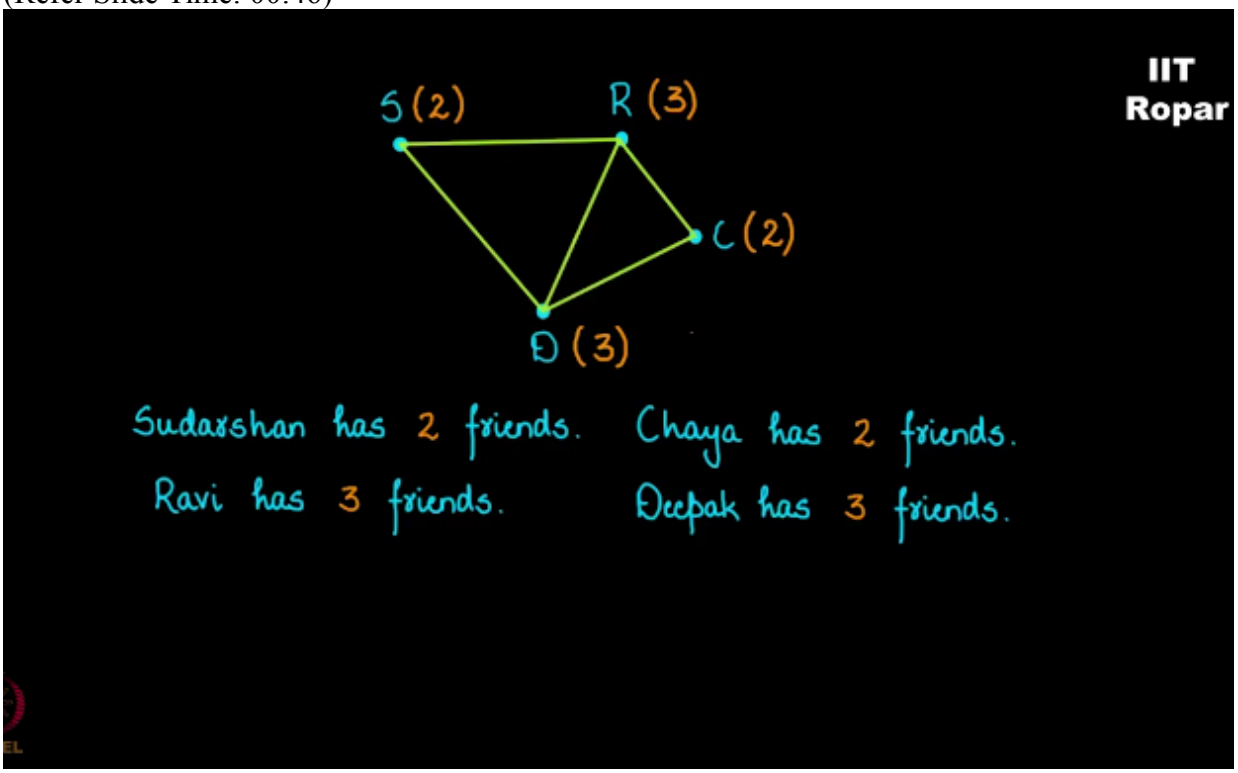

so the number here denotes the number of lines emanating, emanating from that particular point.

From now onwards we will call nodes as vertices,
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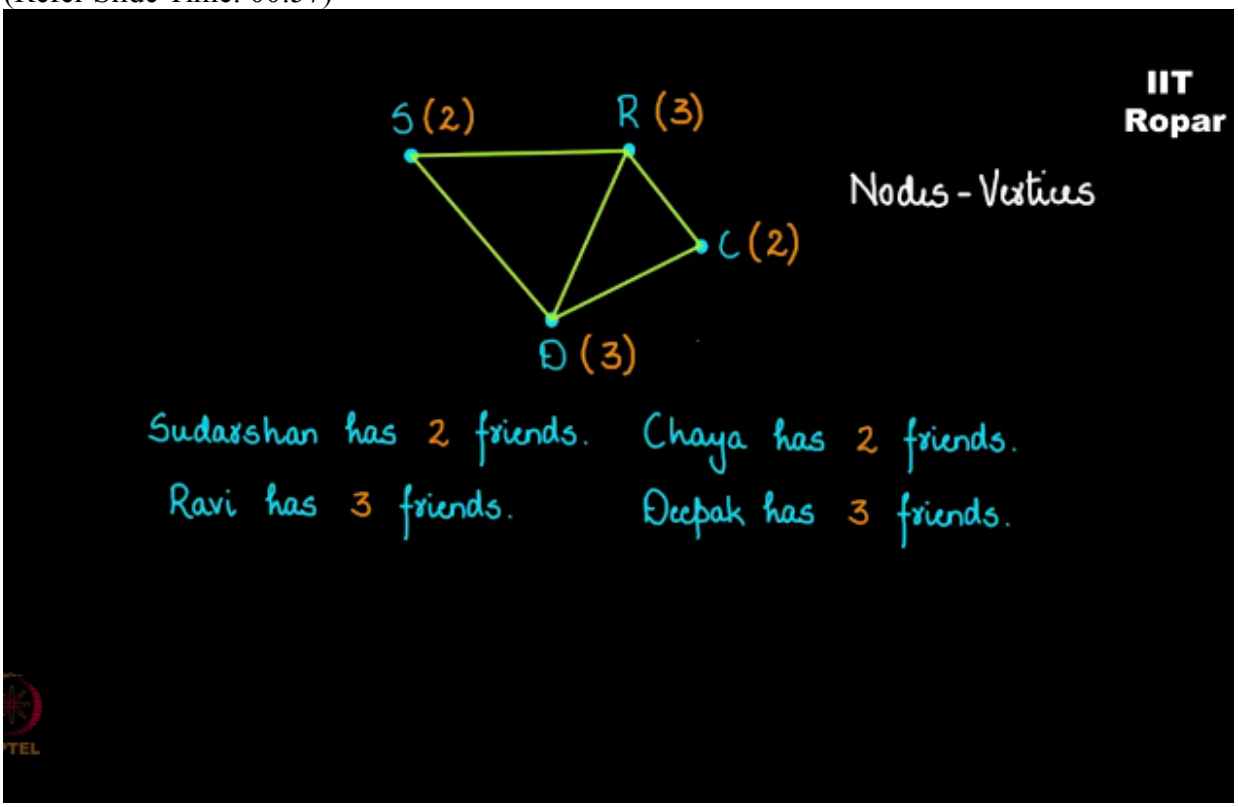

notices and nodes are interchangeable we will keep using nodes and vertices which denotes these points, and the lines are called edges from now onwards,
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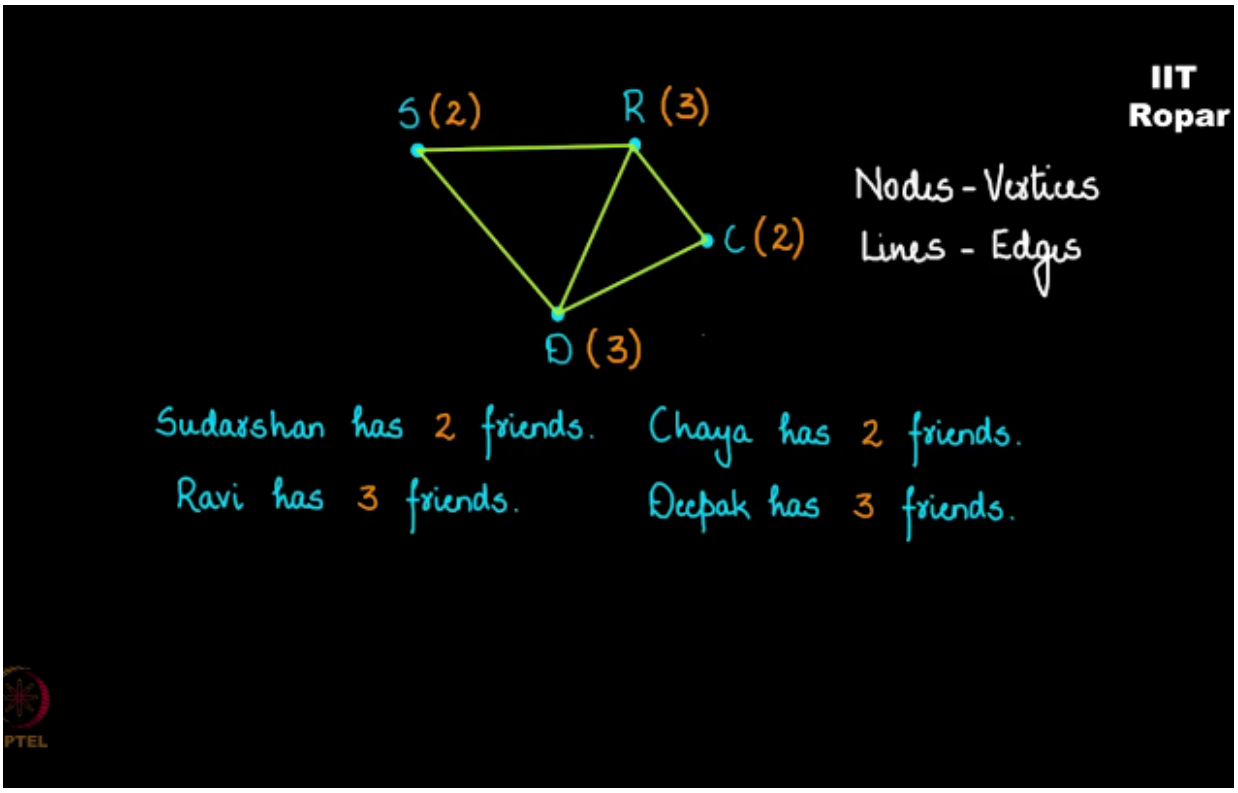

okay, and this number that I have written here as you can see 2, 3, 2, 3 this is called degree of a node,

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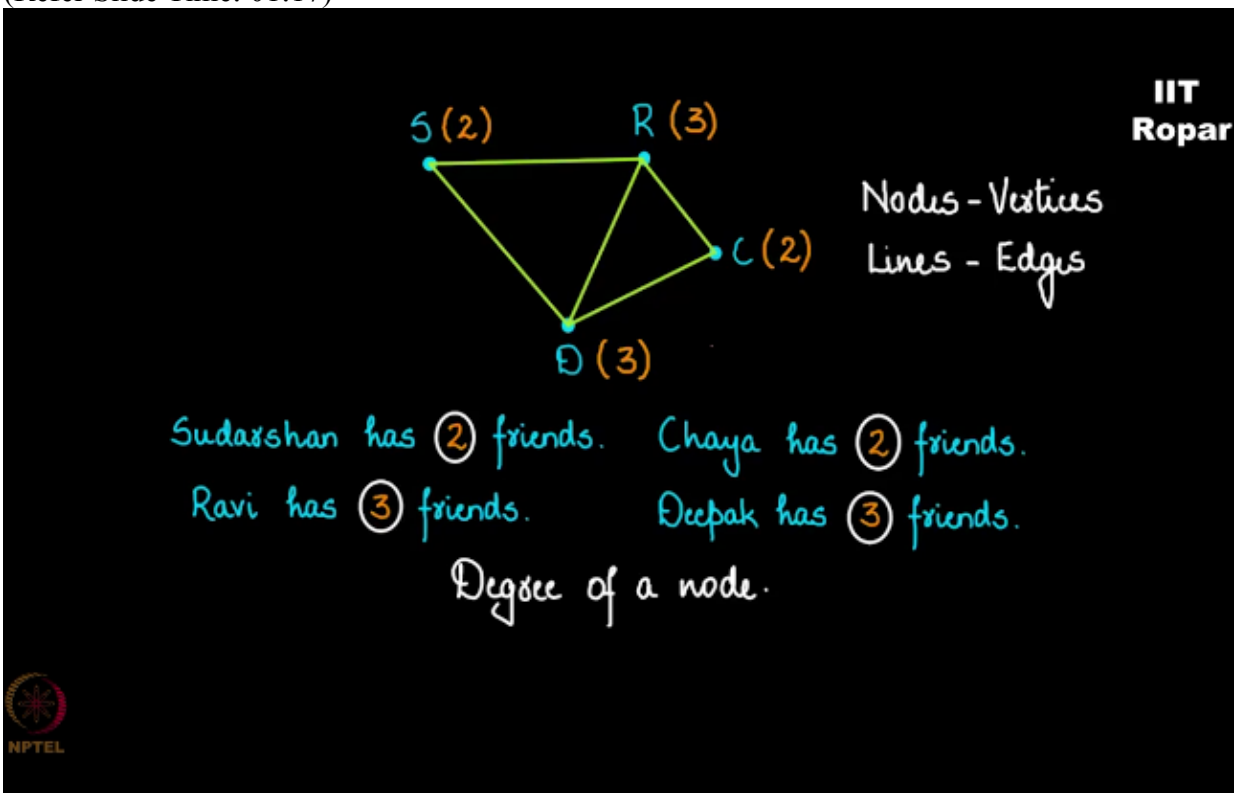

so degree of S is 2, degree of R is 3, degree of C is 2, degree of D is 3, and 2, 3, 2, 3 is called a degree sequence of this graph.
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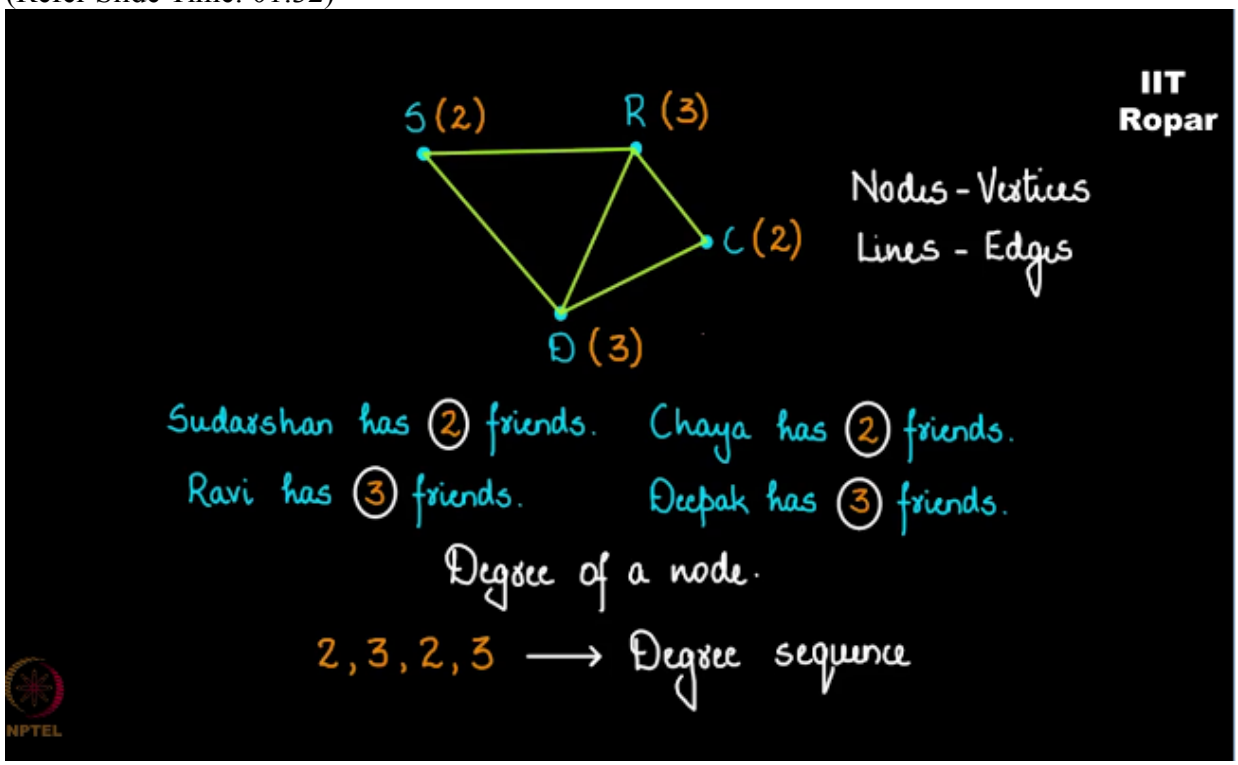

Look at this graph for the sake of clarity,
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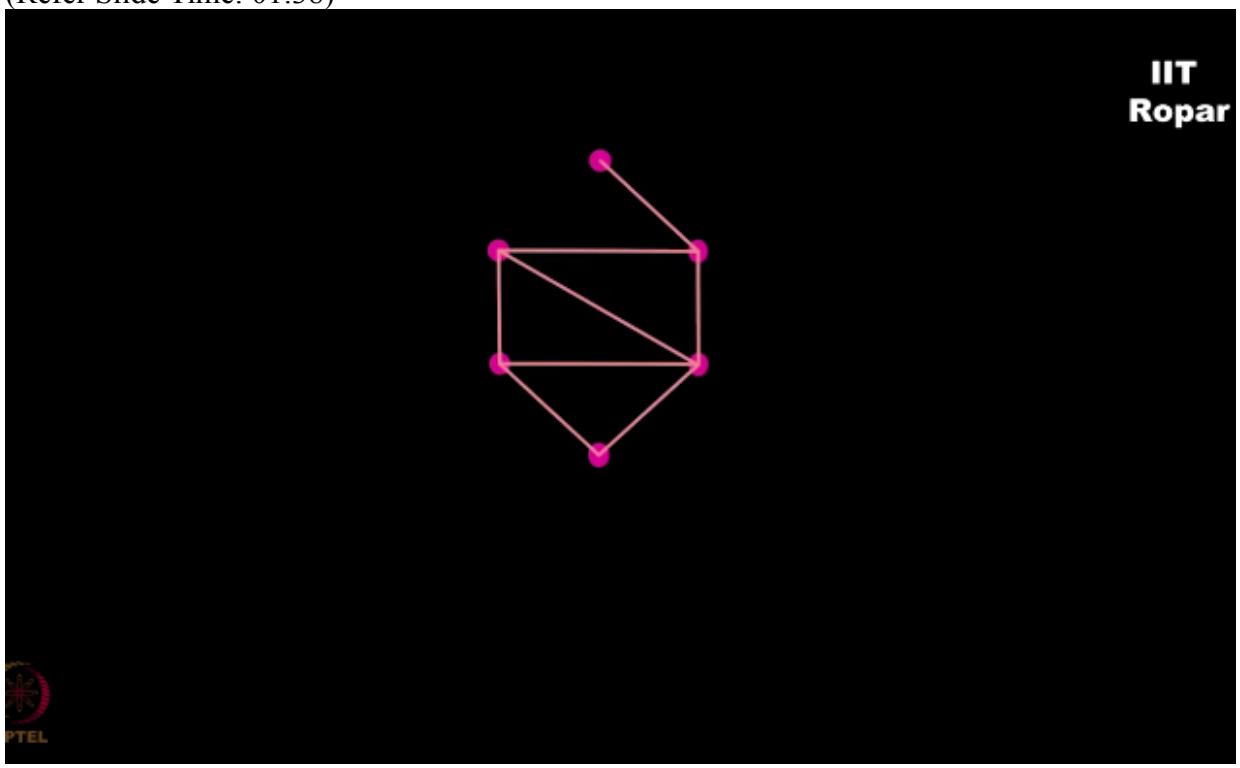

I have written a bigger graph, a graph with 1, 2, 3, 4, 5, 6 vertices and several edges, as you can see the degree here is the first vertex, let's call it A, B, C, D, E, F,
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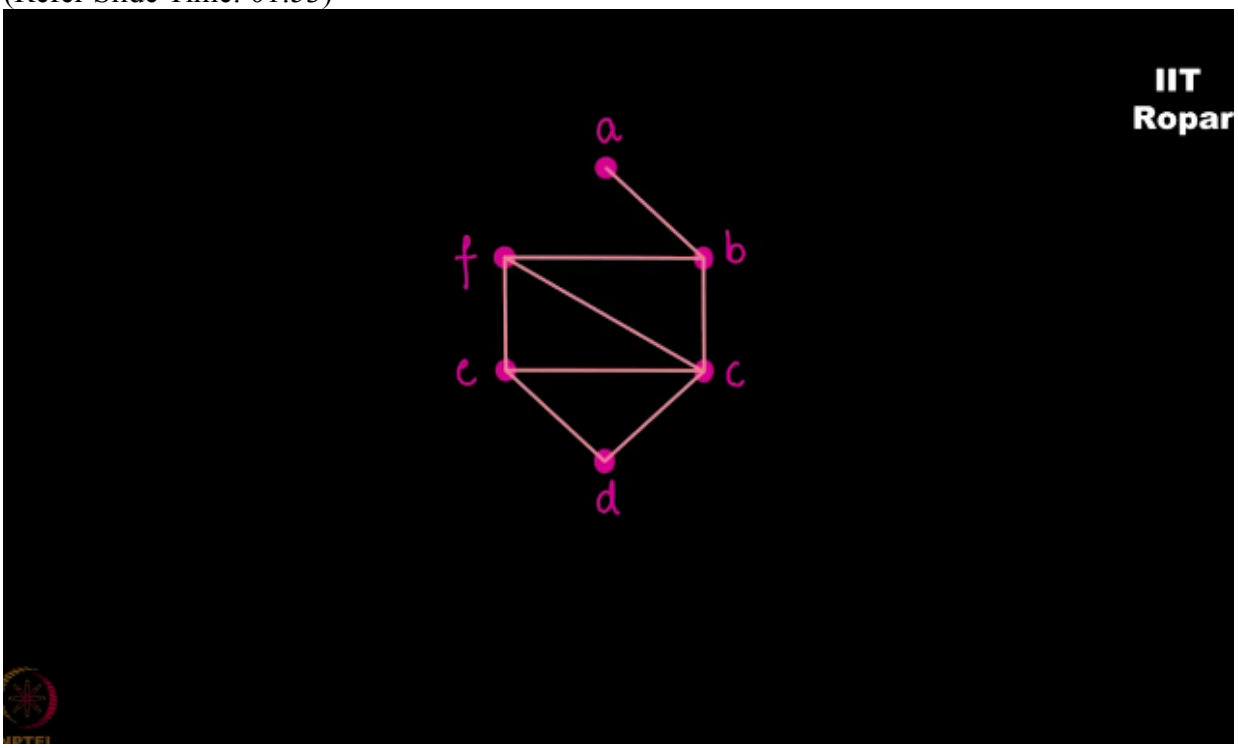

vertex A has degree 1, B has 3, C has 4, D has 2, E has 3, F has 3, and the degree sequence happens to be 1, 3, 4, 2, 3, and 3.
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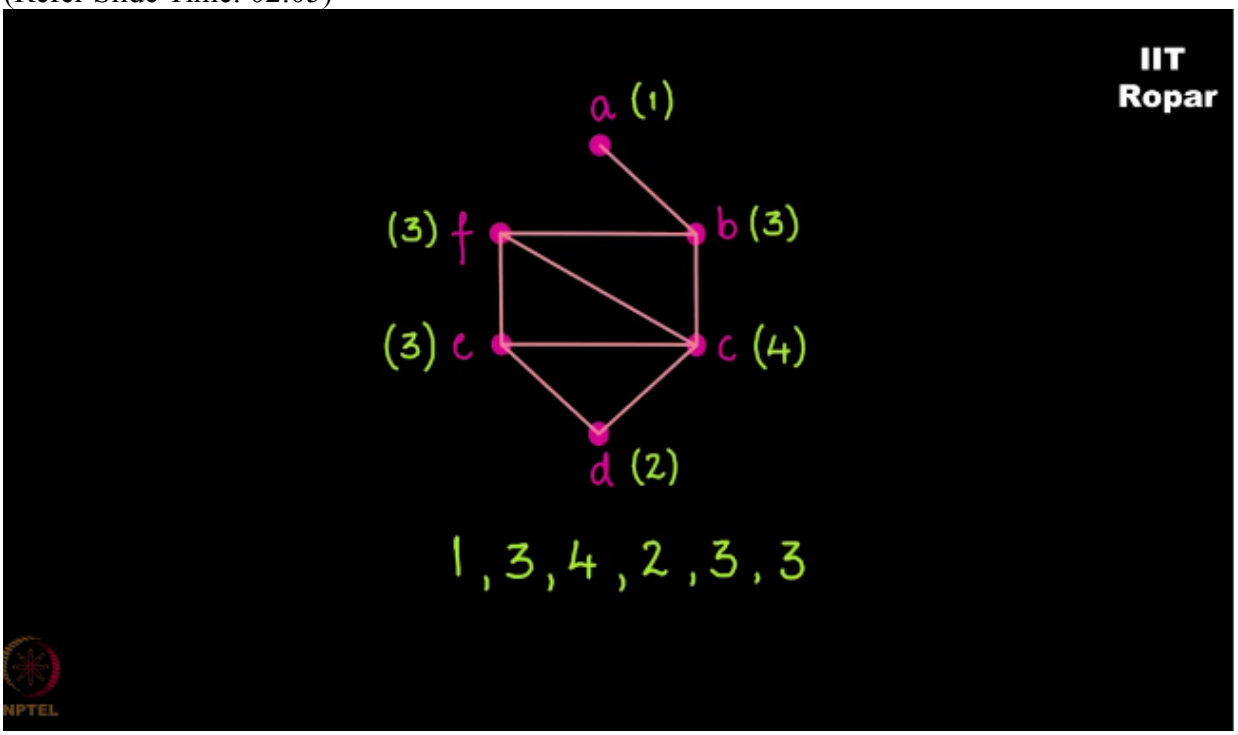

Look at the situation, Akshay knows Bobby, Bobby knows Chaitra, Chaitra knows Dimple, Dimple knows Esha,
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and nobody else knows anyone else outside this friendship, these edges.

Now you see how the graph looks like, it looks like a road starting from one location to another location, this is actually called a path graph, okay,
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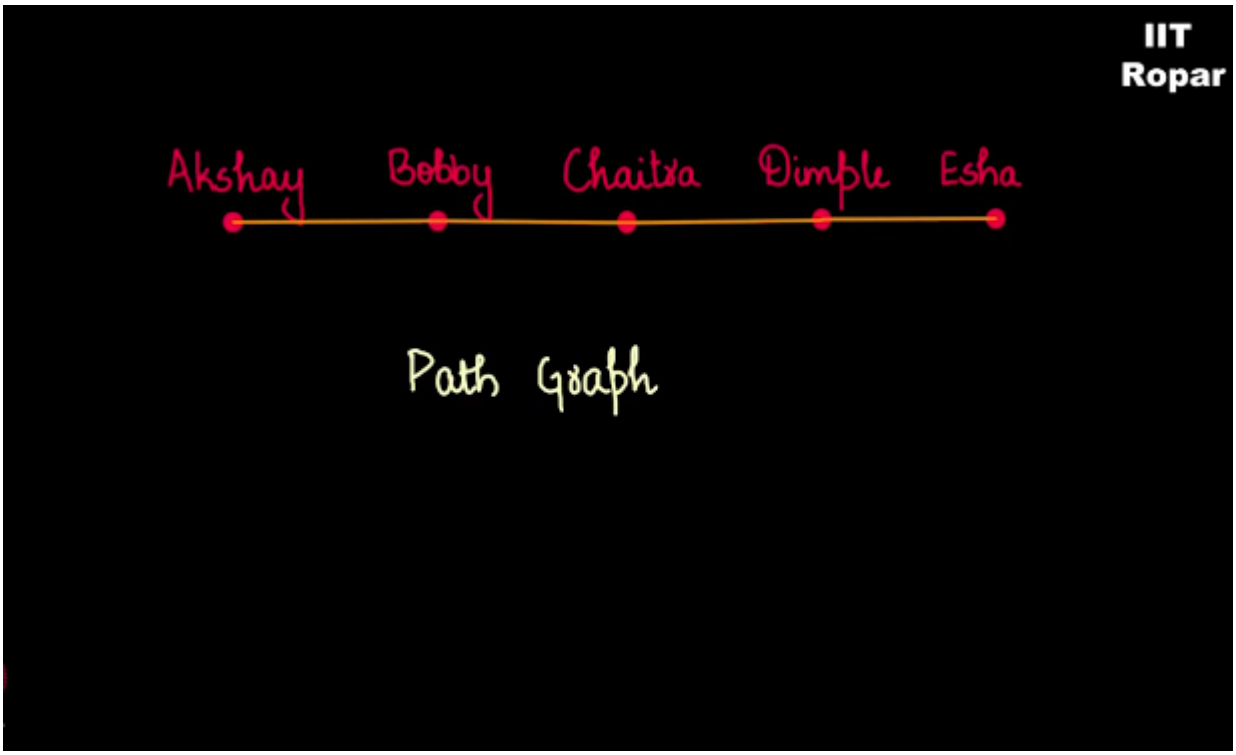

this is called a path graph and we represent it by PK, where K is the number of the nodes here, so how many nodes do you have here? 1, 2, 3, 4, 5

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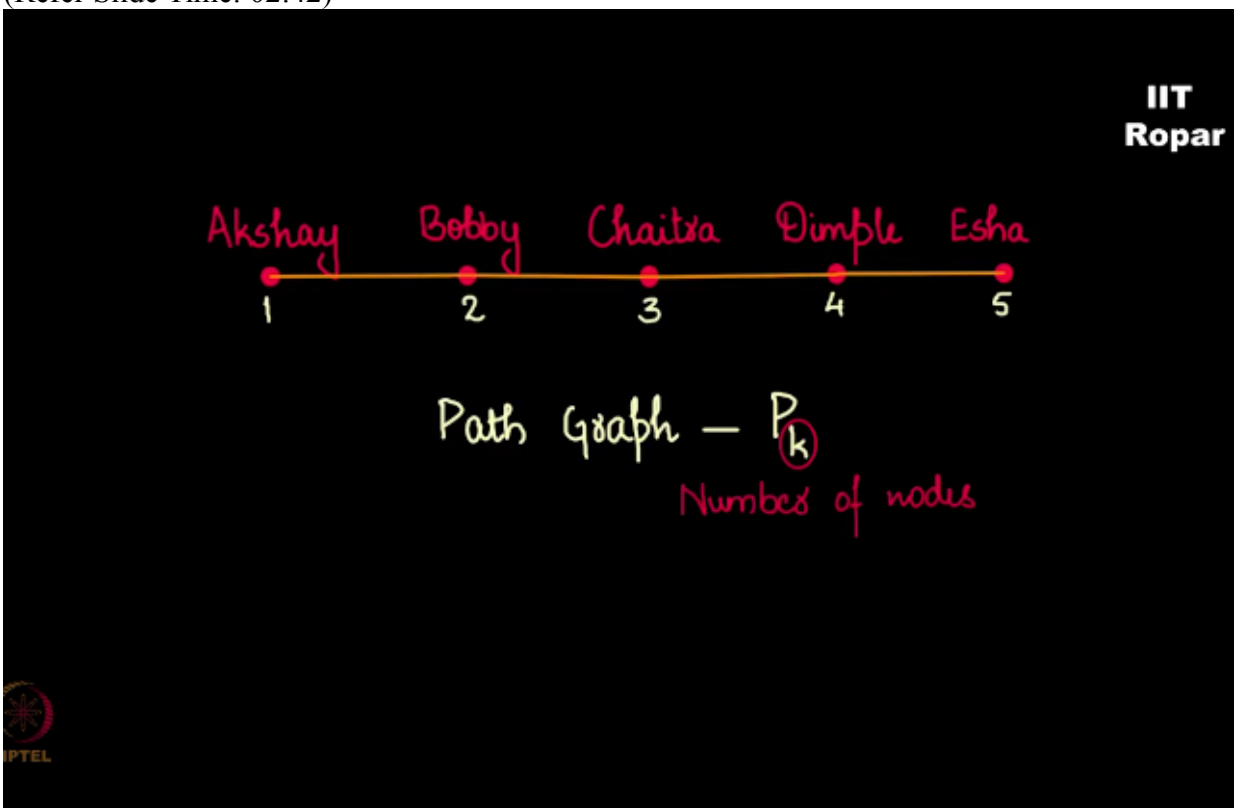

so this is called a P5, this is called a P5 graph, okay,

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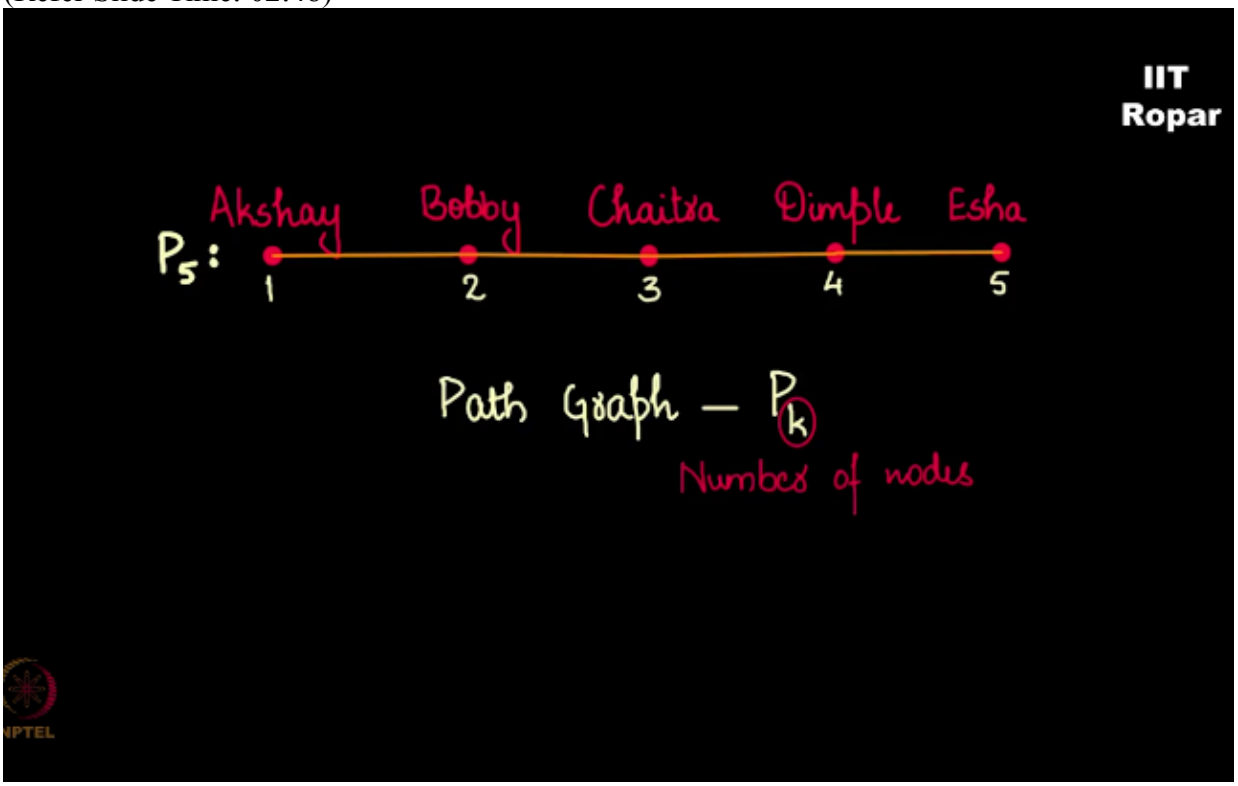

so these are all notations we are just warming you up to understand these notations and examples and definitions so that when we use them you are comfortable knowing what is what, okay.

Let's move on to the next example with the same 5 people, observe that just in case Akshay, Bobby, Chaitra, Dimple, Esha knew each other, they knew each other the graph would look something like this
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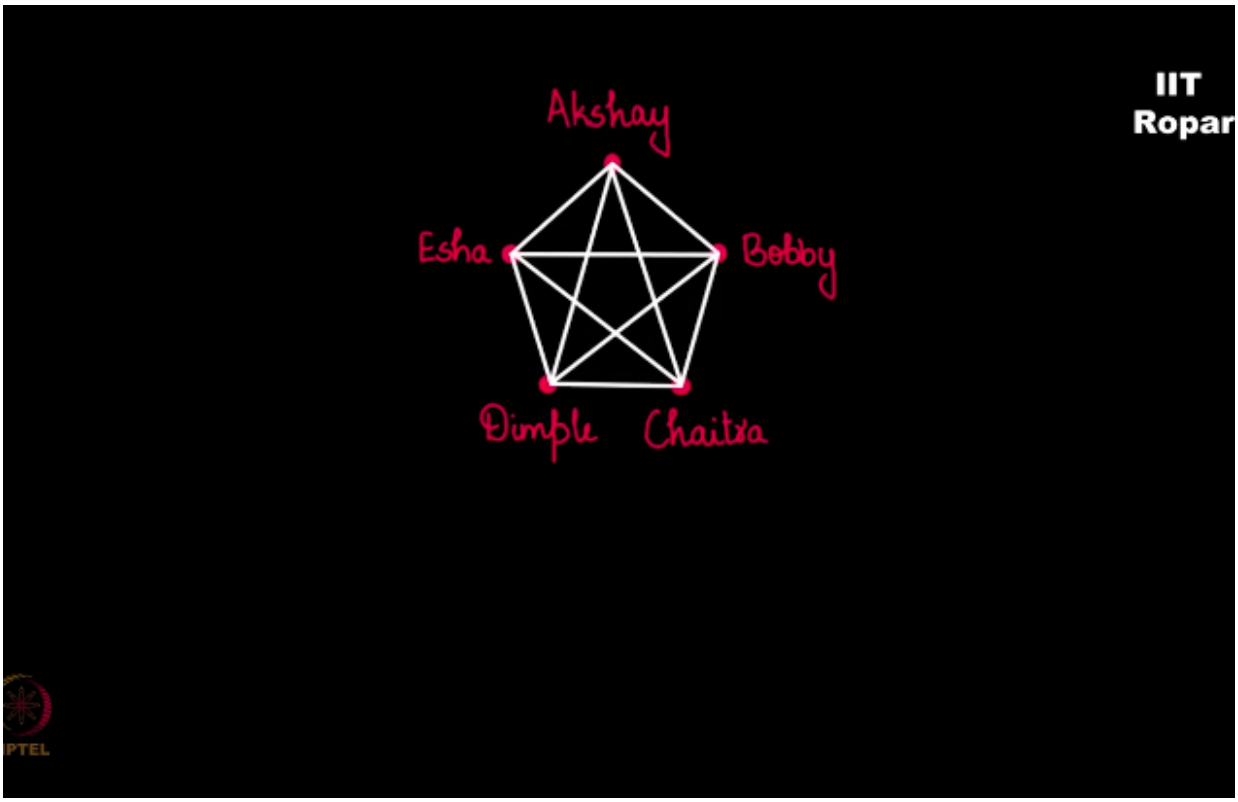

you see what is so nice about this graph, all possible edges are exhausted, you cannot put another edge, let's say as a post of the path graph where you saw that Akshay and Chaitra weren't friends, they could become friends but here

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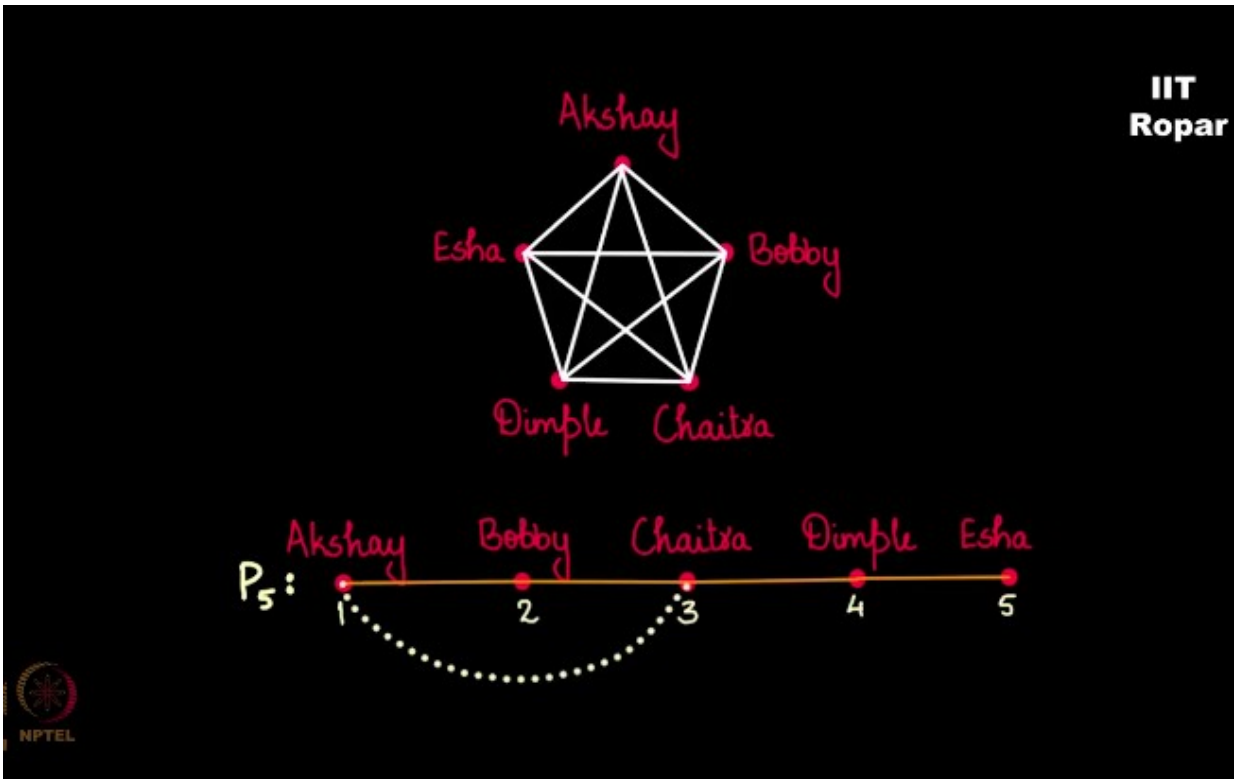

you are saying that everybody is friends with everyone else, when there are 5 people each person will have 4 friends, okay, and this is called a complete friendship network, everybody is friends with everyone else, and such a graph is called a complete graph,

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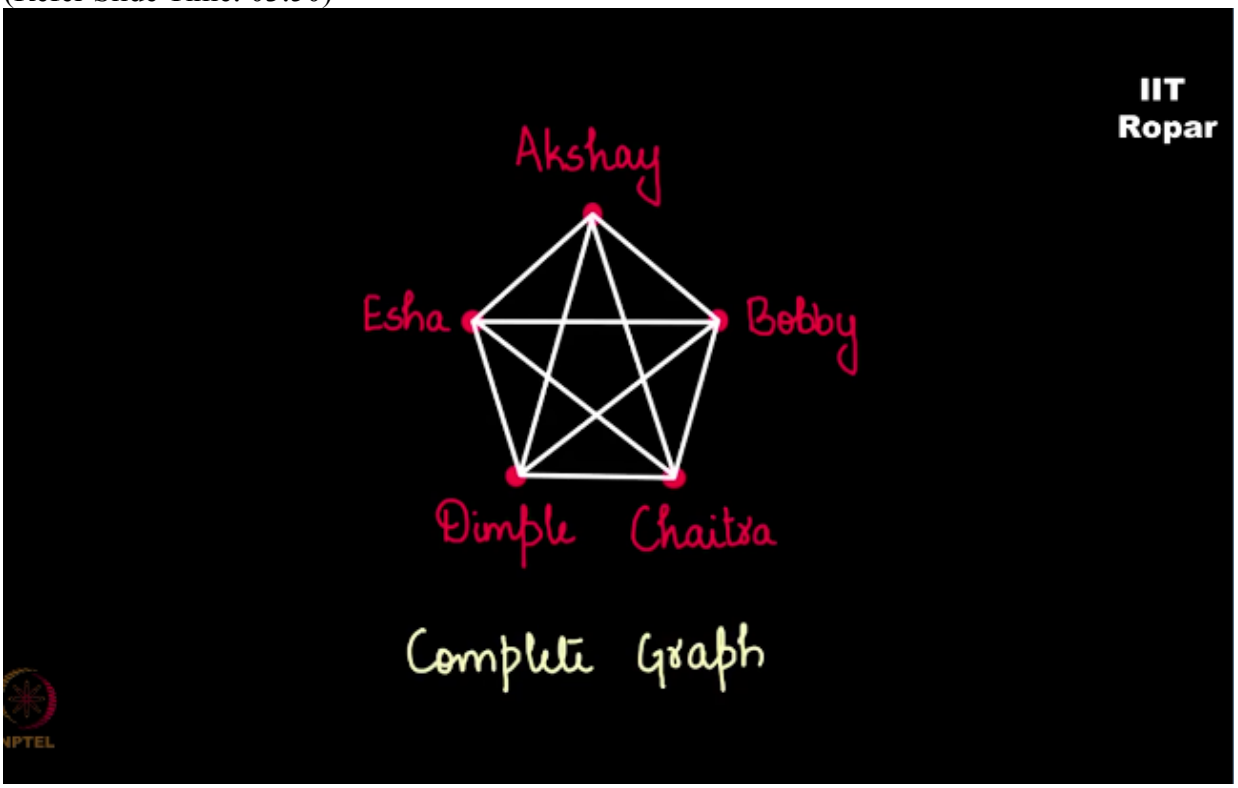

and it is represented by K5, K for complete I know it is C for complete but yeah the conventionally in graph 3 we say K for complete and the complete graph is on 5 nodes, K5.
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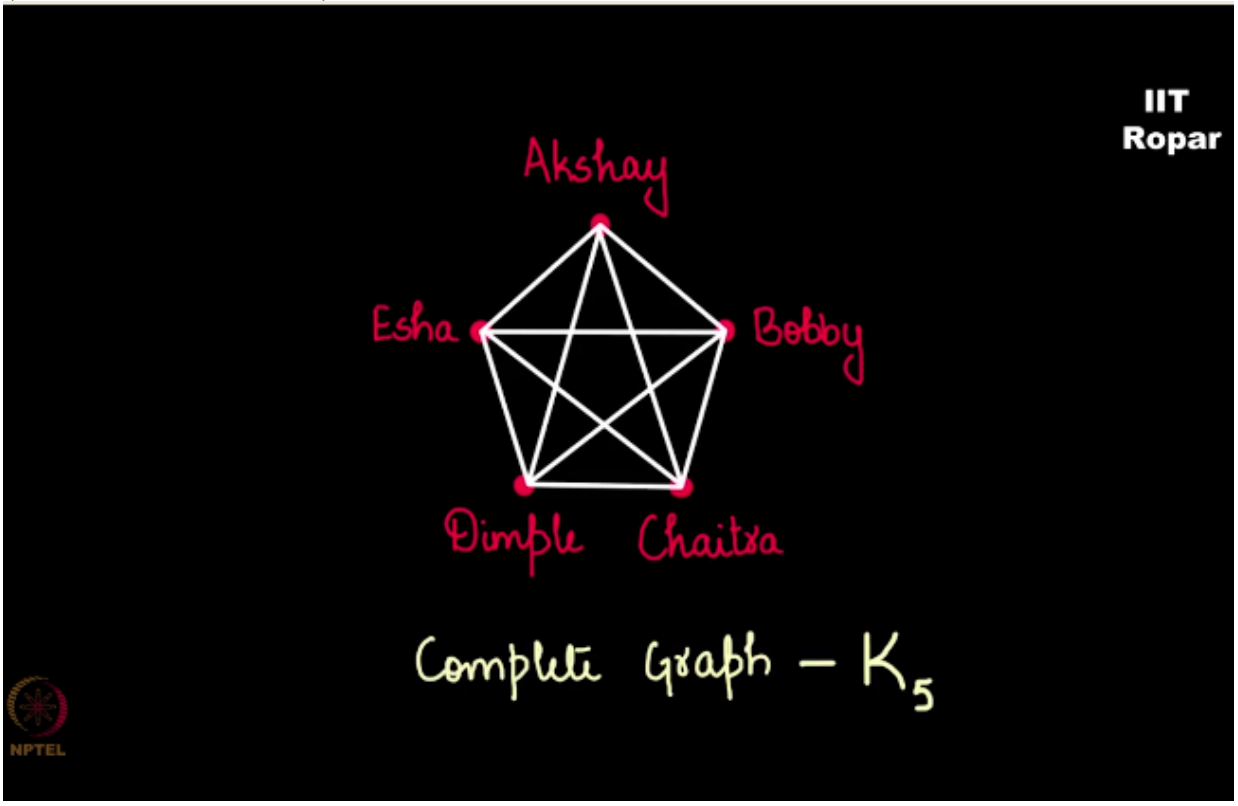

Now here is a question for you all, think about it, a path of length N, PN, I'm sorry it's not length it's a path with number of nodes N will always have N-1 edges,
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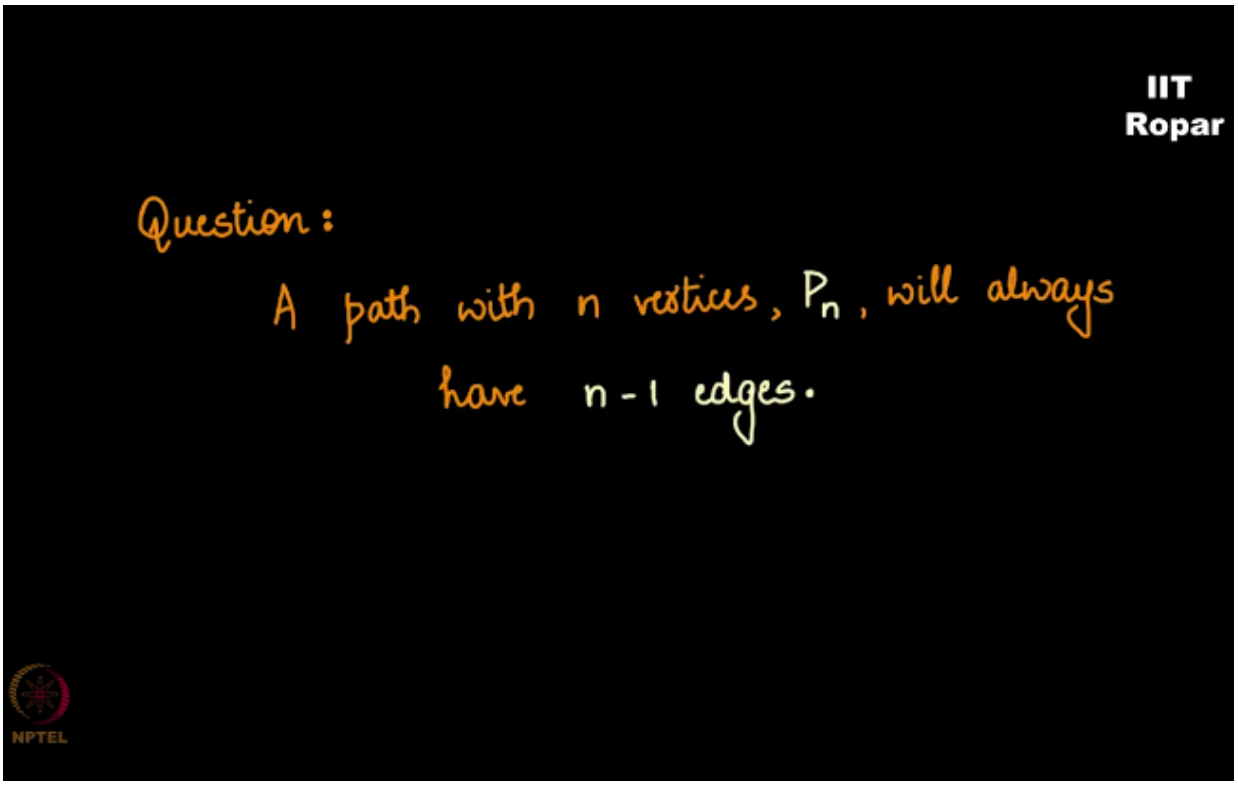

we observe that, a complete graph KN of N nodes will always have, how many edges? Can you guess? It will be N-1, how many edges does K5 have? Let us count, 4, 4, 4, 4, 4, 4 x 5 it had 20 edges

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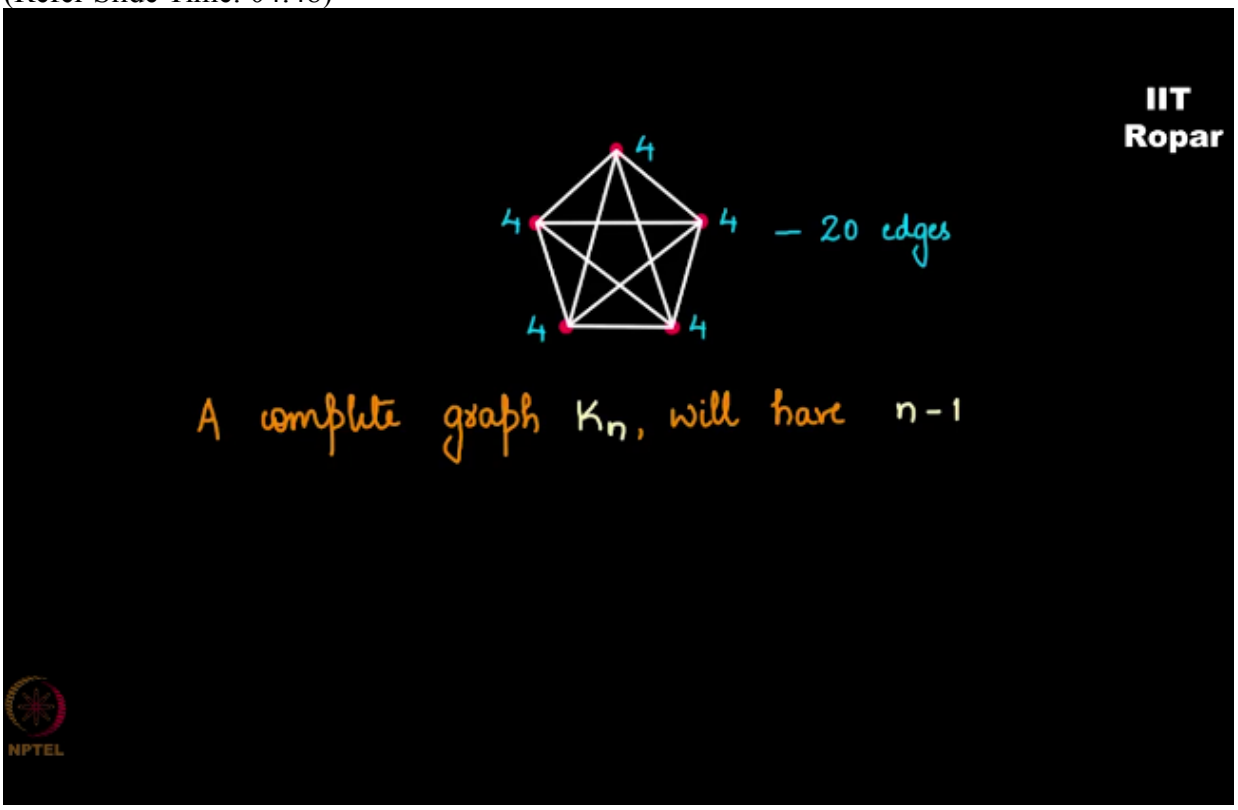

am I right here? Does it have 20 edges, I'm counting 4 from the 4 from C, 4 from B, 4 from A, 4 from E, you see an edge from D is counted once from D and once from B as well, correct, so the total number of edges should not be 20, every edge is counted twice so it should be 10 as you can see, correct
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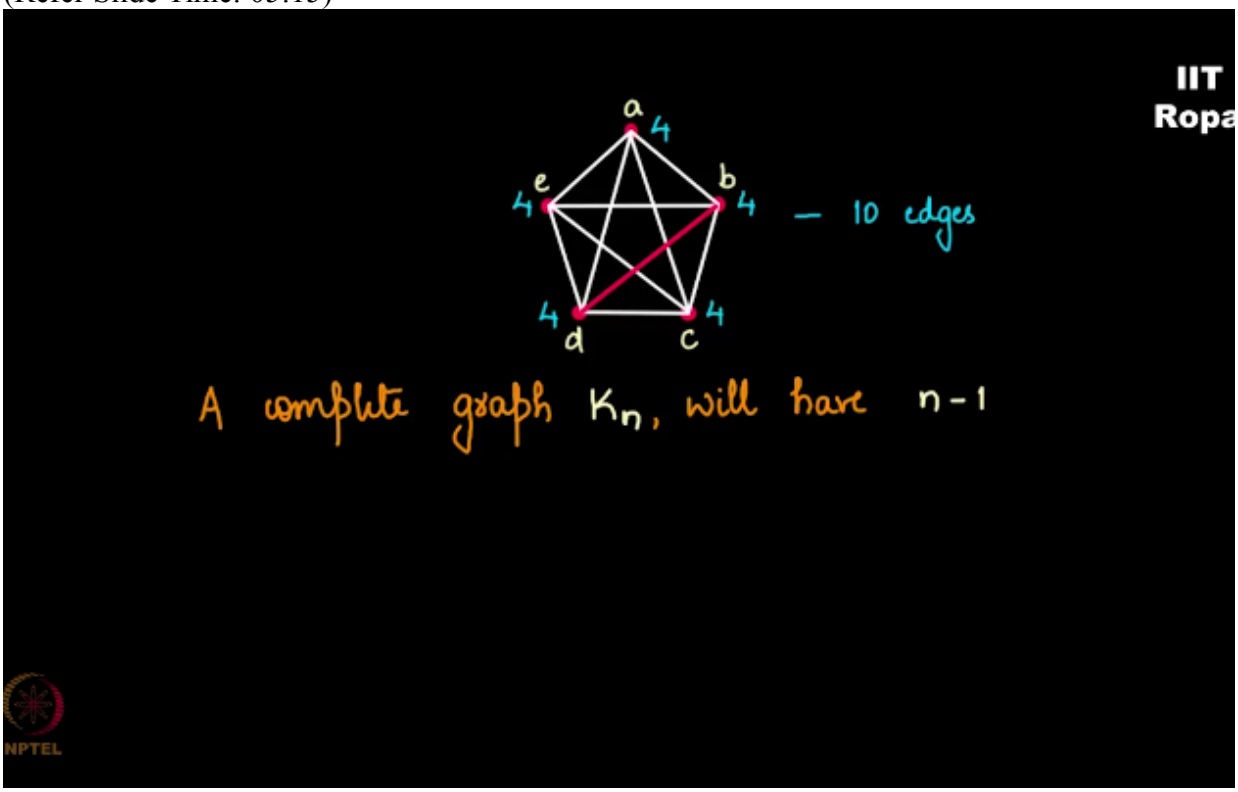

there are 10 edges in K5.

There is another way of seeing it, given 5 people in how many ways can you choose 2 people out of 5 people and that is 5 choose 2, which is again 10,
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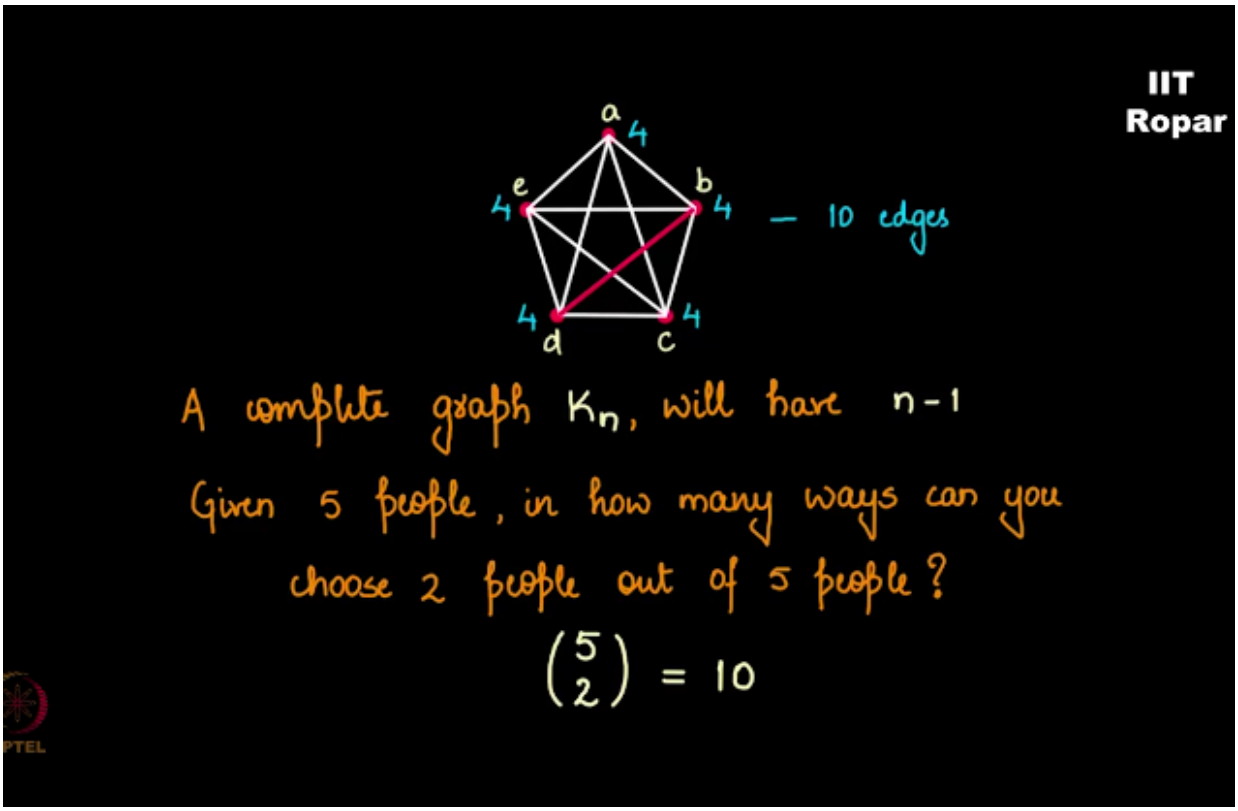

so KN will have, N choose 2 number of edges, right, think about it. Now did you see how I arrived at 10 the first time? I counted the degree of all the nodes, summed it up and divided that by 2, I got the edges, what exactly is this? Think about it.

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